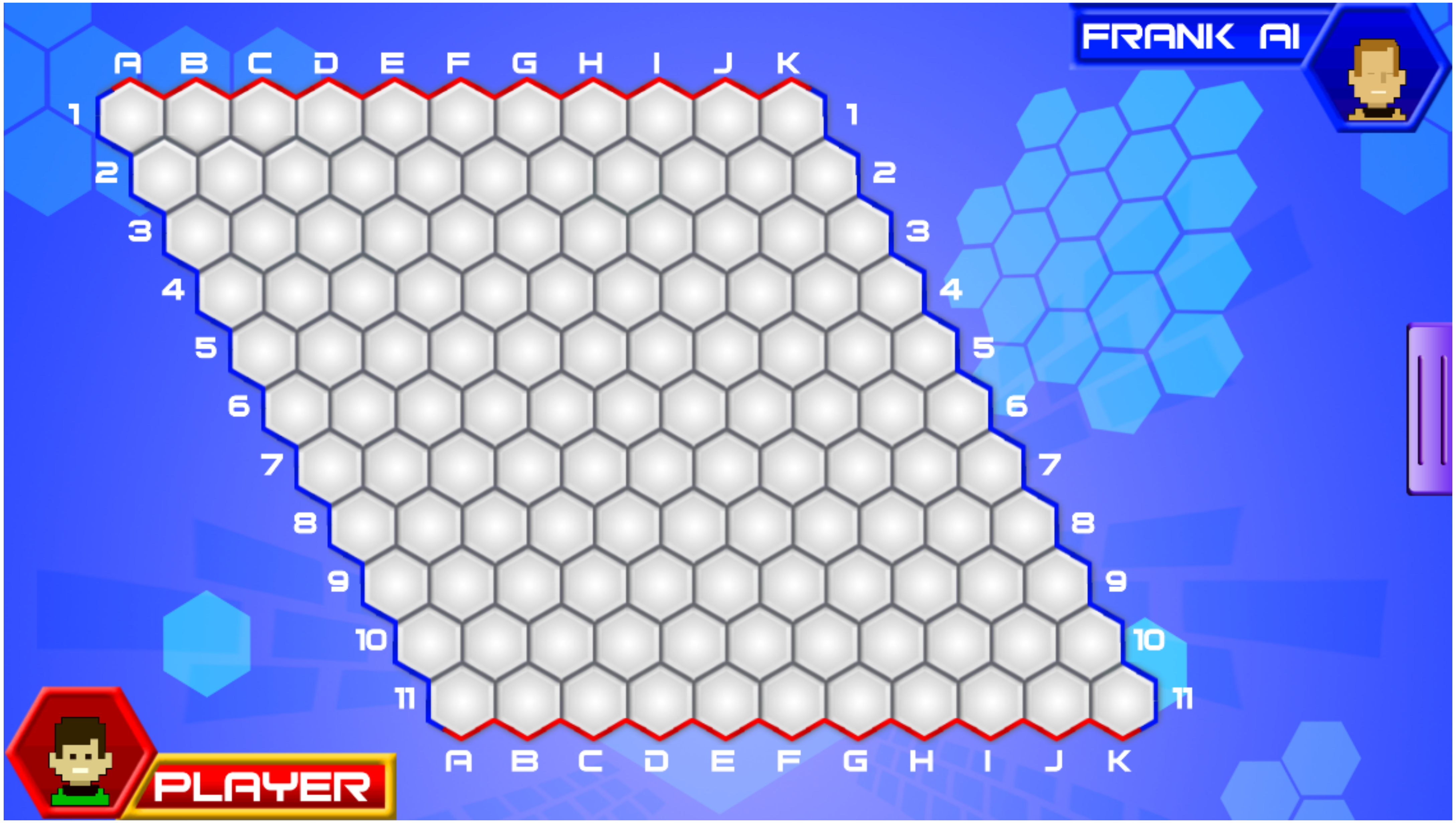
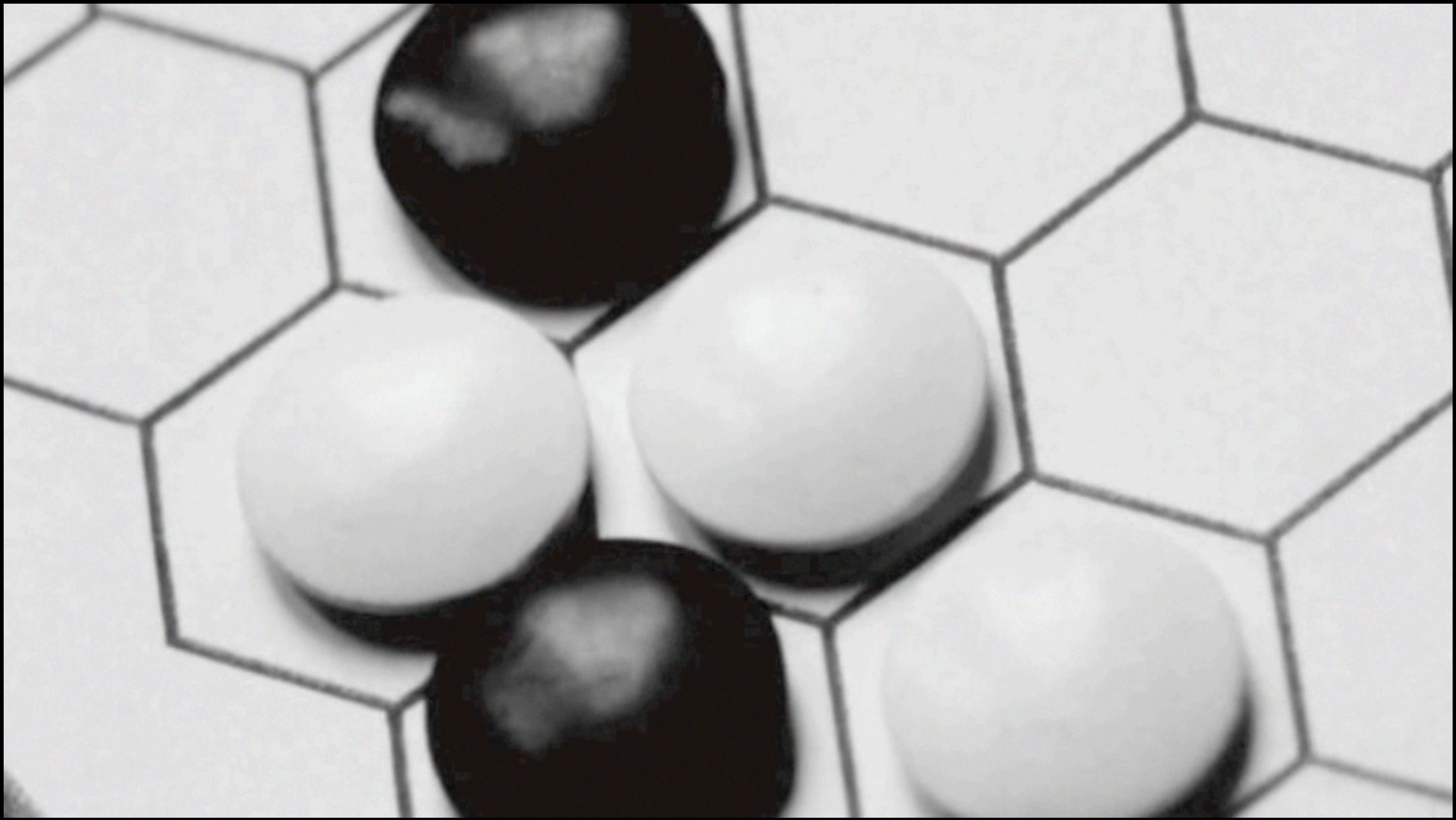
Games on Networks

Jorge Finke (finke@ieee.org)
Dpto. ECE
Universidad Javeriana
Office: 262 Edificio de Ingeniería
http://jfinke.org/course/games-on-networks/

Game of Hex



see also http://www.lutanho.net/play/hex.html

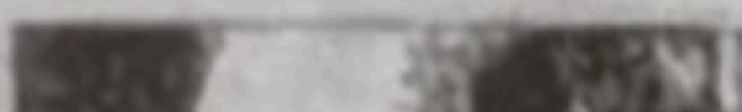


His ideas apply to poker and world trade.

