Lecture 14

Course review

Schedule

PART 1: Graphs (4 weeks)

- Basic definition
- Paths and connectivity
- Distance and breadth-first search
- Statistical and non-statistical network properties (transitivity, community formation, homophily)

PART 2: Game theory (6 weeks)

- What is a game?
- Reasoning about behavior in a game
- Best responses and dominant strategies
- Nash equilibrium
- The Fundamental Theorem of game theory
- Multiple equilibria: coordination games
- Mixed Strategies: examples and empirical analysis
- Pareto-optimality and social optimality

PART 3: Applications of game theory (4 weeks)

- Modeling network traffic
- Auctions
- Bargaining in social networks
- Link analysis and web search
- Multiple-item auctions
- Sponsored search markets
- Modeling the process of decentralized search
- Voting

Framework

- Two or more individuals interact
- Description of strategic interaction (game)
- Strategic interaction: behavior of one person depends on that what person expects others to do
- Include constraints on the actions that the players can take (graphs)
- Goal: Gain insight into how incentives + structure affect the way people behave!

Framework

- Tools for research
- Theoretical foundations, mathematical tools, modeling, equilibrium notions

Focus

- Graph theory: model network structure
- Game theory: model behavior where decisions affect each other's behavior

Related courses

- Behavioral Game Theory
- Information Networks

Basic assumptions:

1. decision-makers are rational
    i. consistent preferences
    ii. maximize utility
2. take into account their knowledge or expectations of other decision-makers behavior

Framework

Systematic approach

- Describe the outcomes that may emerge in a family of games (solutions)
- Predict you opponents' reaction to your behavior!

Good news

- Framework for thinking about strategic interaction
- Provides a way to organize our thinking

