Lecture 8
- epistemic games II -

Review

- Epistemic games
- Incorporating information into the solution
- Formal notation for common knowledge
- An simple epistemic game
  - Possibility operator $\mathbf{P}_i$
  - Knowledge operator $\mathbf{K}_i$

Epistemic game

The normal form game

Epistemic game

- Set of players
- Set of strategies $s = (s_1 , \dots , s_n) \in S$
- Lottery for each player $\pi_i : S \to \mathbb{R}$

The normal form game

Epistemic game

- Set of players
- Set of strategies $s = (s_1 , \ldots , s_n) \in S$
- Lottery for each player $\pi_i : S \to \mathbb{R}$

$\left.\right\}$ The normal form game

- A set of possible states Ω
- A subjective prior $p_i(\cdot;\, \omega)$ over Ω
- A knowledge partition $\mathcal{P}_i$ of Ω for each player i

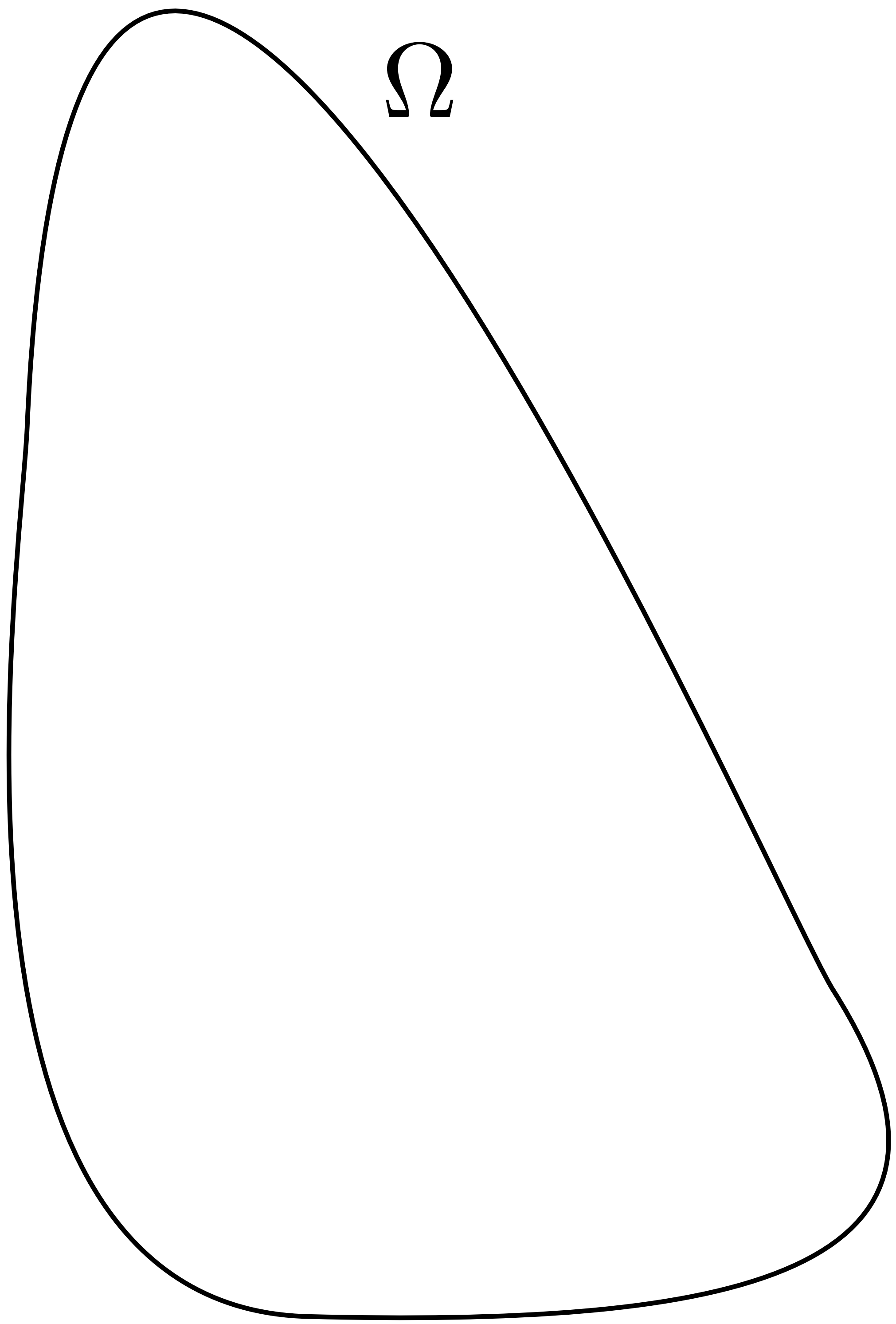
A set of possible states Ω

- Specifies the strategy profile
- Strategy profile $s = \boldsymbol{s}(\omega)$
- Strategy profile of one player $s_i = \boldsymbol{s}_i(\omega)$
- Strategy profile of all other players $s_{\text{-}i} = \boldsymbol{s}_{\text{-}i}(\omega)$

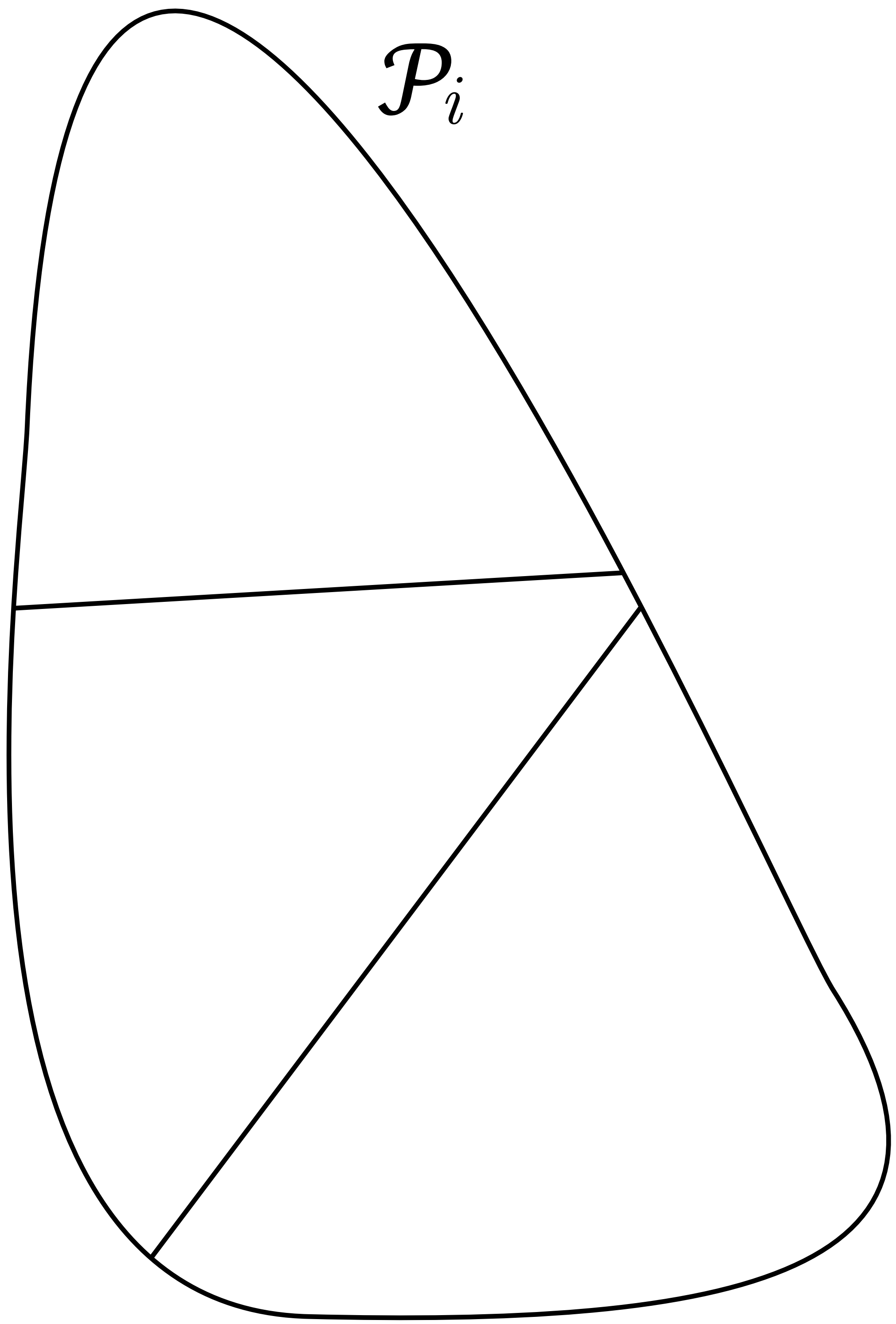
A subjective prior $p_i(\bullet;\ \omega)$

