Lecture 12

- how to design a state feedback? -

Bicycle

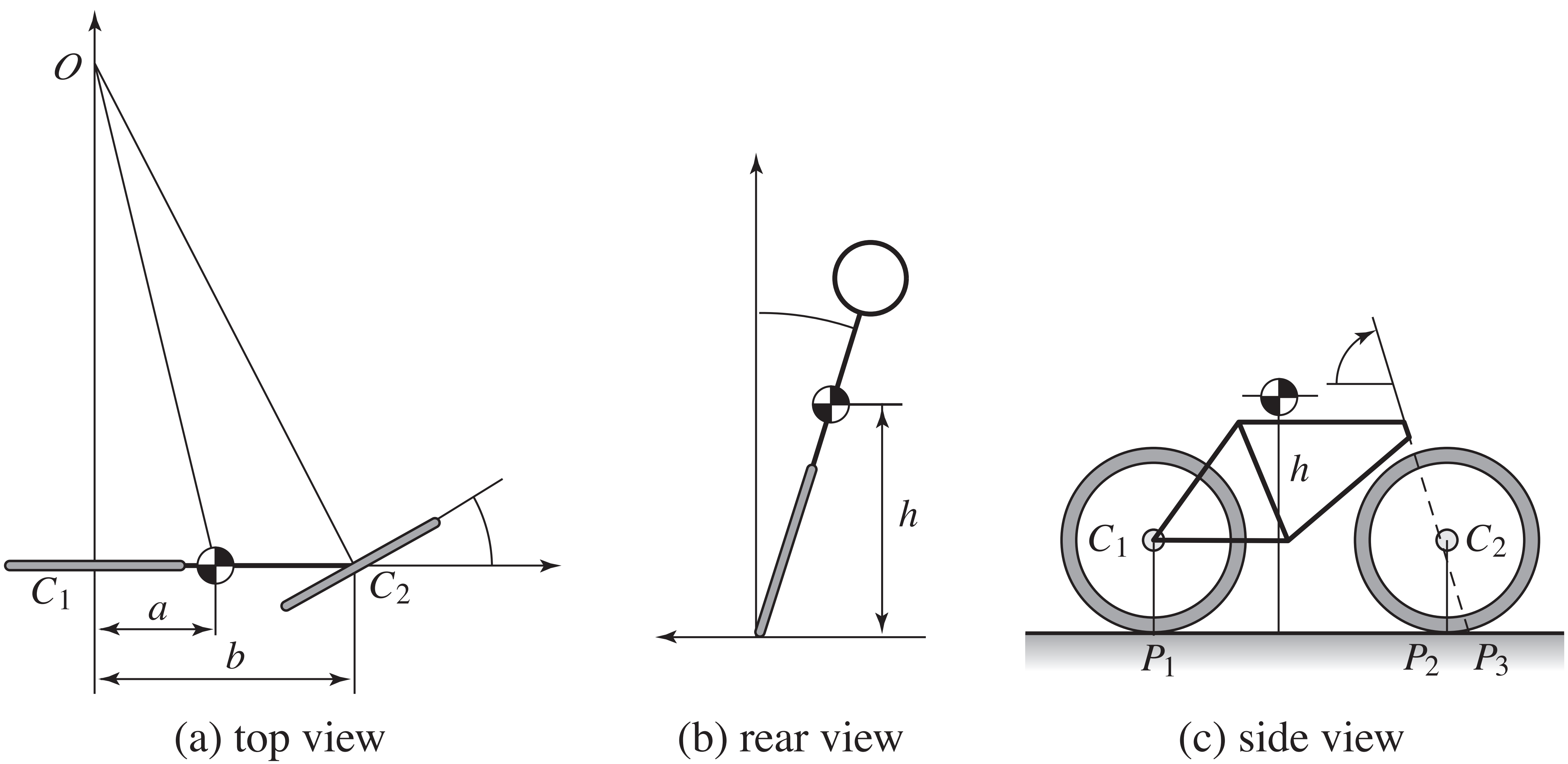


Figure 3.5: Schematic views of a bicycle. The steering angle is , and the roll angle is . The center of mass has height h and distance a from a vertical through the contact point P_1 of the rear wheel. The wheel base is b, and the trail is c.