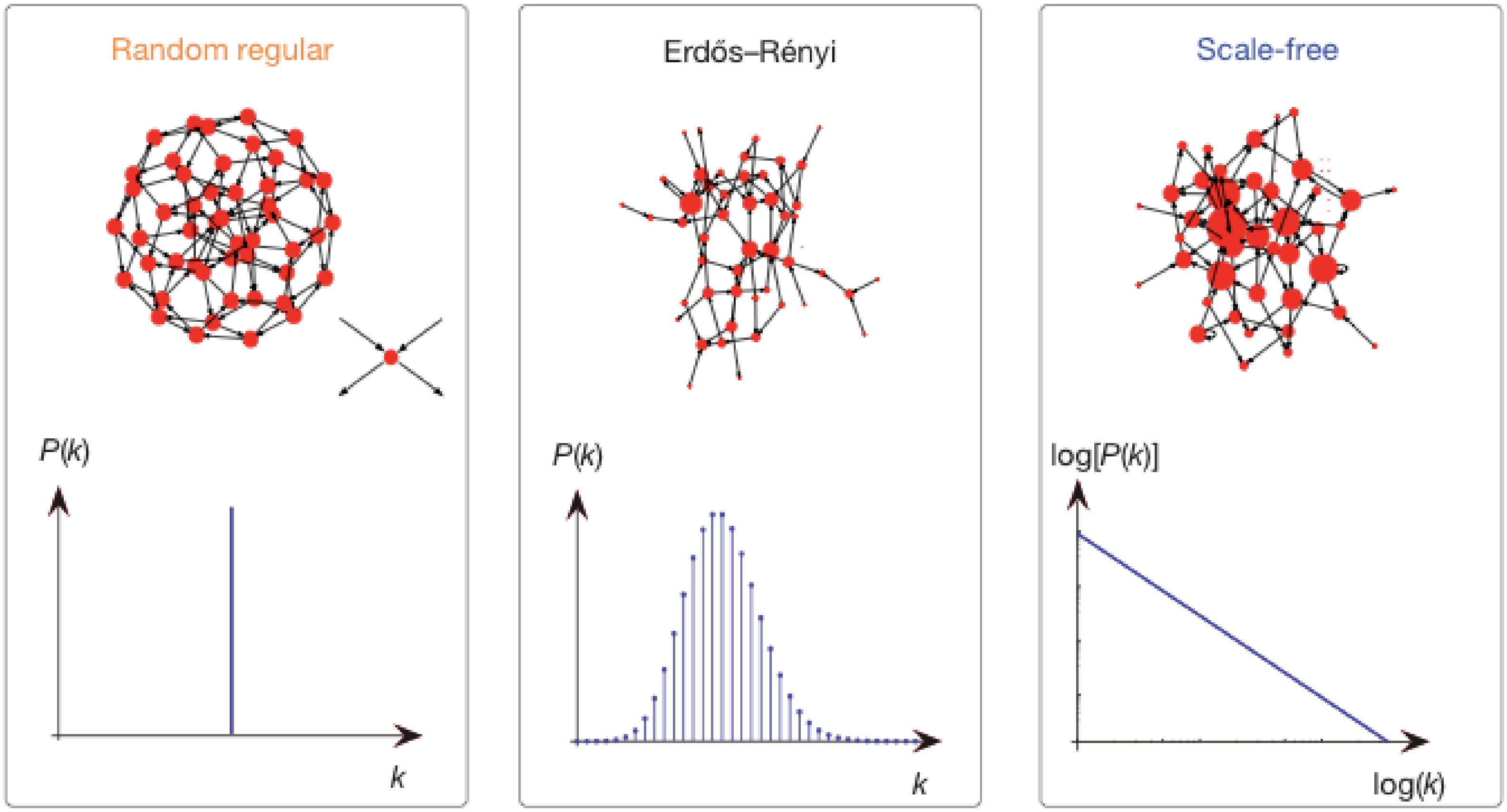
Bad news

- It is the science of behavior, not the art
- Knowing game theory does not guarantee winning (auctions)

Network properties

Degree distributions

1. regular
2. random
3. heavy-tailed
   - power-law
   - exponential



From Yang-Yu Lui et al., Controllability of complex networks

Small-world networks

Research at Facebook

Search publications, blog posts and more

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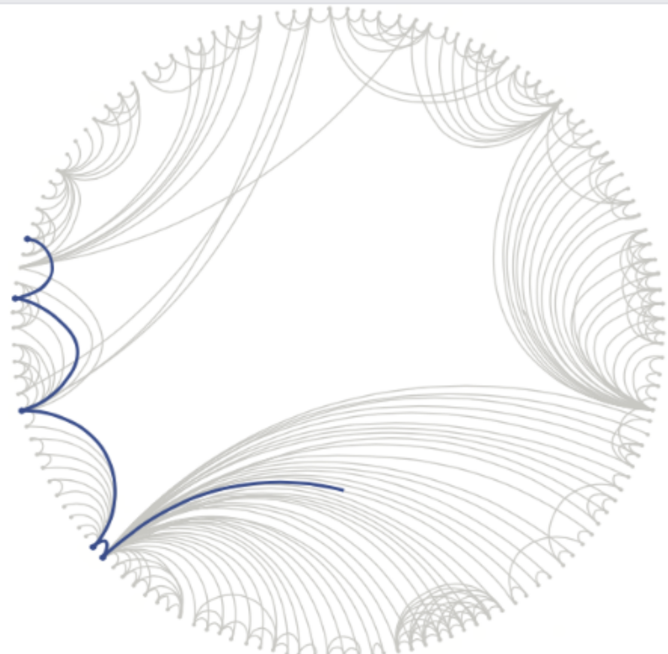
Three and a half degrees of separation

Blog



Sergey Edunov, Carlos Diuk, Ismail Onur Filiz, Smriti Bhagat, Moira Burke
February 4

Core Data Science

Like Share 53k

"I read somewhere that everybody on this planet is separated by only six other people. Six degrees of separation. Between us and everybody else on this planet. The president of the United States. A gondolier in Venice. Fill in the names. . . . How every person is a new door, opening up into other worlds. Six degrees of separation between me and everyone else on this planet. But to find the right six people . . ." - John Guare, Six Degrees of Separation (1990)

How connected is the world? Playwrights [1], poets [2], and scientists [3] have proposed that everyone on the planet is connected to everyone else by six other people. In honor of Friends Day, we've crunched the Facebook friend graph and determined that the number is 3.57. **Each person in the world (at least among the 1.59 billion people active on Facebook) is connected to every other person by an average of three and a half other people.** The average distance we observe is 4.57, corresponding to 3.57 intermediaries or "degrees of separation." Within the US, people are connected to each other by an average of 3.46 degrees.