- Belief concerning the state of the game
- When the actual state is ω
- Probability player i places on the current state being ω' when the actual state is ω is $p_i(\omega';\ \omega)$

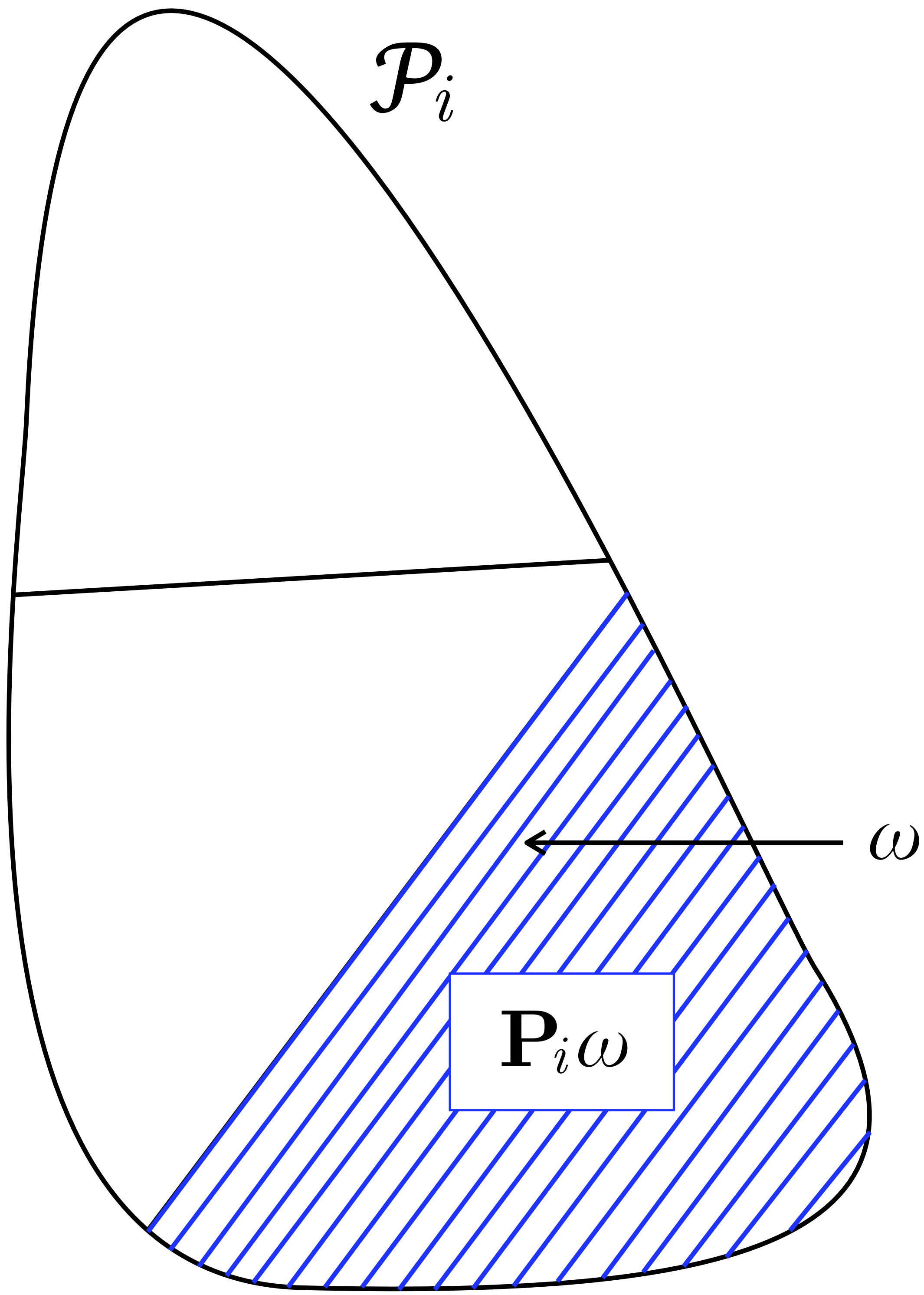
