Lecture 20

- How to tune a PID based on empirical methods? -

Review

- PID (proportional–integral–derivative controller)
- Basic properties
- PI implementation
- PD implementation
- PID implementation

Today

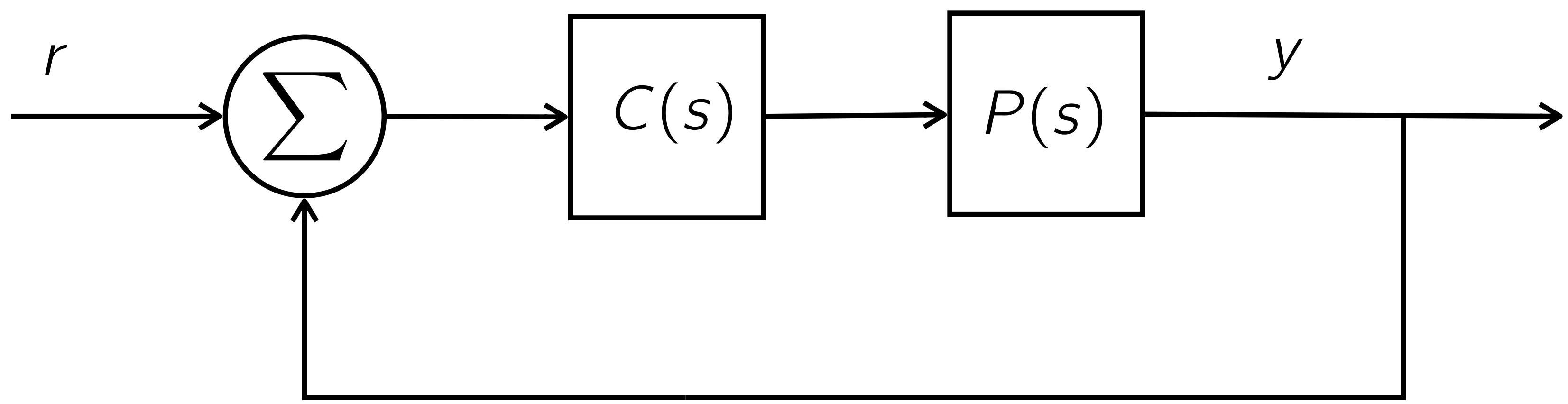
- PID controller (3rd order)
- Empirical tuning methods

Problem

Consider a system with the following system:

$$P(s) = \frac{1}{s^2 + 2s}$$

$$C(s) = \frac{k_d s^2 + k_p s + k_i}{s}$$



Design a PID controller such that:

- Overshoot percentage of 20%
- T_s = 1 second

PID controller

- Compute closed-loop transfer function

$$\frac{C(s)P(s)}{1+C(s)P(s)} = \frac{\frac{k_d s^2+k_p s+k_i}{s}\frac{1}{s^2+2s}}{1+\frac{k_d s^2+k_p s+k_i}{s}\frac{1}{s^2+2s}}$$
$$= \frac{k_d s^2+k_p s+k_i}{s(s^2+2s)+(k_d s^2+k_p s+k_i)}$$

- Steady state error (closed-loop)

$$e_{ss} = \lim_{s\to 0}\left[1-\frac{C(s)P(s)}{1+C(s)P(s)}\right]$$
$$= \lim_{s\to 0}\left[1-\frac{k_d^2+k_p s+k_i}{s(s^2+2s)+(k_d s^2+k_p s+k_i)}\right]$$
$$= 1-\frac{k_i}{k_i} = 0$$

- Third order characteristic equation

$$\Delta = s(s^2+2s)+(k_d s^2+k_p s+k_i)$$
$$= s^3+(2+k_d)s^2+k_p s+k_i$$

PID controller

- Consider a second order system
- Add a third pole
- Third pole further away from the imaginary axis
- Pole location for second order system

$$
\begin{aligned}
p_{1,2} &= -\frac{2\zeta\omega_0 \pm \sqrt{4\zeta^2\omega_0^2 - 4\omega_0^2}}{2} \\
&= -\zeta\omega_0 \pm \omega_0\sqrt{\zeta^2 - 1}
\end{aligned}
$$

- Third pole at

$$
p_3 < -\zeta\omega_0
$$

Meet system specifications based on behavior of second order system

PID controller

- Desired behavior of second order system

$$\frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

Settling time

$$T_s = 1 \approx \frac{4}{\zeta\omega_0} \implies \zeta\omega_0 = 4$$

Percentage of overshoot

$$P.O. = 0.2 = 100e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$
$$\implies \zeta = 0.89$$
$$\implies \omega_0 = 4.5$$