Our collective "degrees of separation" have shrunk over the past five years. In 2011, researchers at Cornell, the Università degli Studi di Milano, and Facebook computed the average across the 721 million people using the site then, and found that it was 3.74 [4,5]. Now, with twice as many people using the site, we've grown more interconnected, thus shortening the distance between any two people in the world.

Calculating this number across billions of people and hundreds of billions of friendship connections is challenging; we use statistical techniques described below to precisely estimate distance based on de-identified, aggregate data.

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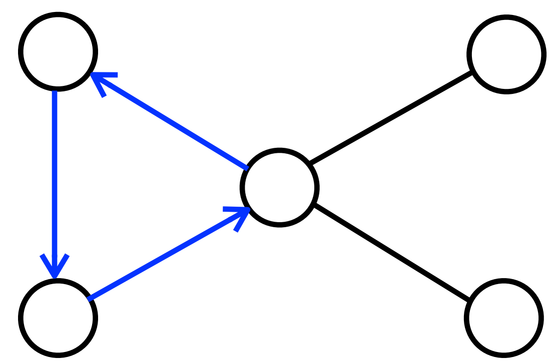
by Moira Burke, Robert Kraut · January 22

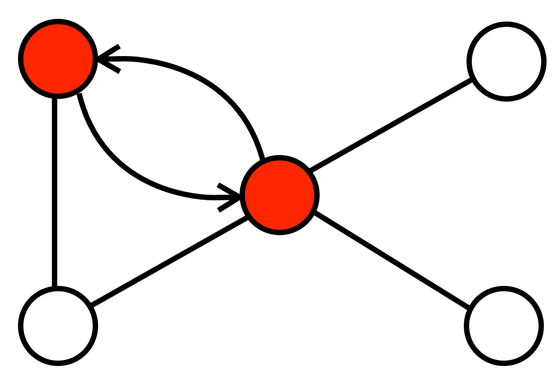
Transitivity or clustering

- Considerably higher than for random graphs
- Measures density of triangles
- Generalize to four and longer loops
- Important in directed graphs