State feedback controller

- Consider an open-loop system of the form

$$\frac{dx}{dt} = Ax + Bu$$

- Let $u = -Kx$. Then closed-loop system is given by

$$\frac{dx}{dt} = Ax + B(-Kx) = (A - BK)x$$

Theorem
Consider a linear system of the form

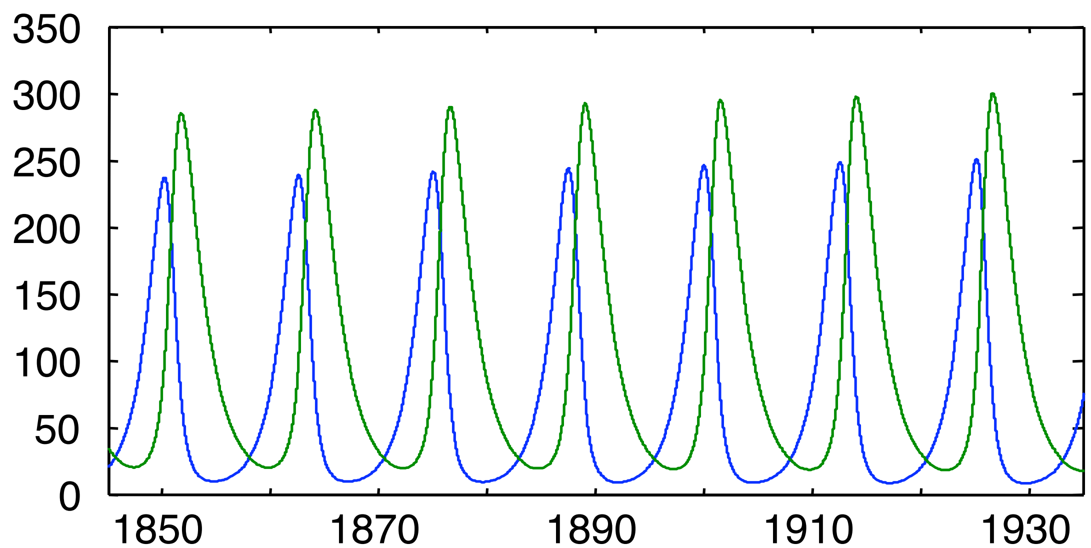
$$\frac{dx}{dt} = Ax + Bu$$

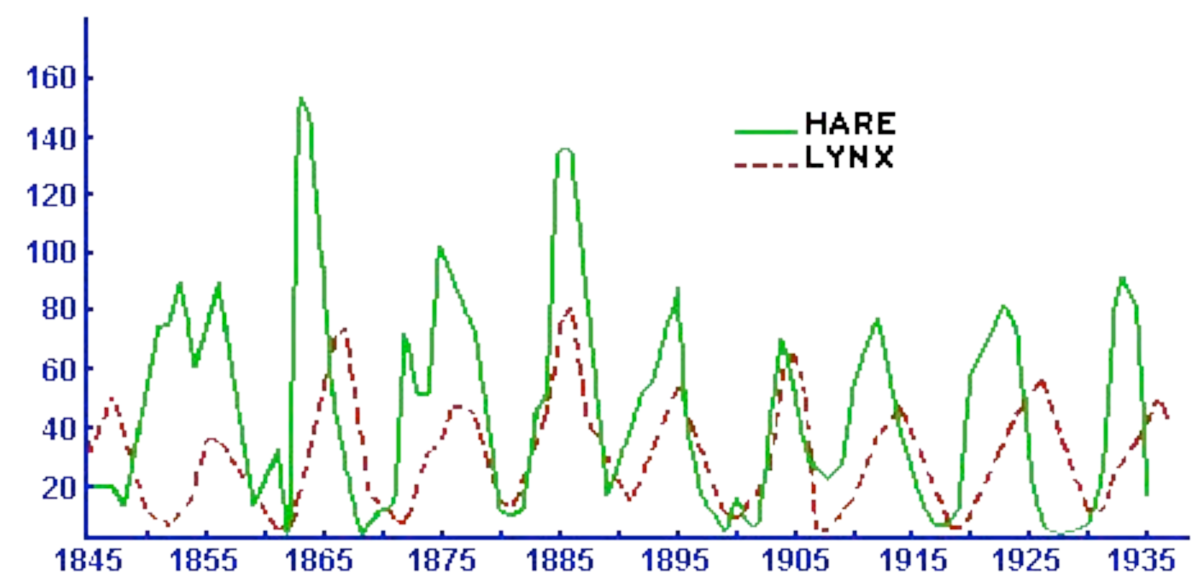
with input $u = -Kx$. The eigenvalues of $(A - BK)$ can be set to arbitrary values if and only if the pair (A, B) is reachable.