- Adding a third pole at $p_3 = -10$

$$\frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \frac{1}{s - p_3} = \frac{20.25}{s^2 + 8s + 20.25} \frac{1}{s + 10}$$
$$= \frac{20.25}{s^3 + 18s^2 + 100.25s + 202.5} \longleftarrow \text{desired characteristic equation}$$

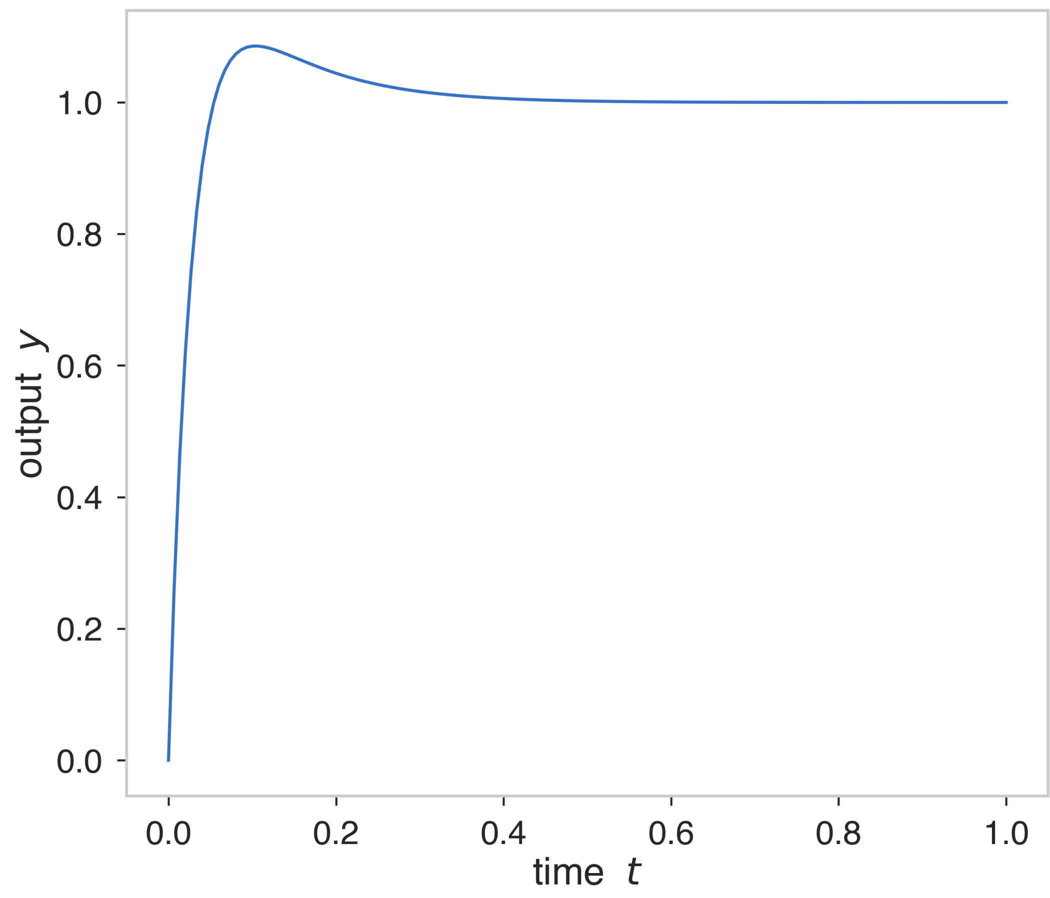
PID controller

$$\Delta_{\text{desired}} = \Delta$$

$$s^3 + 18s^2 + 100.25s + 202.5 = s^3 + (2 + k_d)s^2 + k_p s + k_i$$

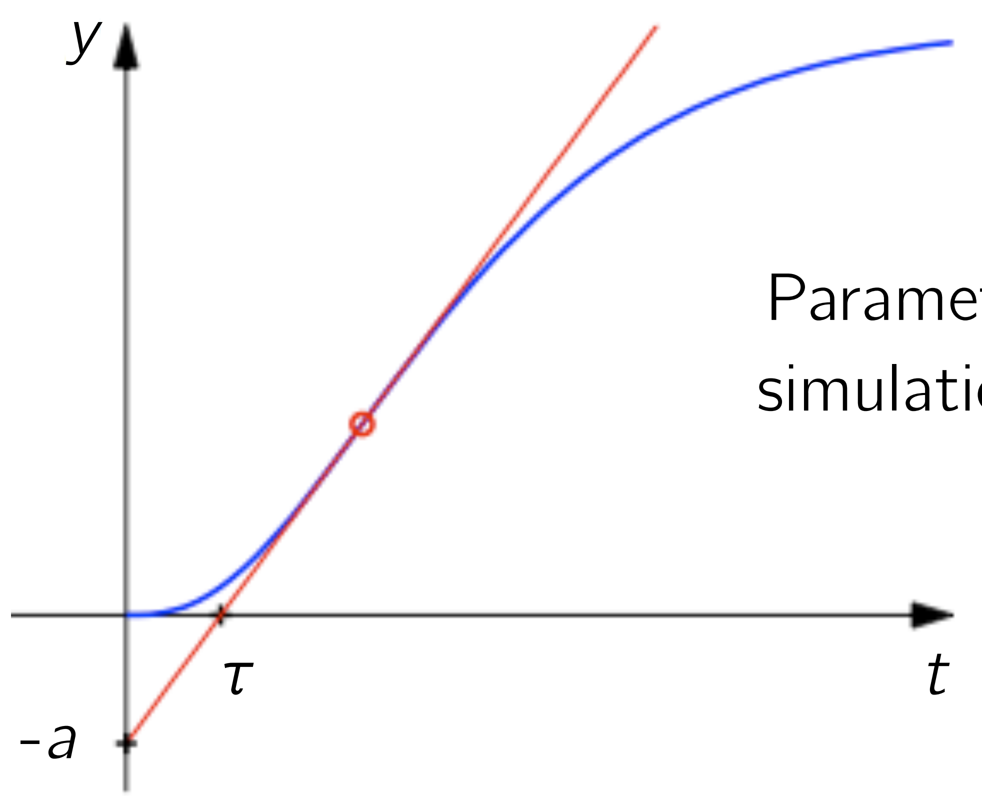
Controller gains

$$k_p = 100.25$$
$$k_d = 18 - 2 = 16$$
$$k_i = 202.5$$



The time domain method

- Open loop unit step response of the process
- Steepest slope of the step response: a/τ