feedback cycle of length 3

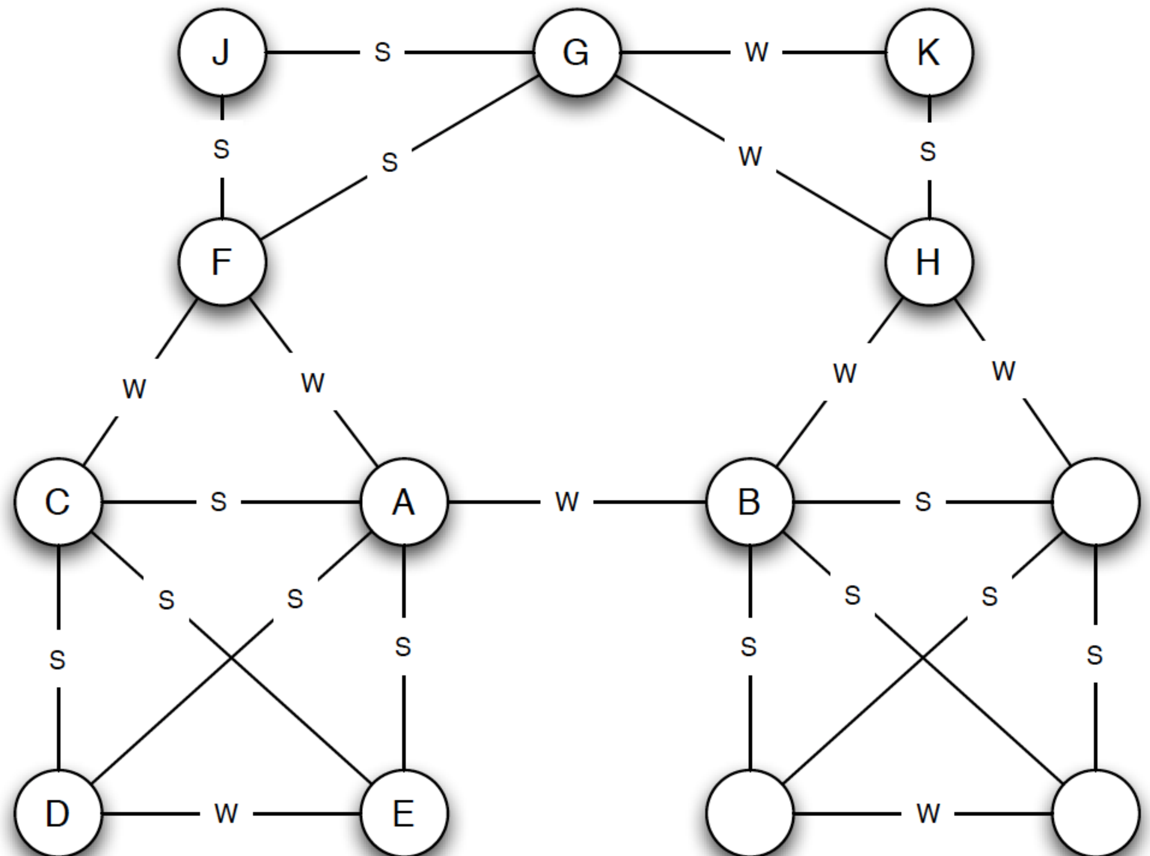


reciprocity



			number of nodes	number of edges	average degree	average distance	scaling exponent	global clustering	av. local clustering	deg. corr.
	network	type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r
social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208
	company directors	undirected	7 673	55 392	14.44	4.60	–	0.59	0.88	0.276
	math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15	0.34	0.120
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45	0.56	0.363
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088	0.60	0.127
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1			
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0		0.16	
	email address books	directed	16 881	57 029	3.38	5.22	–	0.17	0.13	0.092
	student relationships	undirected	573	477	1.66	16.01	–	0.005	0.001	−0.029
	sexual contacts	undirected	2 810				3.2			
information	WWW `nd.edu`	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	−0.067
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7			
	citation network	directed	783 339	6 716 198	8.57		3.0/–			
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	–	0.13	0.15	0.157
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7		0.44	
technological	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	−0.189
	power grid	undirected	4 941	6 594	2.67	18.99	–	0.10	0.080	−0.003
	train routes	undirected	587	19 603	66.79	2.16	–		0.69	−0.033
	software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.070	0.082	−0.016
	software classes	directed	1 377	2 213	1.61	1.51	–	0.033	0.012	−0.119
	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	−0.154
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	−0.366
biological	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	−0.240
	protein interactions	undirected	2 115	2 240	2.12	6.80	2.4	0.072	0.071	−0.156
	marine food web	directed	135	598	4.43	2.05	–	0.16	0.23	−0.263
	freshwater food web	directed	92	997	10.84	1.90	–	0.20	0.087	−0.326
	neural network	directed	307	2 359	7.68	3.97	–	0.18	0.28	−0.226