Population model

Questions we want to answer:

- Given the current population of rabbits and foxes, what will it be next year
- If we hunt down lots of foxes in a given year what will the effect on the rabbit and fox population be?
- Can we stabilize the population? (new)

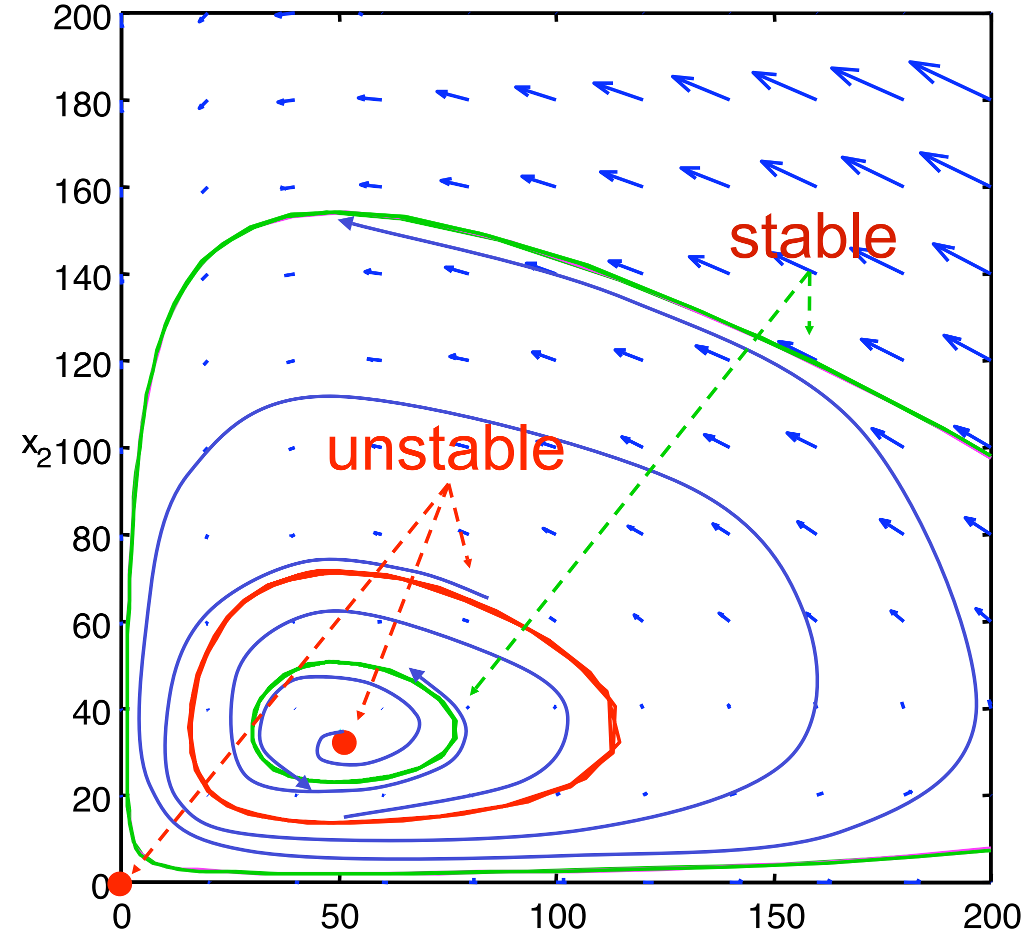




Example

Natural dynamics

$$\dot{x}_1 = b_r x_1 - a x_1 x_2$$
$$\dot{x}_2 = a x_1 x_2 - d_f x_2$$



Try to answer question locally, that is, around the natural equilibrium point

Controlled dynamics: modulate food supply

$$\dot{x}_1 = b_r(1+u)x_1 - a x_1 x_2$$
$$\dot{x}_2 = a x_1 x_2 - d_f x_2$$

Equilibrium point calculation

x_e = (50, 35)

Linearization

- Compute linearization around equil. point, x_e:

$$A = \left.\frac{\partial f}{\partial x}\right|_{(x_e,u_e)} \qquad B = \left.\frac{\partial f}{\partial u}\right|_{(x_e,u_e)}$$

- Redefine local variables: $z = x - x_e$, $v = u - u_e$

$$\frac{d}{dt}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} -b_r - a x_{2,e} & -a x_{1,e} \\ a x_{2,e} & -d_f + a x_{1,e} \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} b_r x_{1,e} \\ 0 \end{bmatrix} v$$

Linearized dynamics:

$$\frac{d}{dt}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} -b_r - ax_{2,e} & -ax_{1,e} \\ ax_{2,e} & -d_f + ax_{1,e} \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} b_r x_{1,e} \\ 0 \end{bmatrix} v$$

Control design:

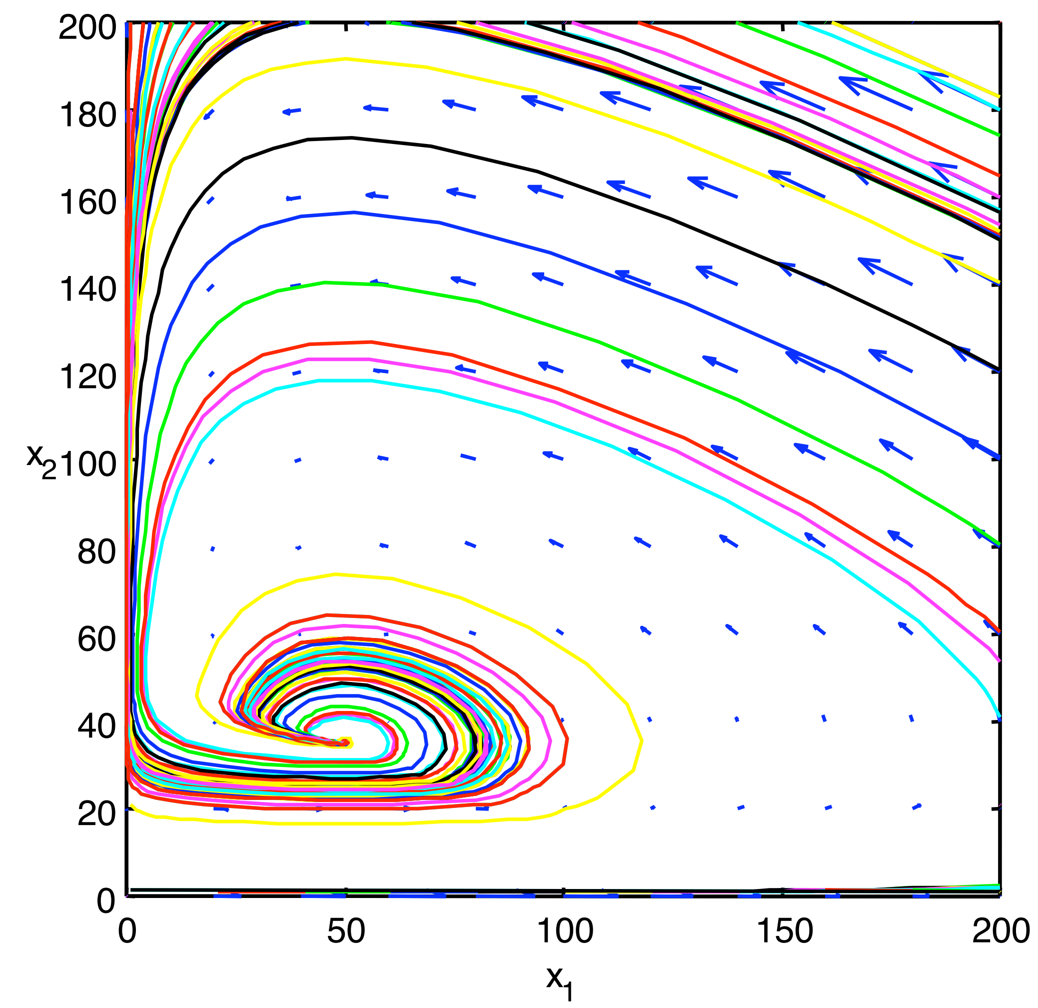
$$v = -Kz = -K(x - x_e)$$
$$u = u_e + v = u_e - K(x - x_e)$$