Princeton mathematician wins a Nobel in economics

By Anthony Gnoffo Jr.
INQUIRER STAFF WRITER

PRINCETON — A couple is negotiating over vacation plans. He wants Alaska. She wants Europe. He appeals to her sense of adventure, then to her love of the



cian at Princeton University, yesterday won a share of the Nobel Memorial Prize in economics.

It was a long time coming: Nash first proposed the theory in a 23-page doctoral thesis at Princeton in 1950. He was 22

What is game theory?

Only with two or more players does a problem becomes game theoretical!

Framework

- Two or more individuals interact
- Description of strategic interaction (game)
- Strategic interaction: behavior of one person depends on that what person expects others to do
- Includes constraints on the actions that the players can take (strategy)
- Goal: Gain insight into how incentives affect the way people behave!

Game Theory

- Branch of applied mathematics
- Capture multi-person behavior/decision problems
- Analyze and predict behavior in strategic situations

Basic assumptions:

1. decision-makers are rational
   i. consistent preferences
   ii. e.g., maximize utility
2. take into account their knowledge or expectations of other decision-makers behavior

Why call it a game?

Game theory models

- Step back!
- Throw out what is irrelevant to the problem
- Define a model that captures the essence
- Reduce the elements of interactions to forms that resemble parlor games
- Family of games

What is a solution?

Solution to a game

Systematic approach

- Describe the outcomes that may emerge in a family of games
- Predict you opponents' reaction to your behavior!

Good news

- Framework for thinking about strategic interaction
- Provides a way to organize our thinking

Bad news

- It is the science of behavior, not the art
- Knowing game theory does not guarantee winning

What this course is about...

- Tools for research
- Theoretical foundations, mathematical tools, modeling, equilibrium notions

Focus

- Graph theory: model network structure
- Game theory: model behavior where decisions affect each other's behavior

Related courses

- Behavioral Game Theory
- Information Networks

Strategic Interactions

~~Games~~ on Networks

Schedule (tentative)

PART 1: Graphs (4 weeks)

- Basic definition
- Paths and connectivity
- Distance and breadth-first search
- Statistical network properties

PART 2: Game theory (6 weeks)

- What is a game?
- Reasoning about behavior in a game
- Best responses and dominant strategies
- Nash equilibrium
- Multiple equilibria: coordination games
- Mixed Strategies: examples and empirical analysis
- Pareto-optimality and social optimality

PART 3: Applications of game theory (4 weeks)

- Modeling network traffic
- Link analysis and web search
- Sponsored search markets
- Modeling the process of decentralized search
- Voting

Grading

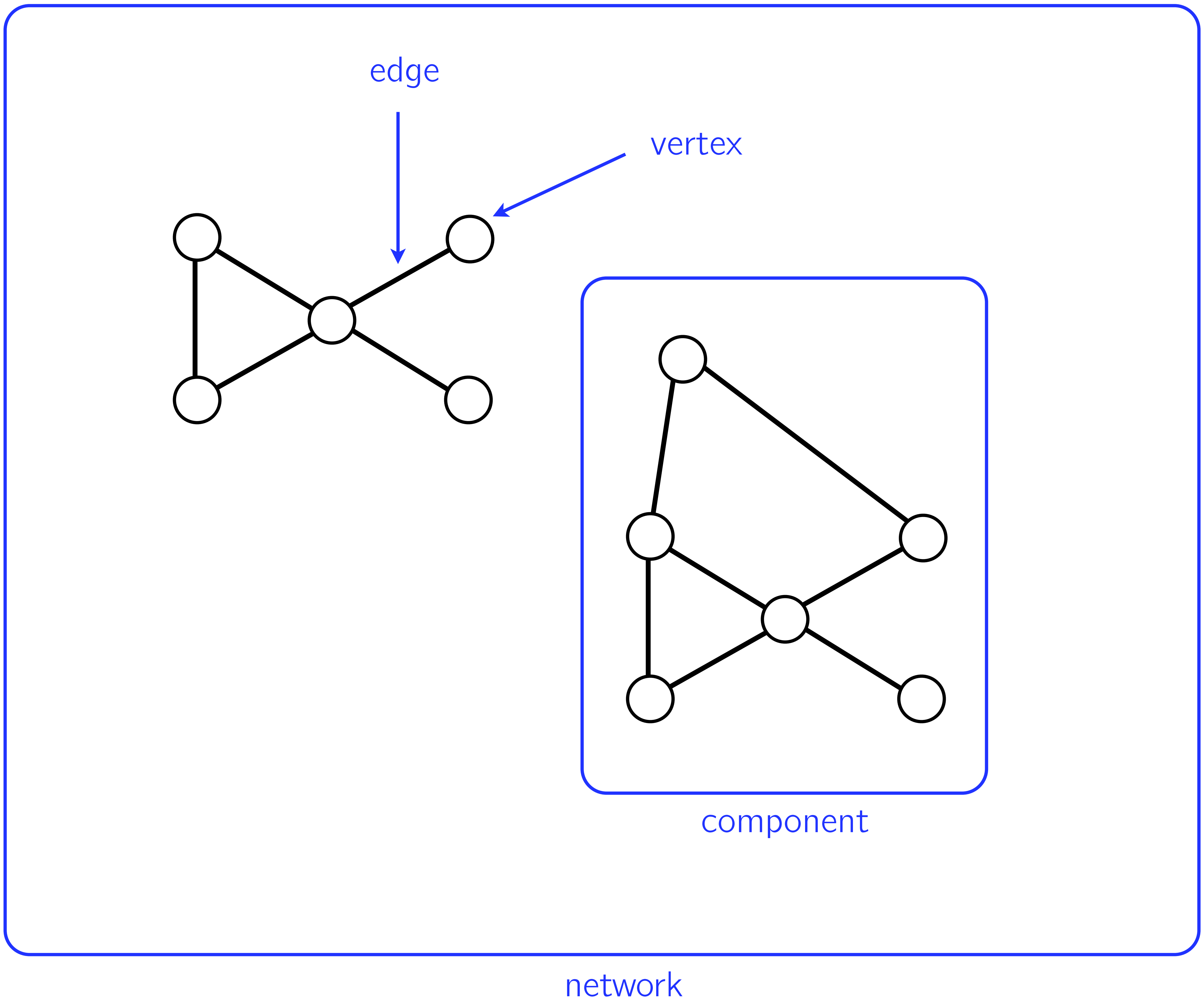
- Homework - 20%
- Project 1 - 30%
    - choosing a topic of your interest
    - pick 2-3 papers
    - understand, evaluate, and recreate research
    - project report + presentation
- Project 2 - 50%
    - develop game-theoretic framework
    - contrast with actual data
    - project report + presentation

References

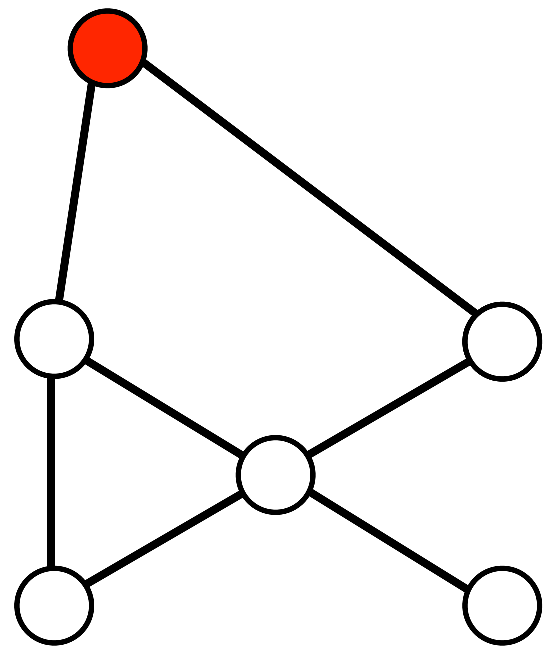
- Drew Fudenberg. Game Theory (MIT Press, 1991)
- Martin J. Osborne. An Introduction to Game Theory (Oxford University Press, 2009)
- David Easterly and Jon Kleinberg. Networks, Crowds and Markets (Cambridge Univesity Press, 2010)
- Herbert Gintis. The Bounds of Reason: Game Theory and the Unification of the Behavioral Sciences (Princeton University Press, 2009)
- Herbert Gintis. Game Theory Evolving: Second Edition (Princeton University Press, 2009)