Strong and weak ties



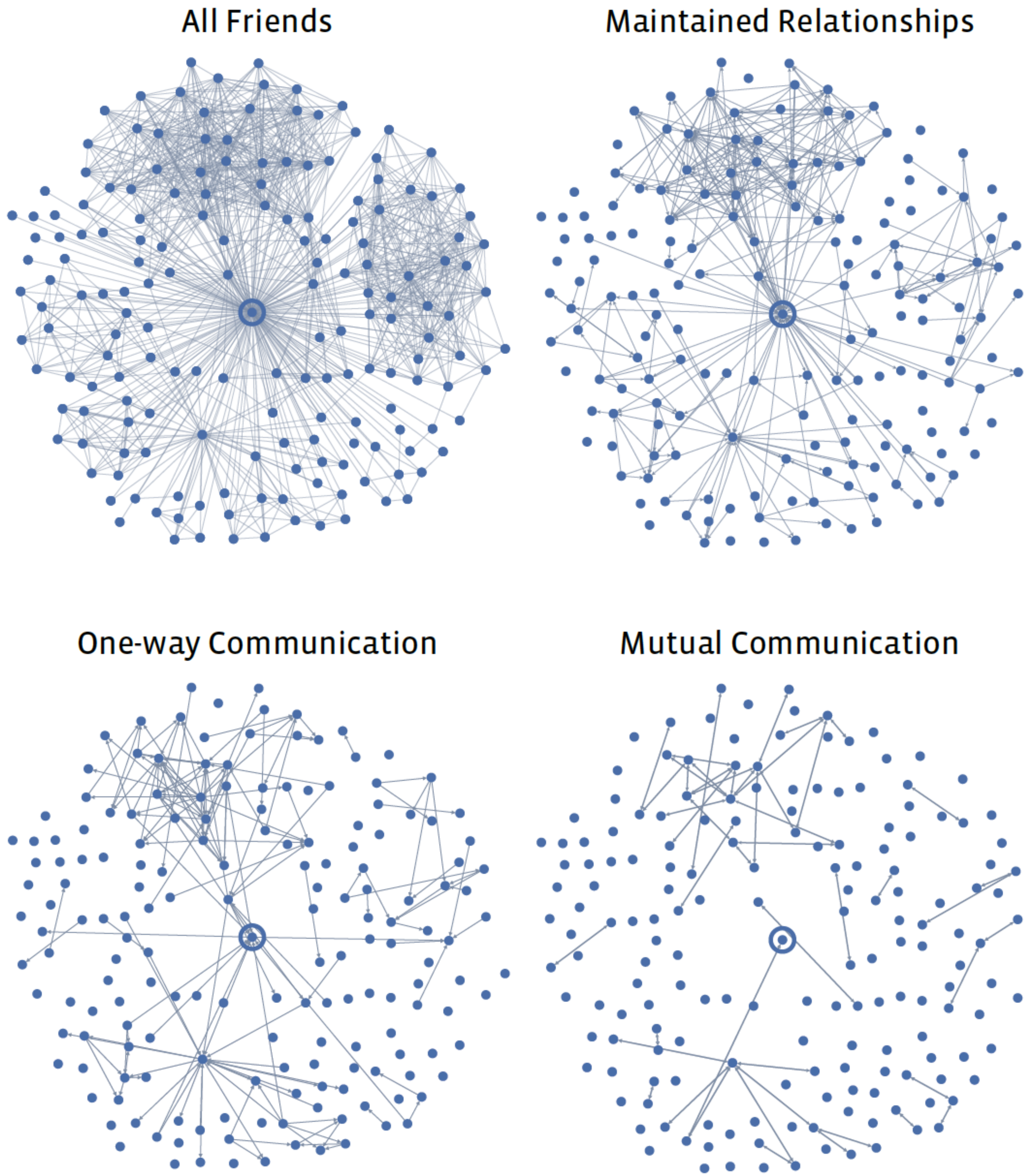
David Easley and Jon Kleinberg. Networks, Crowds, and Markets: Reasoning about a Highly Connected World. Cambridge University Press, 2010.

Strong Triadic Closure

We say that a node A violates the Strong Triadic Closure Property if it has strong ties to two other nodes B and C, and there is no edge at all (either a strong or weak tie) between B and C. We say that a node A satisfies the Strong Triadic Closure Property if it does not violate it.

Claim

If a node A in a network satisfies the Strong Triadic Closure Property and is involved in at least two strong ties, then any local bridge it is involved in must be a weak tie.