Place poles at stable values

- Choose λ=-1, -2
- K = place(A, B, [-1; -2]);

Modify dynamics to include control

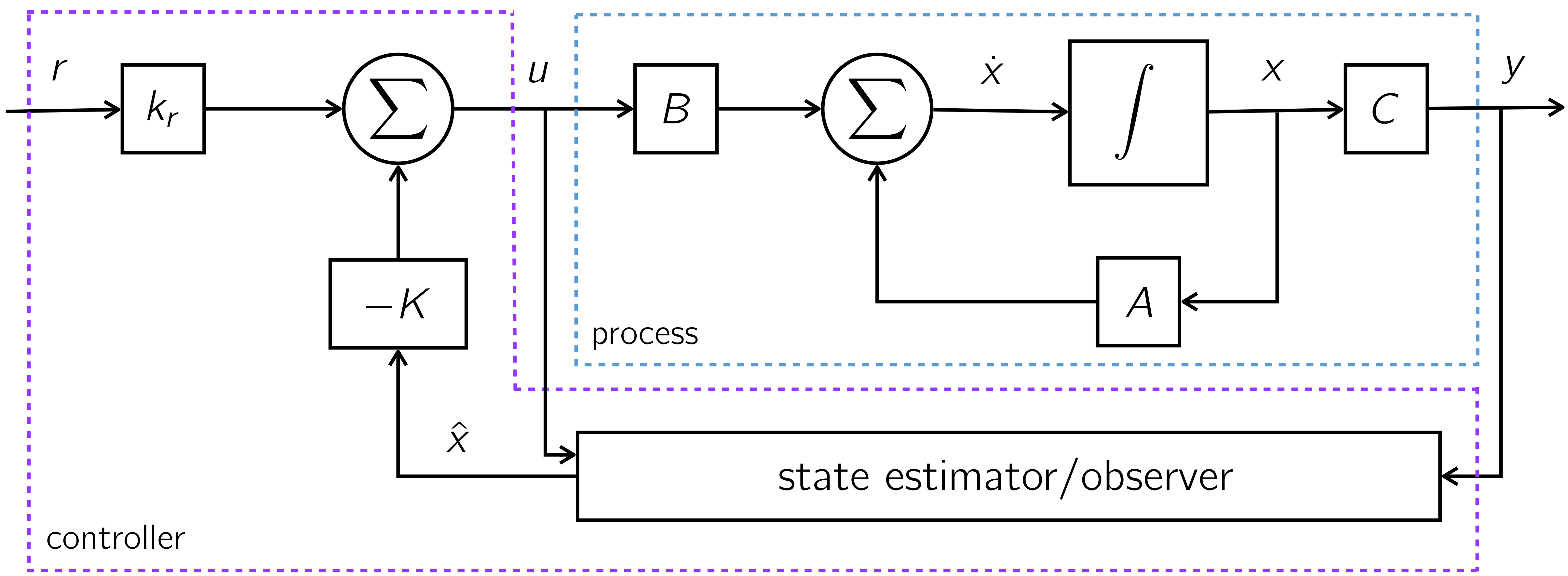
$$\dot{x}_1 = b_r(1 - K(x - x_e))x_1 - ax_1x_2$$
$$\dot{x}_2 = ax_1x_2 - d_f x_2$$