- The parameter τ is an approximation of the time delay of the system



Parameters obtained by extensive simulation of a range of processes

Type	k_p	T_i	T_d
P	1/a		
PI	0.9/a	3τ	
PID	1.2/a	2τ	0.5τ

and

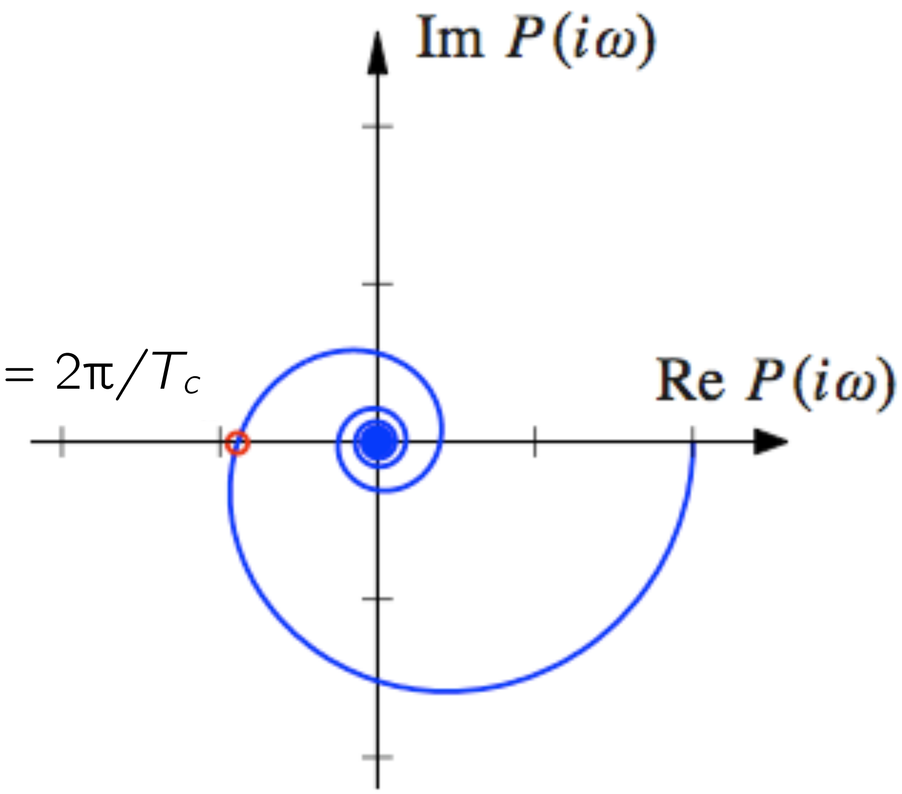
$$k_i = \frac{k_p}{T_i}$$

$$k_d = k_p T_d$$

Frequency domain method

- Controller's integral and derivative gains are set to zero
- Proportional gain is increased until the system starts to oscillate
- Critical value of $k_p = k_c$ is observed together with the period of oscillation T_c

critical point at $\omega = \omega_c = 2\pi/T_c$



Type	k_p	T_i	T_d
P	$0.5k_c$		
PI	$0.4k_c$	$0.8T_c$	
PID	$0.6k_c$	$0.5T_c$	$0.125T_c$

Ziegler–Nichols method