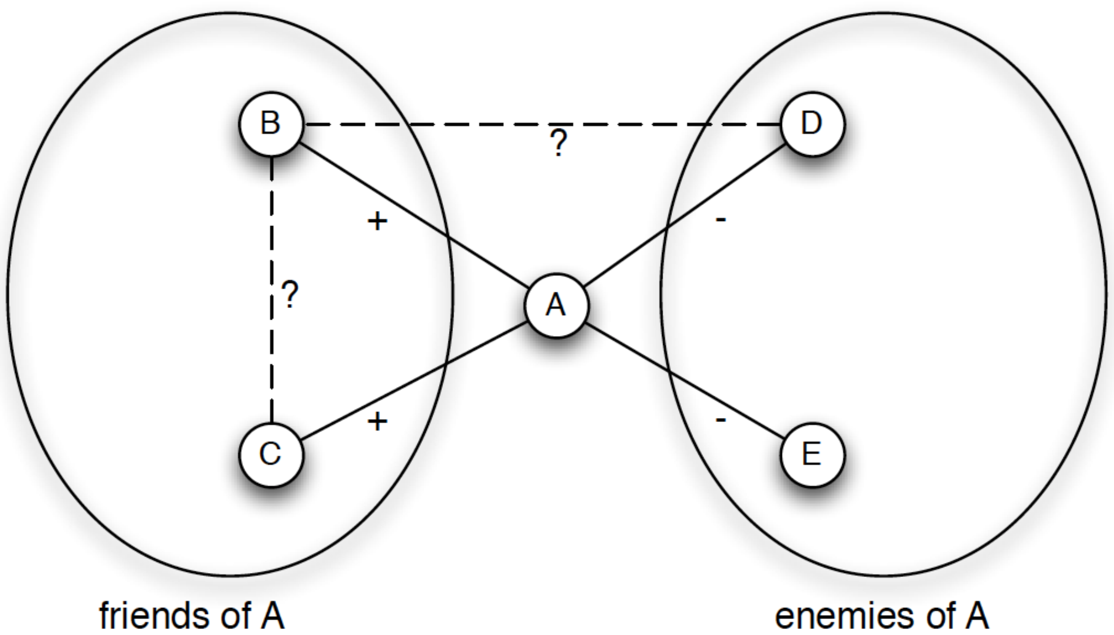


Cameron Marlow, Lee Byron, Tom Lento, and Itamar Rosenn. Maintained relationships on Facebook, 2009. On-line at http://overstated.net/2009/03/09/maintainedrelationships-on-facebook.

Other properties

- Homophily
- Community structure
- Positive and negative relationships

Weakly Balance Theorem
If a labeled complete graph is weakly balanced, then its nodes can be divided into groups in such a way that every two nodes belonging to the same group are friends, and every two nodes belonging to different groups are enemies.