Networks

A framework to formally quantify underlying structure that may impact strategic behavior


edge
vertex
component
network

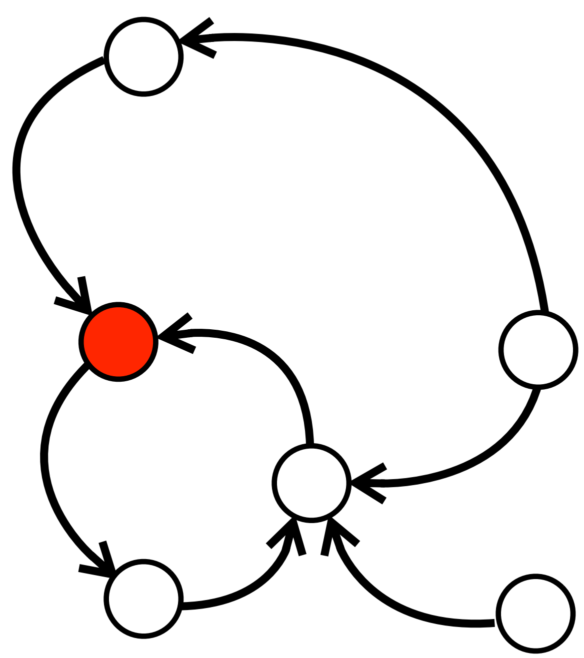
degree = 2



undirected

in-degree = 2

out-degree = 1



directed

degree:

number of edges connected to a vertex

in-degree:

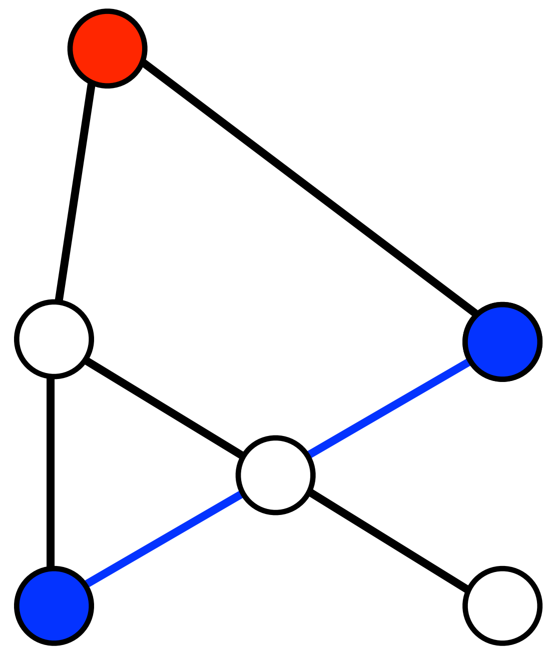
number of coming edges connected to a vertex

out-degree:

number of outgoing edges connected to a vertex

degree = 2

path = 2

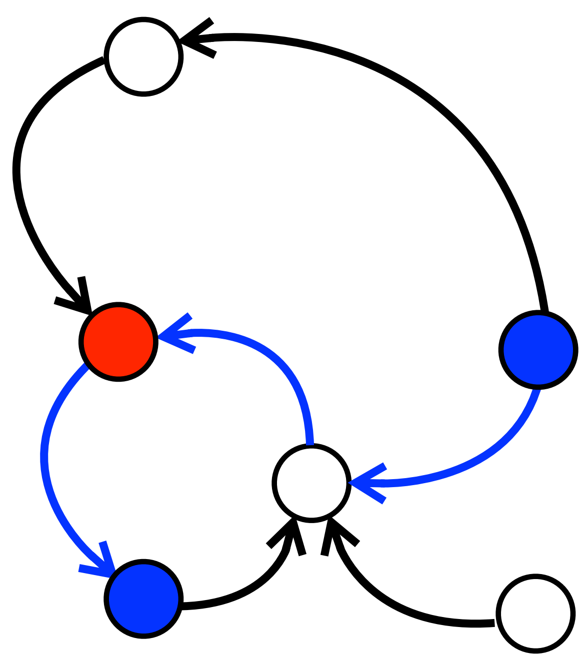


undirected

in-degree = 2

out-degree = 1

path = 2



directed

degree:
number of edges
connected to a vertex

in-degree:
number of coming edges
connected to a vertex

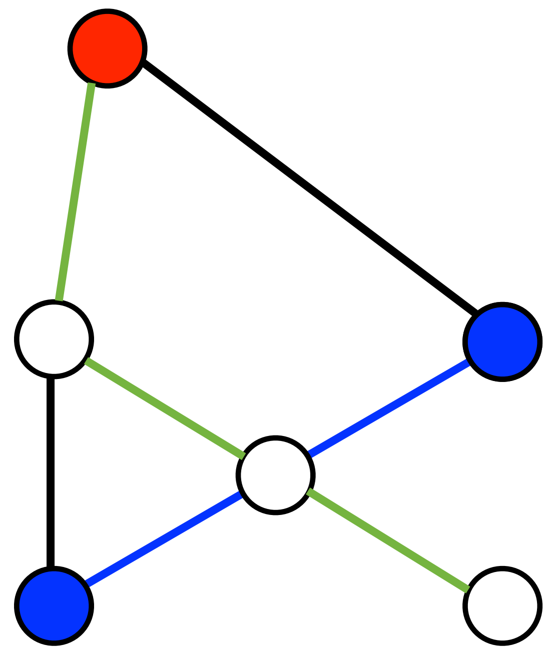
out-degree:
number of outgoing edges
connected to a vertex