General time-invariant control law

- Consider the generalized form

$$
\begin{aligned}
u &= \alpha(x, r) \\
&= -Kx + k_r r
\end{aligned}
$$



For what value of k_r does the output y track the reference r?

General time-invariant control law

- Consider the generalized form

$$\begin{aligned} u &= \alpha(x, r) \\ &= -Kx + k_r r \end{aligned}$$

- Applied to a linear system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ &= Ax + B(-Kx + k_r r) \\ &= (A - BK)x + Bk_r r \end{aligned}$$

- At equilibrium

$$0 = (A - BK)x_e + Bk_r r \implies x_e = -(A - BK)^{-1}Bk_r r$$

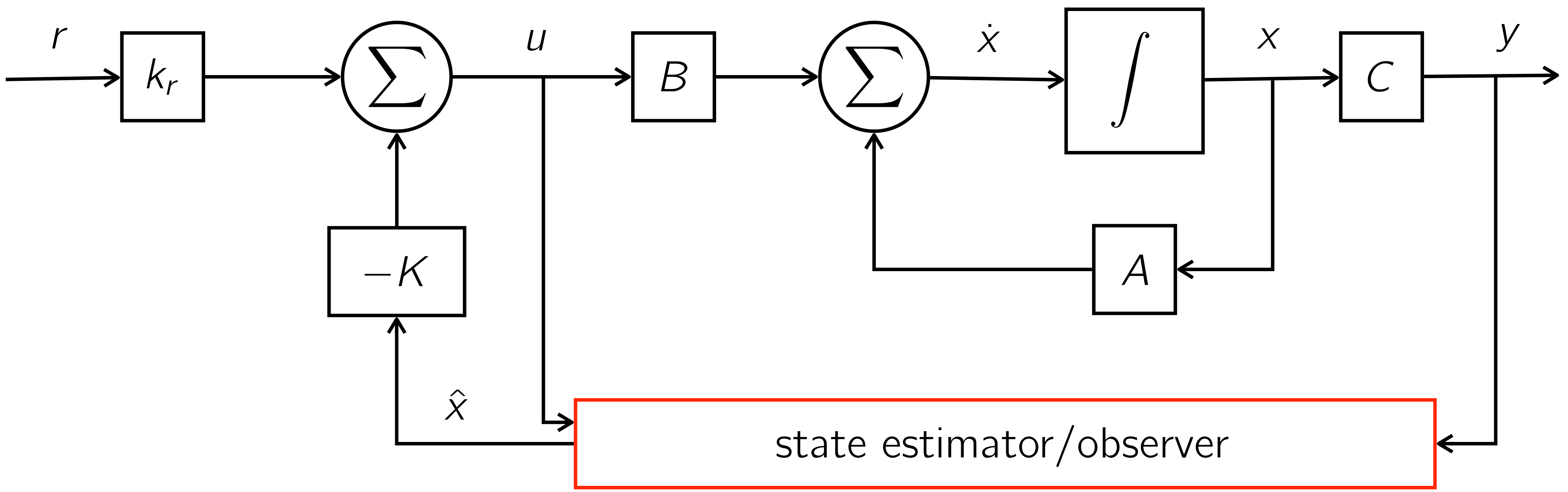
- The output (at equilibrium)

$$y_e = Cx_e = -C(A - BK)^{-1}Bk_r r$$

Condition for $y_e = r$?

General time-invariant control law

$$y_e = Cx_e = -C(A - BK)^{-1}Bk_r r = r \ \text{ if } \ k_r = -\frac{1}{C(A - BK)^{-1}B}$$



How to define state estimator/observer?