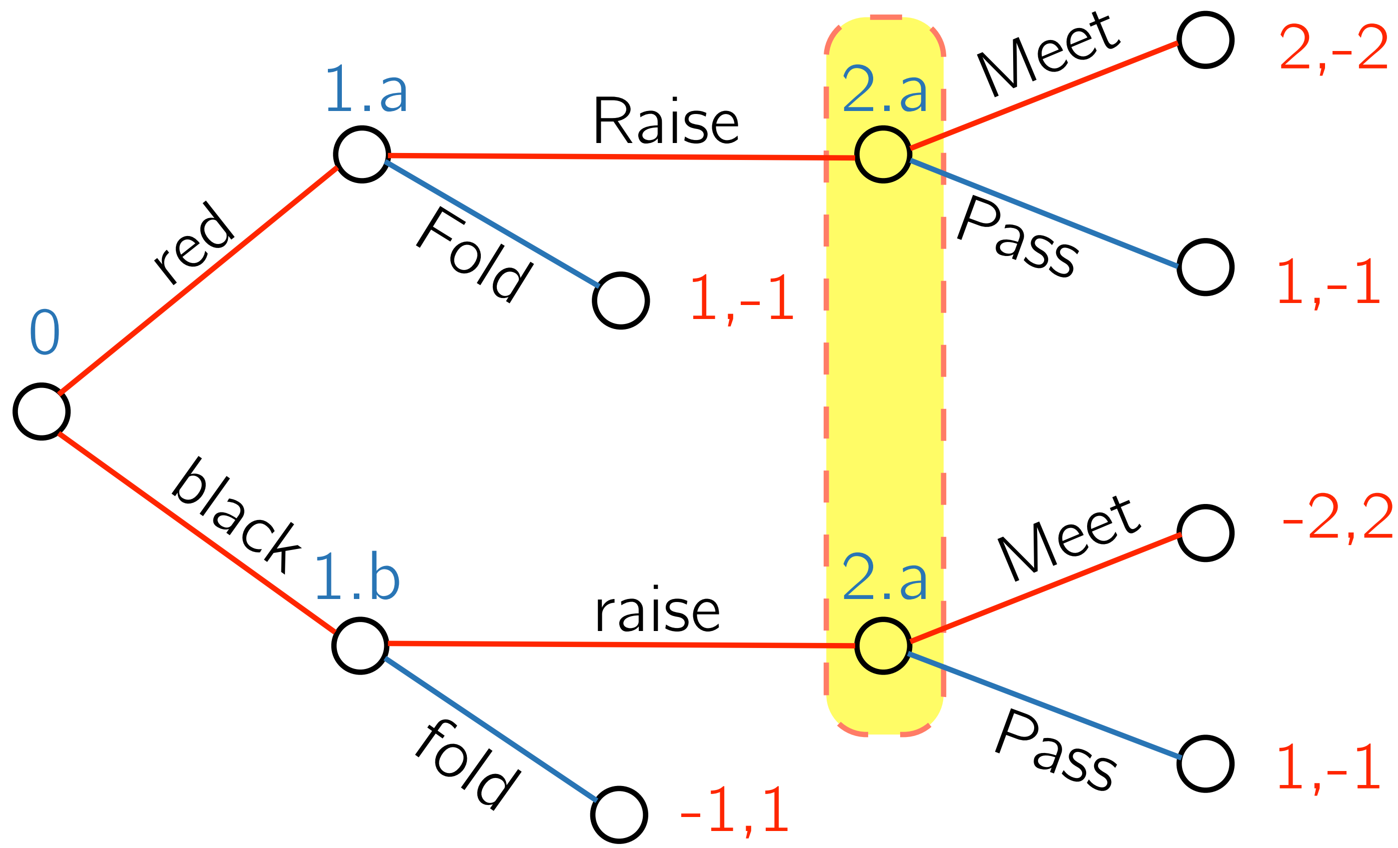
Idea: commonly observed network patterns help to understand interactions

Games

Overview

- Extensive v. normal form games
- Best responses and dominant strategies
- Notion of Nash Equilibrium
- Multiple Equilibria (coordination games)
- Formalize Mixed Strategies
- Formalize Nash Equilibrium
- The Fundamental Theorem of game theory
- Nash v. Pareto-Optimality
- Evolutionary game theory (evolutionarily stable strategies)

Example



P$_2$

P$_1$

	M	P
Rr	0,0	1,-1
Rf	0.5,-0.5	0,0
Fr	-0.5,0.5	1,-1
Ff	0,0	0,0

$$S_1 = \{Rr, Rf, Fr, Ff\}$$

$$S_2 = \{M, P\}$$

$$s = \{Rr, M\}$$

$$\boxed{\begin{aligned} \pi_1(s) &= 0 \\ \pi_2(s) &= 0 \end{aligned}}$$

P$_2$

P$_1$

	s_{12}	s_{22}
s_{11}	0,0	1,-1
s_{21}	0.5,-0.5	0,0
s_{31}	-0.5,0.5	1,-1
s_{41}	0,0	0,0

Normal game form

$P = \{1, \ldots, n\}$: set of players

S_i : possible strategies for player i

$s = \{s_1, \ldots, s_n\}$: (pure) strategy profile of the game

S : set of strategy profiles

$\pi_i : S \mapsto \mathbb{R}$: payoff to player i when profile s is chosen by all players

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$s_{1i}, \ldots, s_{ki}$: pure strategies for player i