geodesic path:
shortest path from one vertex
to another

degree = 2

path = 2

diameter = 3



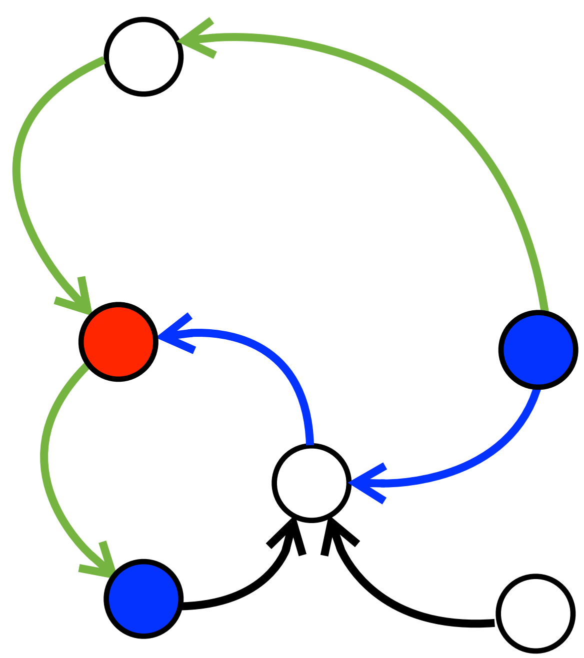
undirected

in-degree = 2

out-degree = 1

path = 2

diameter = 3



directed

degree:
number of edges
connected to a vertex

in-degree:
number of coming edges
connected to a vertex

out-degree:
number of outgoing edges
connected to a vertex

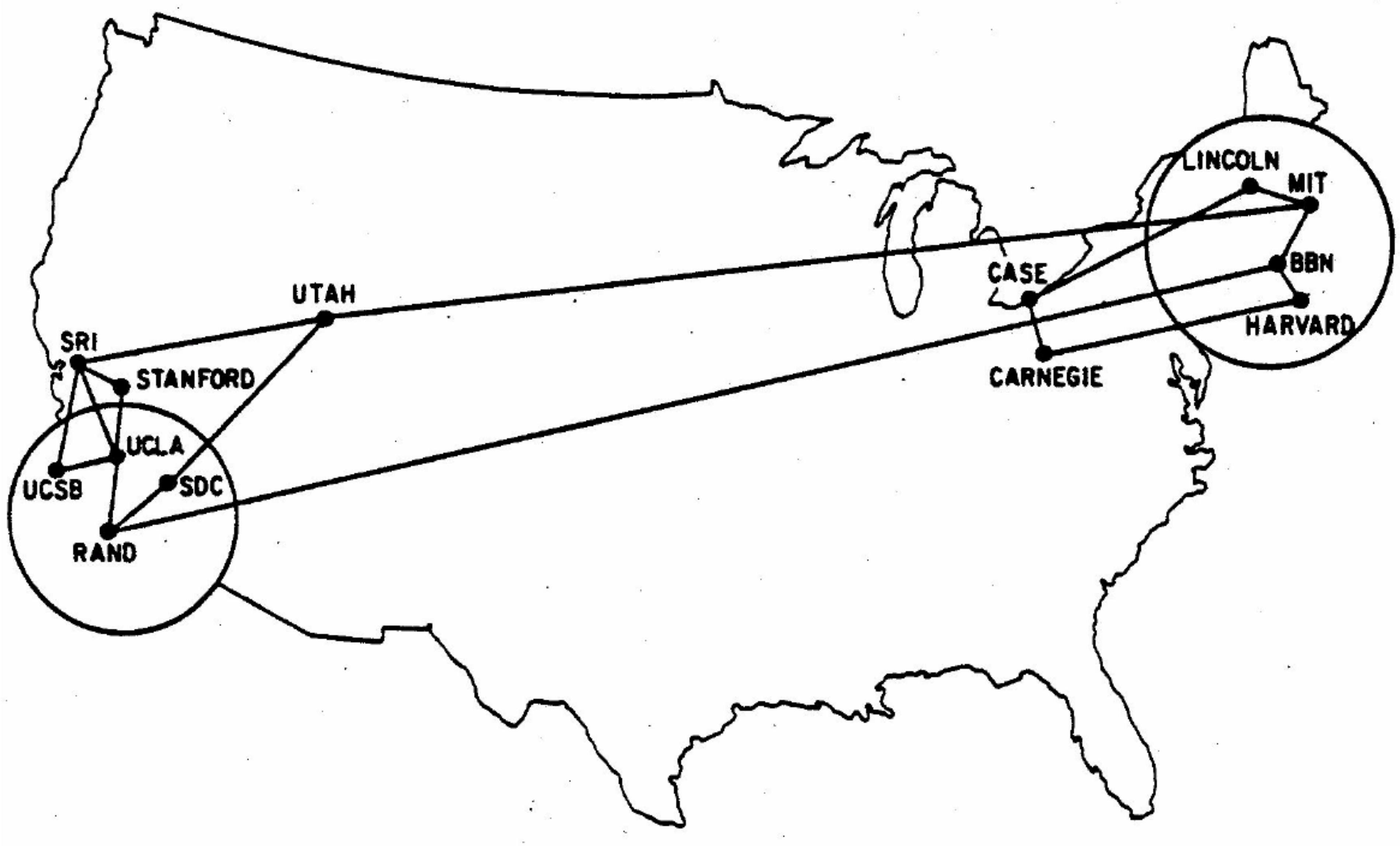
geodesic path:
shortest path from one vertex
to another

diameter:
length in number of edges
of the longest geodesic path

Many complex systems are actually well-structured networks

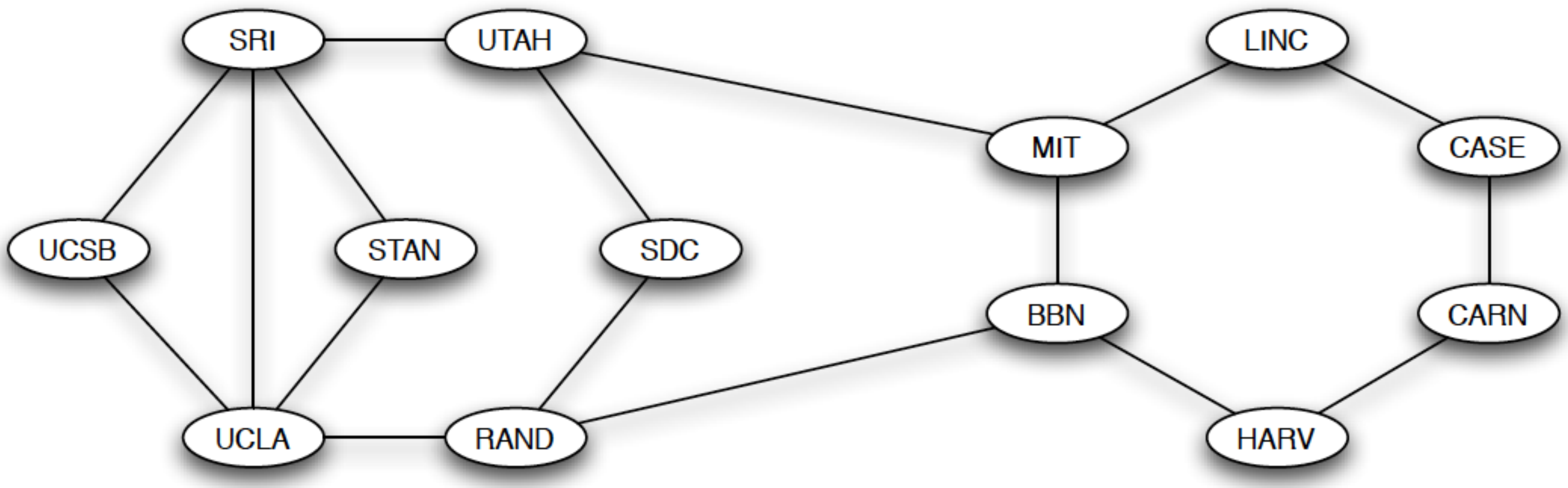
César A. Hidalgo

The internet in 1970

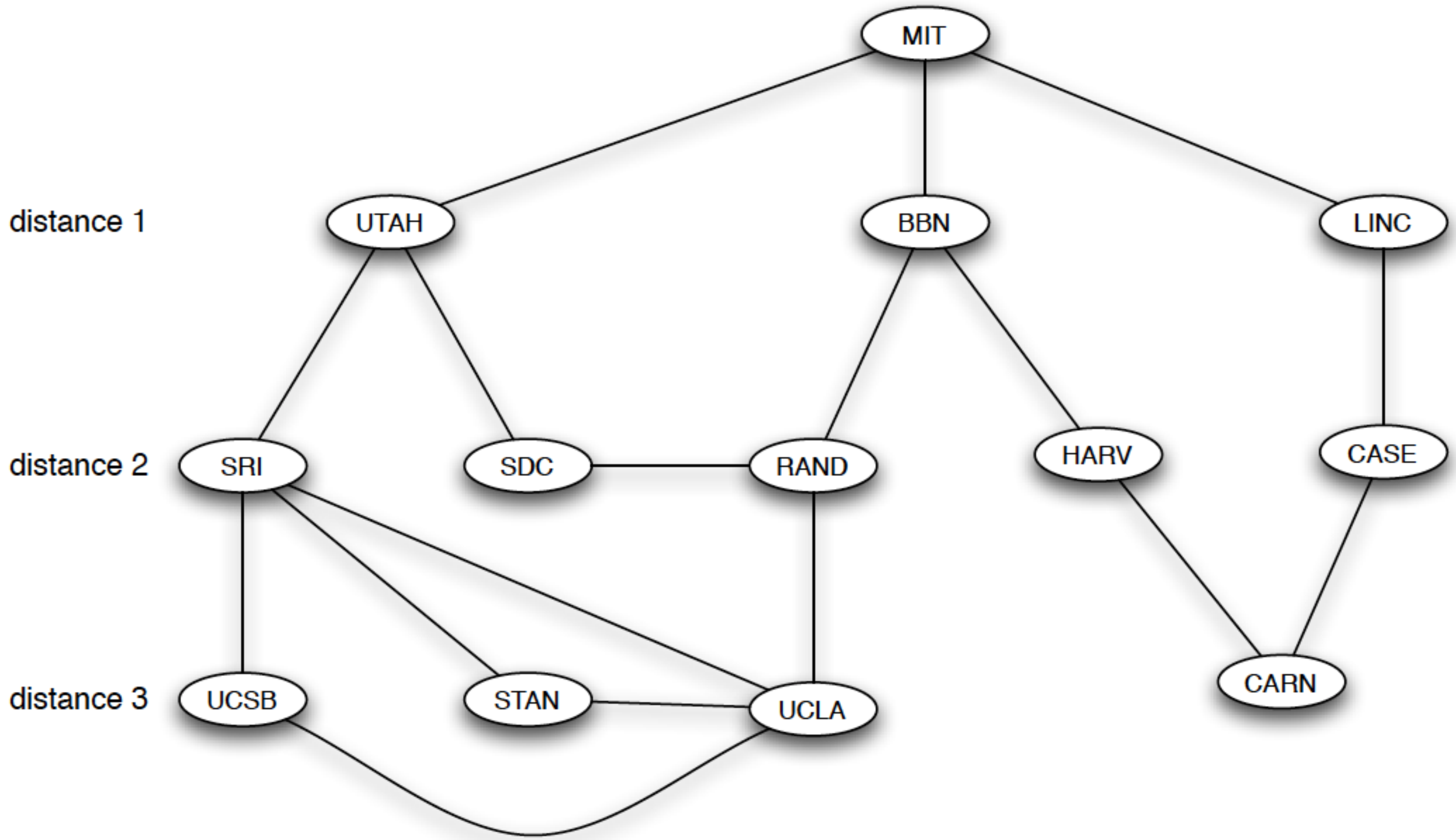


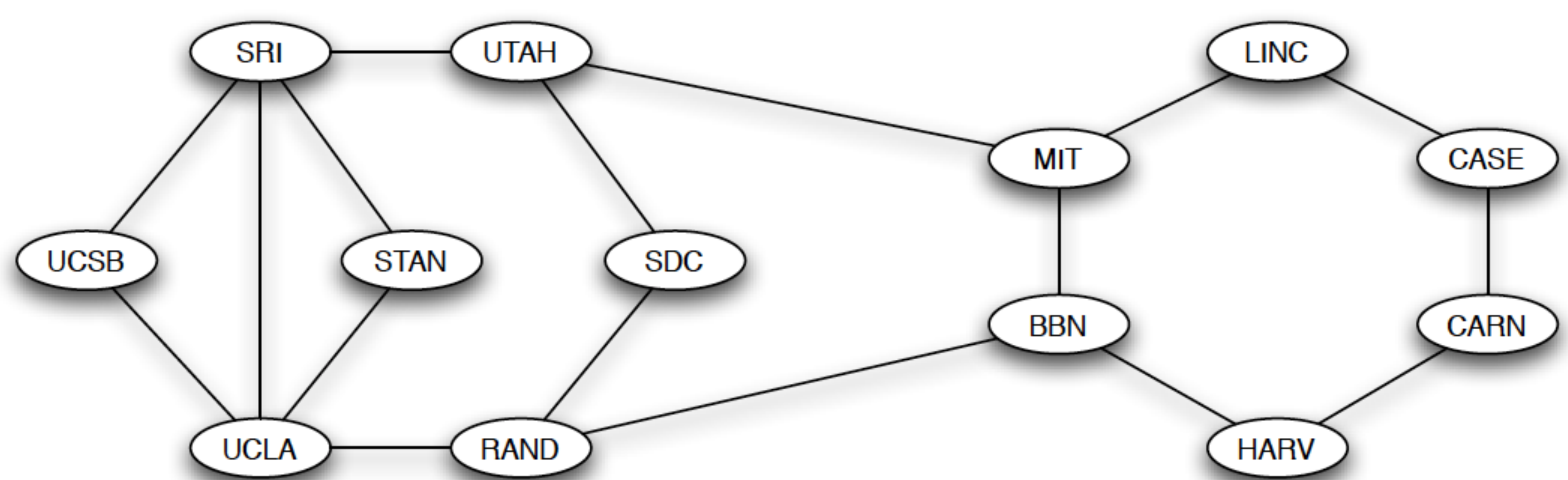
http://som.csudh.edu/cis/lpress/history/arpamaps