$\sigma_i = p_{1i}s_{1i} + \ldots + p_{ki}s_{ki}$: mixed strategy for player i

p_{ji} : weight of strategy s_{ji} on σ_i

$\sigma = (\sigma_1, \ldots, \sigma_n)$: mixed strategy profile

Payoffs of mixed strategies

$$s_{1i}, \ldots, s_{ki} \; : \; \text{pure strategies for player } i$$

$$\sigma_i = p_{1i}s_{1i} + \ldots + p_{ki}s_{ki} \; : \; \text{mixed strategy for player } i$$

$$p_{ji} \; : \; \text{weight of strategy } s_{ji} \text{ on } \sigma_i$$

$$\sigma = (\sigma_1, \ldots, \sigma_n) \; : \; \text{mixed strategy profile}$$

Payoff for player 1

$$\pi_1(\sigma) = \sum_{s_{j1} \in S_1} \sum_{s_{j2} \in S_2} \ldots \sum_{s_{jn} \in S_n} p_{j1} p_{j2} \ldots p_{jn} \pi_1(s_{j1}, s_{j2}, \ldots, s_{jn})$$

Definition (Nash): A strategy profile $\sigma^* = (\sigma_1^*, \ldots, \sigma_n^*)$ is a Nash equilibrium if for every player i and every $\tau_i \in \Delta S_i$

$$\pi_i(\sigma^*) \geq \pi_i(\sigma_{-i}^*, \tau_i)$$

Remarks

- Choosing Nash is at least as good as any other strategy
- Best response to the strategies chosen by the other players

Theorem (Nash): If each player in an n-player game has a finite number of pure strategies, then the game is a Nash equilibrium

Remarks

- Nash equilibrium is not necessarily unique
- Nash equilibrium may be a mixed strategy profile

Evolutionary game theory

- Genetically-determined characteristics are like strategies
- Fitness is a result of interactions
- An individual's fitness is like its payoff

Evolutionary forces

- Plays do not get to choose strategies
- Example: genetically wired!
- "what do I want my body size to be?" makes no sense!
- Does Nash still make sense?

Idea: look at how strategies operate over long time scales

Evolutionarily stable strategies (ESS)

- Analogous notion to Nash
- Nash:
  - No incentive to deviate given everyone's choice of strategies
  - Tends to persist onces players are using it

> **Definition (Pure Nash):** A pure strategy profile $s^* = (s_1^*, \ldots, s_n^*) \in S$ is a Nash equilibrium if for every player i and every $\tau_j \in S_i$
>
> $$\pi_i(s^*) \geq \pi_i(s^*_{-j}, \tau_j)$$

- ESS: A (genetically-determined strategy) that tends to persist once it is prevalent

Definition (ESS): Let $s^* = (s_1^*, s_2^*) = s_{11}, s_{12}$. A strategy profile s^* is an ESS if

$$\pi_1(s_{11}, s_{12}) > \pi_1(s_{21}, s_{12})$$
$$\pi_2(s_{11}, s_{12}) > \pi_2(s_{11}, s_{22})$$

strict pure Nash

or

if $\pi_1(s_{11}, s_{12}) = \pi_1(s_{21}, s_{12})$ then $\pi_1(s_{11}, s_{22}) > \pi_1(s_{21}, s_{22})$
if $\pi_2(s_{11}, s_{12}) = \pi_2(s_{11}, s_{22})$ then $\pi_2(s_{21}, s_{12}) > \pi_2(s_{21}, s_{22})$

Maynard Smith's condition

Remarks

- Strategy 1 is neutral with respect to the payoff against strategy 2
- But the population of players who continue to play strategy 1 have an advantage when playing against strategy 2

Applications

- Modeling network traffic
- Auctions
- Bargaining in social networks
- Link analysis and web search
- Multiple-item auctions
- Sponsored search markets

Common assumptions

- Decisions based on payoff of players

Common questions:

- Does an equilibrium exist?
- What are the properties of that equilibrium?

Applications

- Modeling network traffic (Nash at most 2x Pareto optimum)
- Auctions (bid true value)
- Bargaining in social networks (stable and balanced outcome - outside option)
- Link analysis and web search
- Multiple-item auctions (bid true value)
- Sponsored search markets (bid true value)

Common assumptions

- Decisions based on payoff of players

Common questions:

- Does an equilibrium exist?
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