The internet in 1970

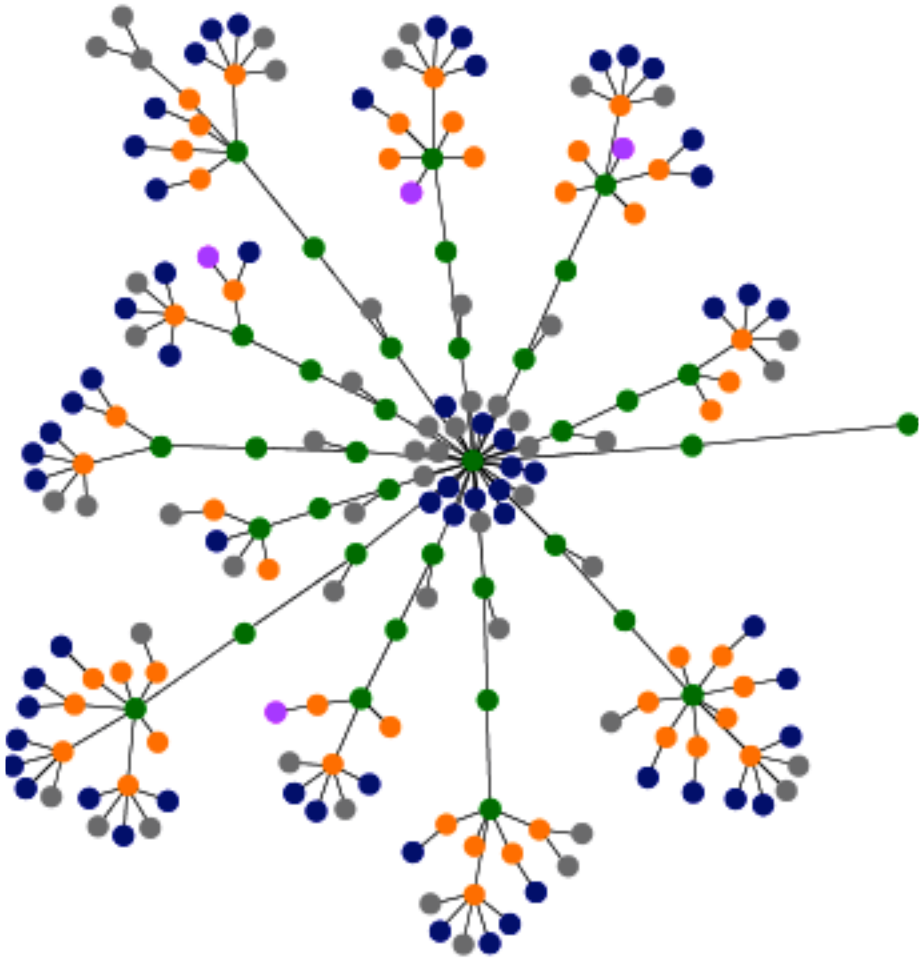


Breadth-first search

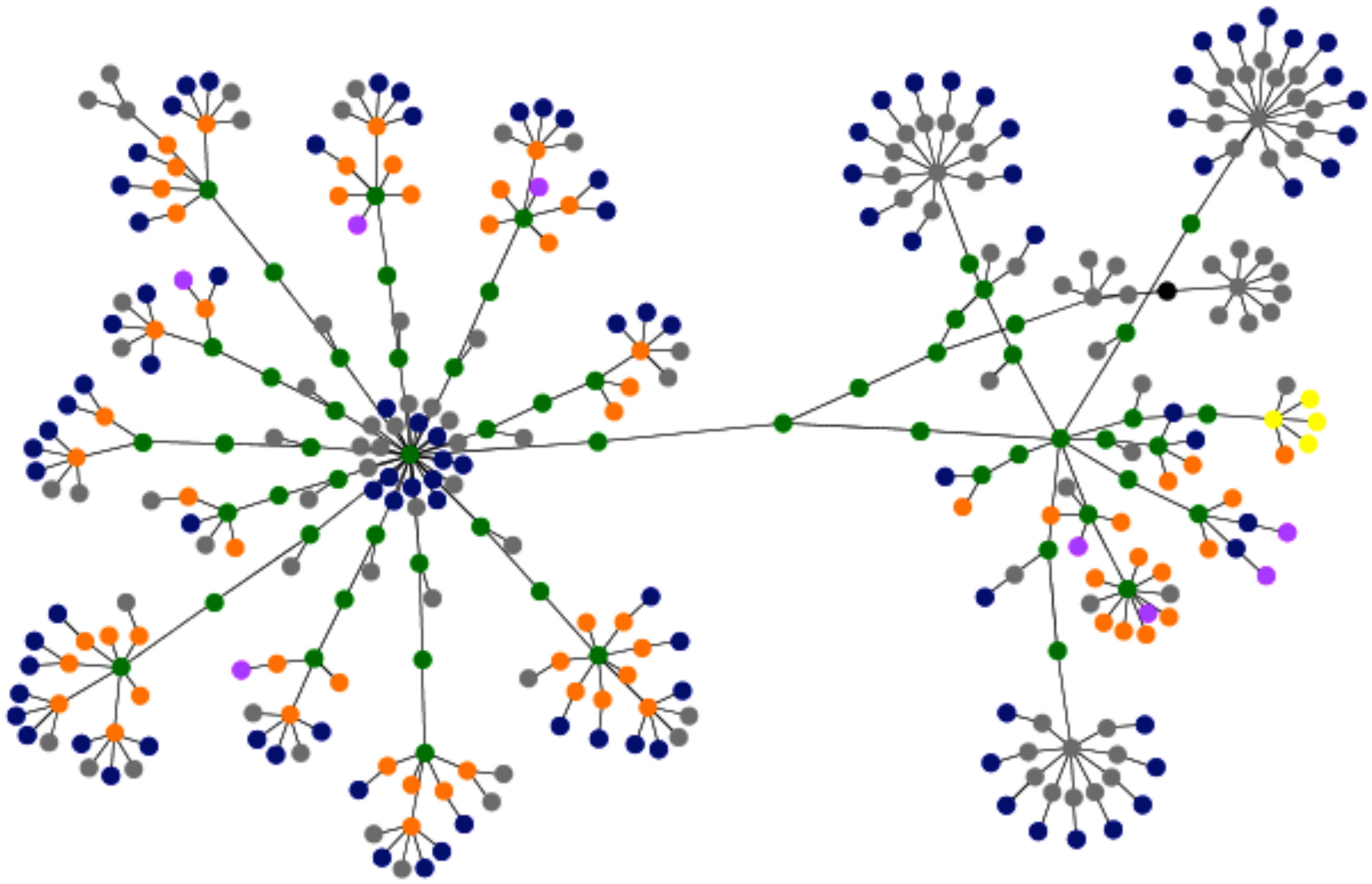




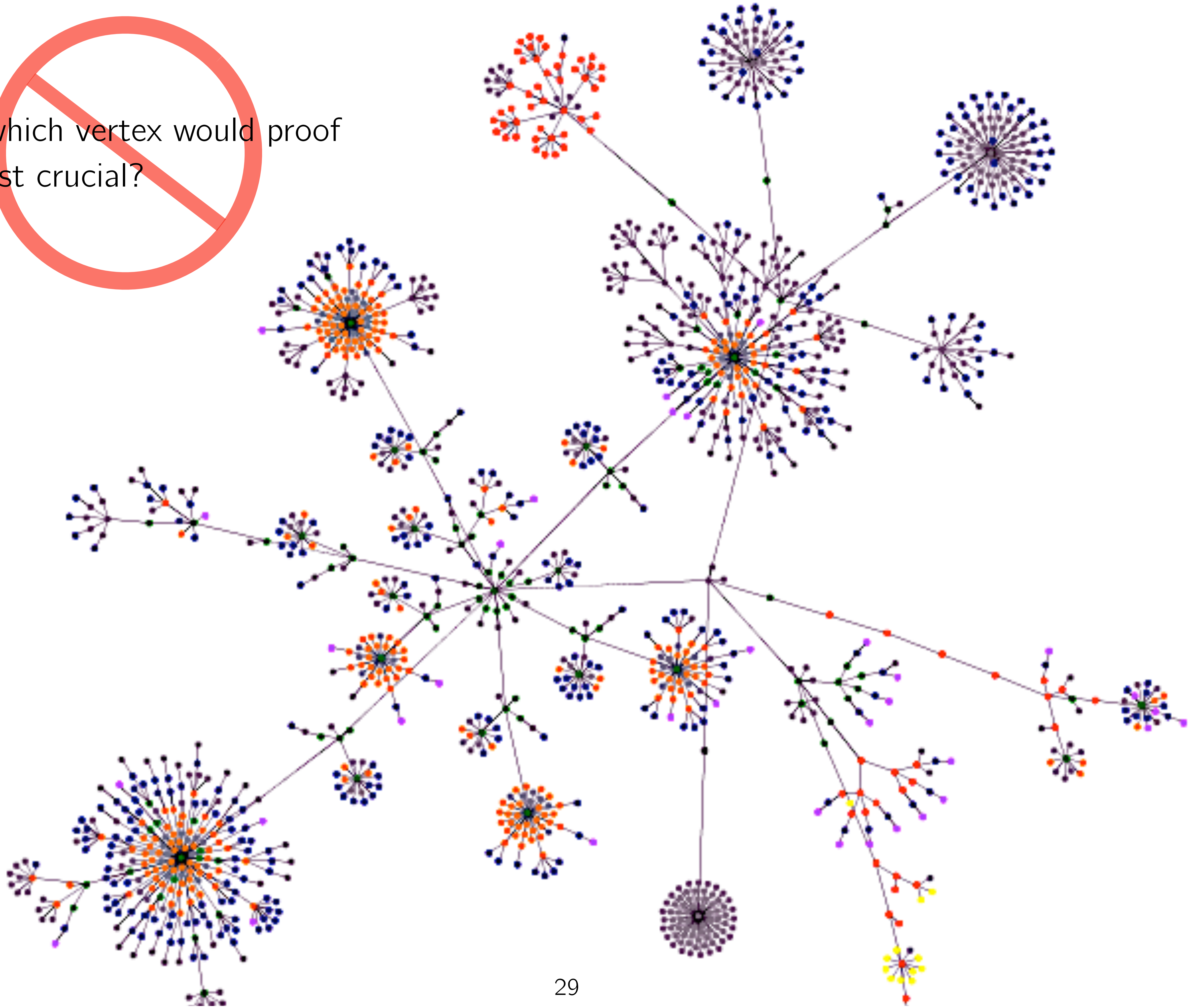
As networks grow...



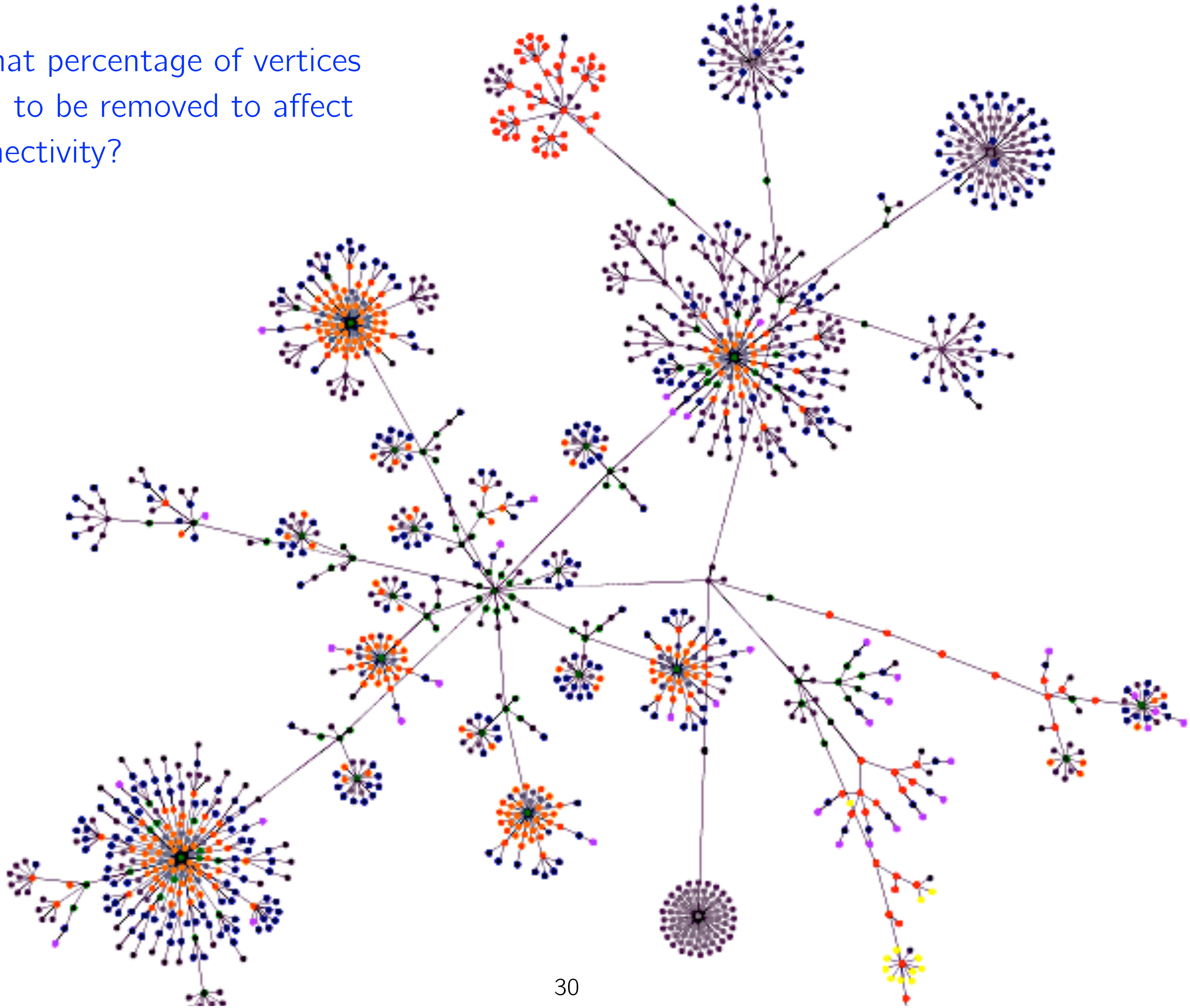
As networks grow...



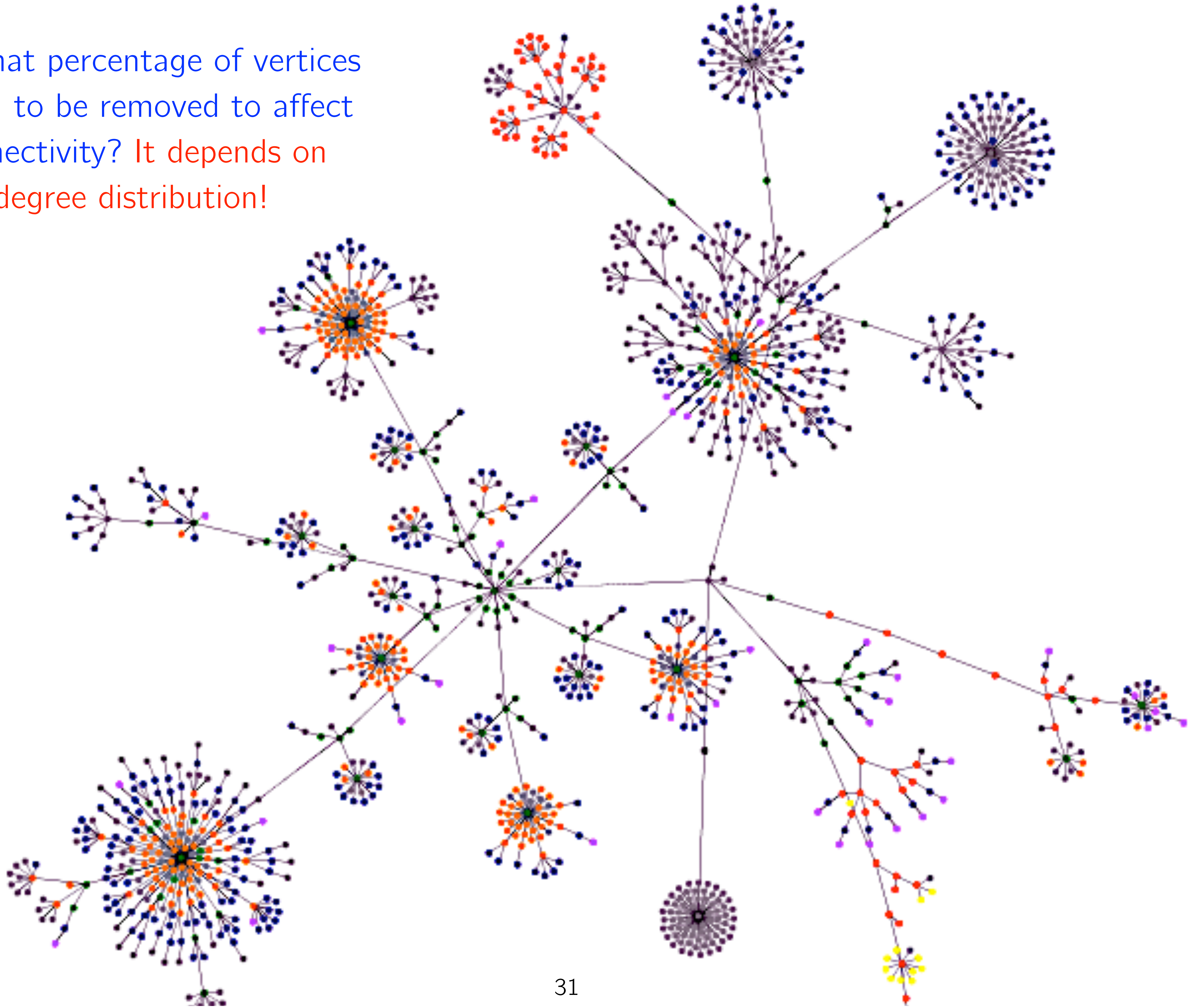
...which vertex would proof most crucial?



...what percentage of vertices need to be removed to affect connectivity?



...what percentage of vertices need to be removed to affect connectivity? It depends on the degree distribution!

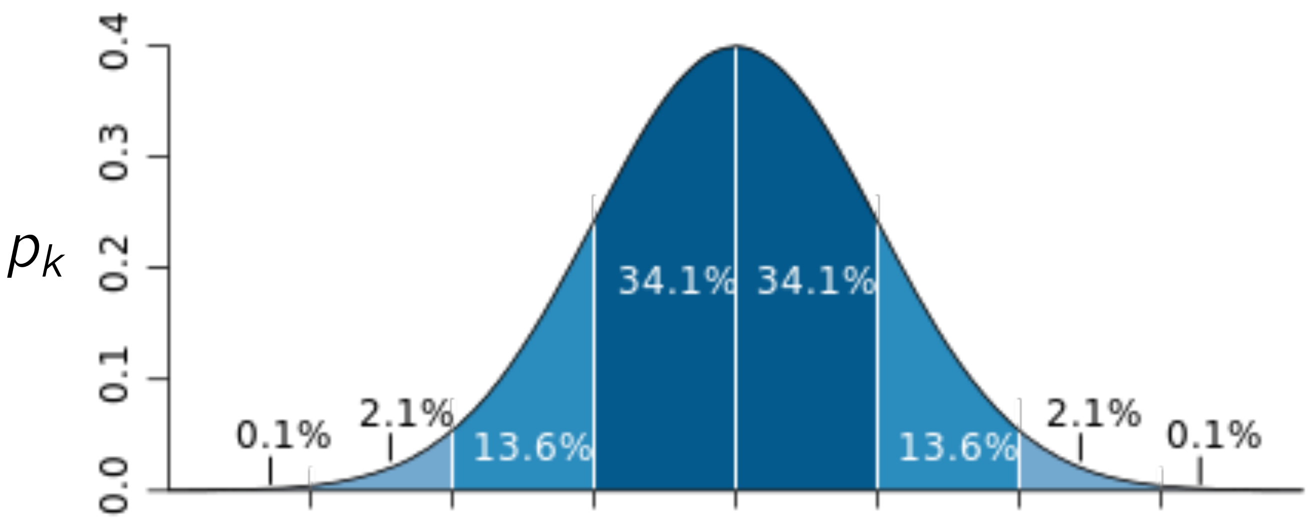


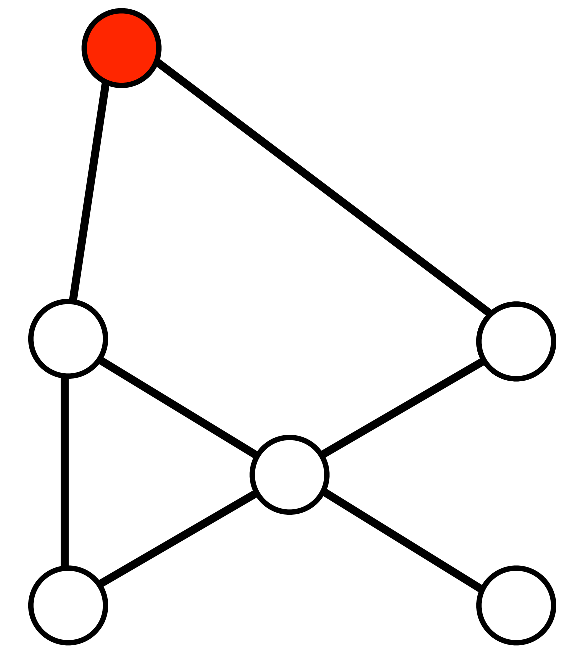
Degree distributions

+

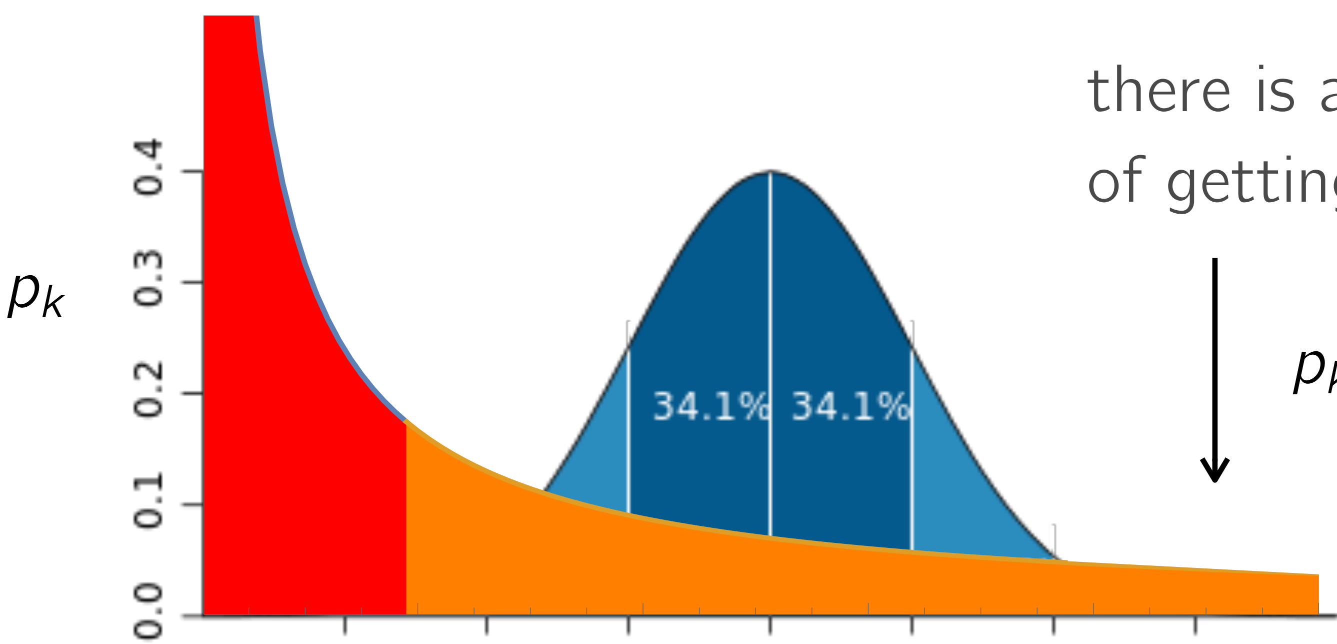
K = degree of a randomly selected node

$p_k = P[K = k]$



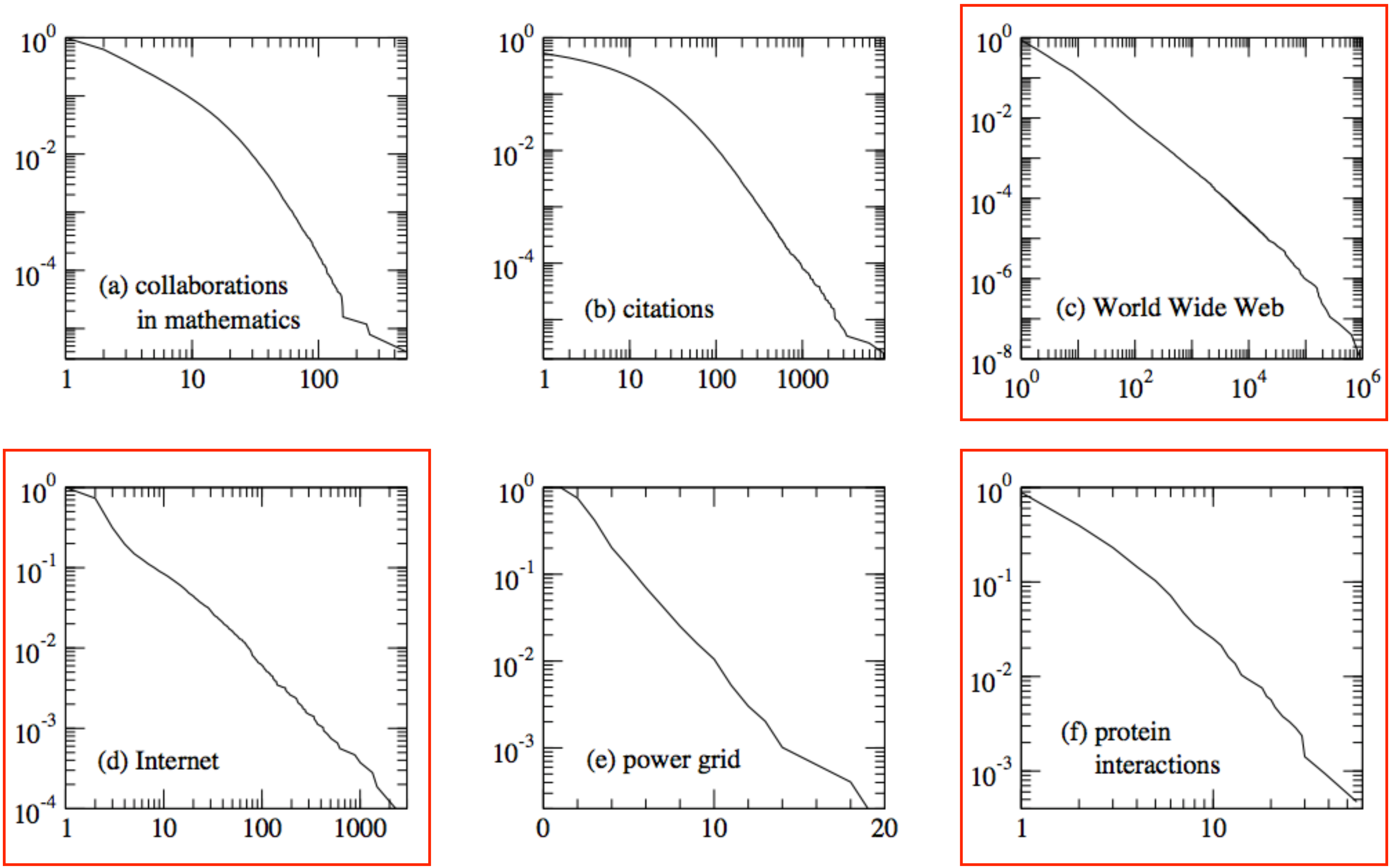


undirected



there is a larger probability of getting very large values

$p_k \propto k^{-\alpha}$



(a) collaborations in mathematics
(b) citations
(c) World Wide Web
(d) Internet
(e) power grid
(f) protein interactions

			number of nodes	number of edges	average degree	average distance	scaling exponent	global clustering	av. local clustering	deg. corr.
	network	type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r
social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208
	company directors	undirected	7 673	55 392	14.44	4.60	–	0.59	0.88	0.276
	math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15	0.34	0.120
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45	0.56	0.363
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088	0.60	0.127
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1			
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0		0.16	
	email address books	directed	16 881	57 029	3.38	5.22	–	0.17	0.13	0.092
	student relationships	undirected	573	477	1.66	16.01	–	0.005	0.001	−0.029
	sexual contacts	undirected	2 810				3.2			
information	WWW `nd.edu`	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	−0.067
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7			
	citation network	directed	783 339	6 716 198	8.57		3.0/–			
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	–	0.13	0.15	0.157
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7		0.44	
technological	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	−0.189
	power grid	undirected	4 941	6 594	2.67	18.99	–	0.10	0.080	−0.003
	train routes	undirected	587	19 603	66.79	2.16	–		0.69	−0.033
	software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.070	0.082	−0.016
	software classes	directed	1 377	2 213	1.61	1.51	–	0.033	0.012	−0.119
	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	−0.154
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	−0.366
biological	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	−0.240
	protein interactions	undirected	2 115	2 240	2.12	6.80	2.4	0.072	0.071	−0.156
	marine food web	directed	135	598	4.43	2.05	–	0.16	0.23	−0.263
	freshwater food web	directed	92	997	10.84	1.90	–	0.20	0.087	−0.326
	neural network	directed	307	2 359	7.68	3.97	–	0.18	0.28	−0.226

Next class

- Small-world effect
- Transitivity or clustering
- Strong and weak ties