A knowledge partition $\mathcal{P}_i$



A knowledge partition $\mathcal{P}_i$



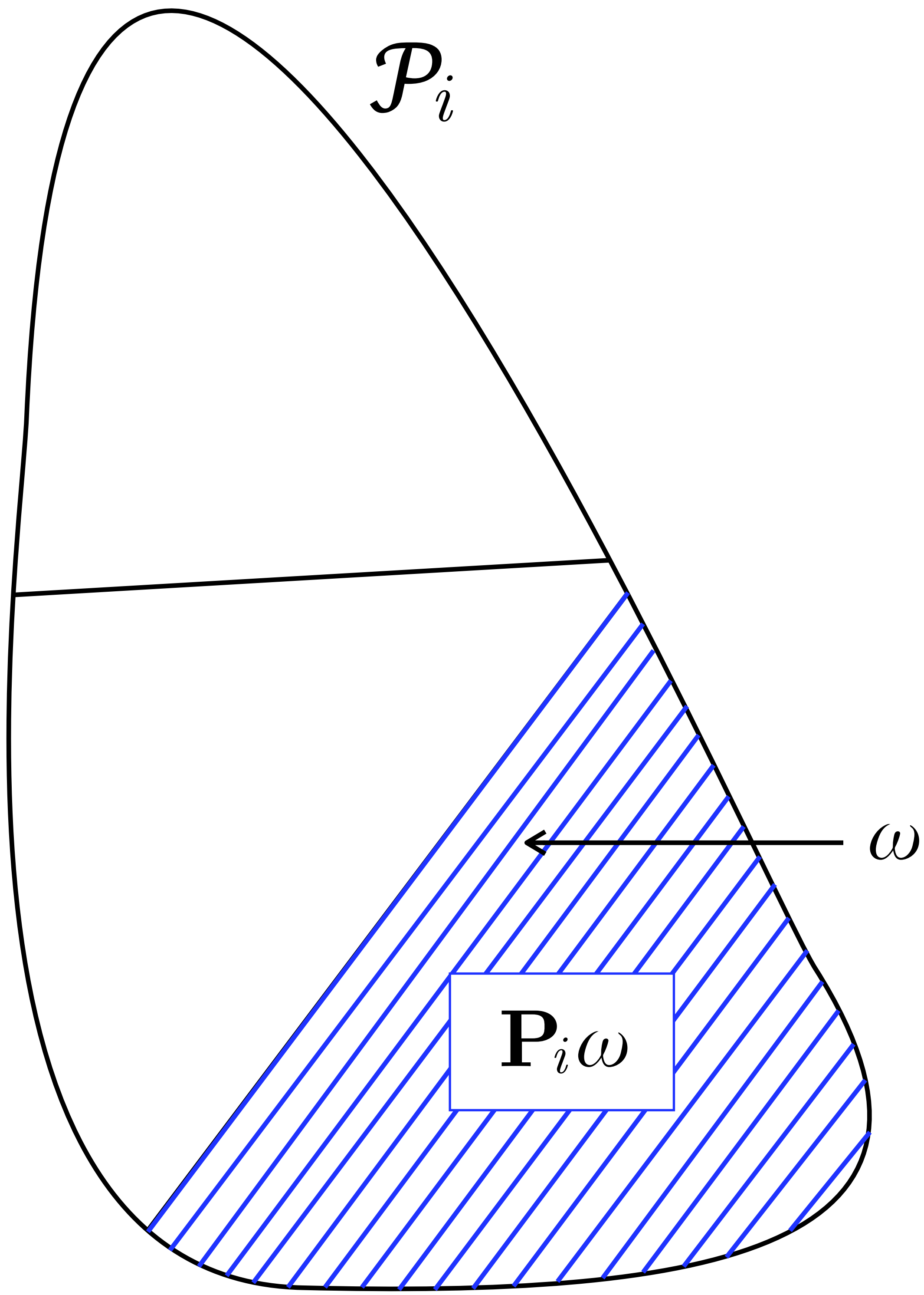
A knowledge partition $\mathcal{P}_i$



$\mathbf{P}_i\omega$ set of states player i considers possible when the actual state is ω:

$$\mathbf{P}_i\omega = \{\omega' \in \Omega : p_i(\omega'; \omega) > 0\}$$

A knowledge partition $\mathcal{P}_i$



$\mathbf{P}_i\omega$ set of states player i considers possible when the actual state is ω:

$$\mathbf{P}_i\omega = \{\omega' \in \Omega : p_i(\omega';\omega) > 0\}$$

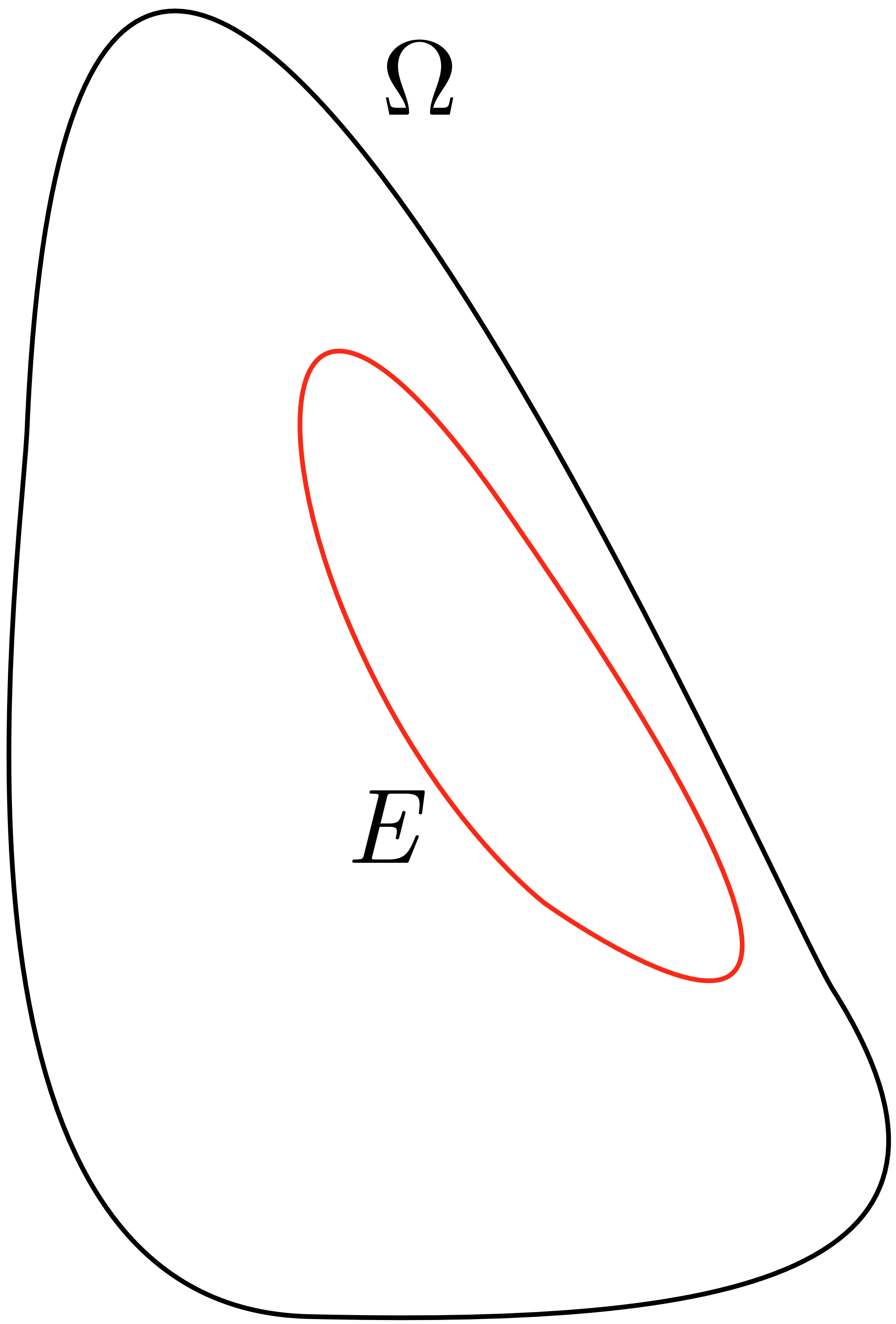
Assumptions

$\omega \in \mathbf{P}_i\omega$

$\omega' \in \mathbf{P}_i\omega \to \mathbf{P}_i\omega' = \mathbf{P}_i\omega$

$\mathbf{P}_i$: Possibility operator

Knowledge operator



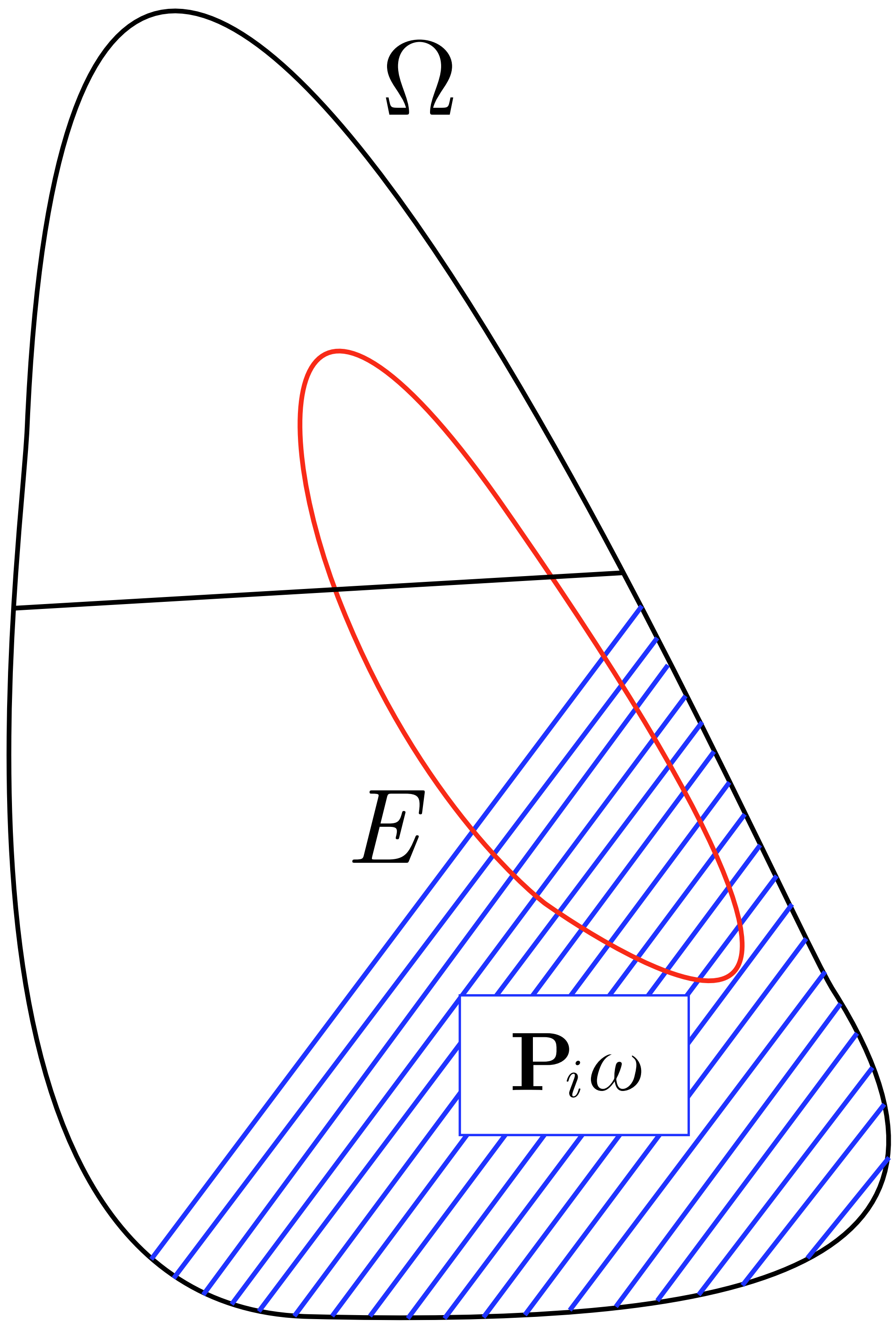
Event $E \subseteq \Omega$

A player knows an event if:

$$\mathbf{P}_i\omega \subseteq E$$

$$\forall \omega \in \mathbf{P}_i\omega \to \omega \in E$$

Knowledge operator



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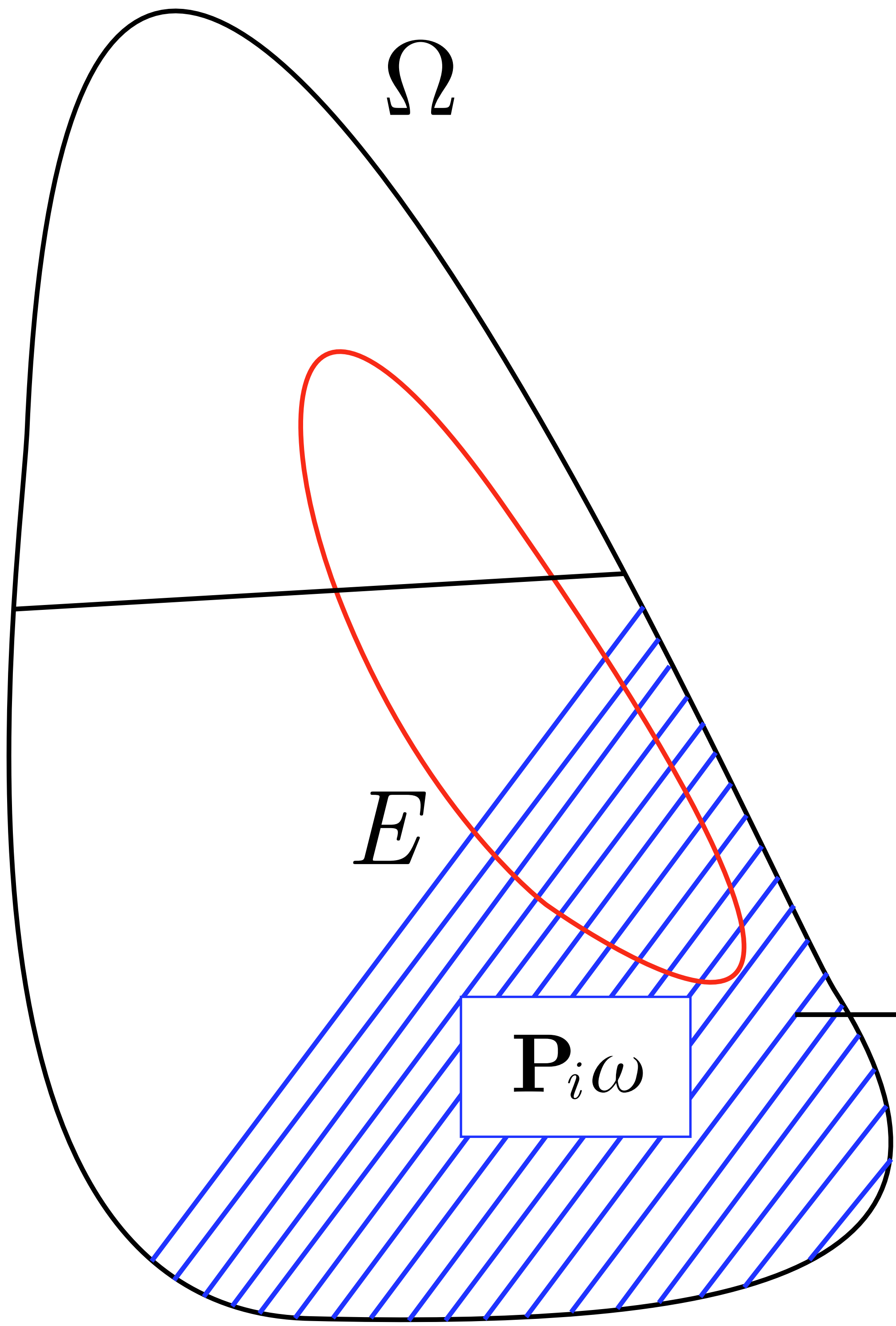
Knowledge operator



Event $E \subseteq \Omega$

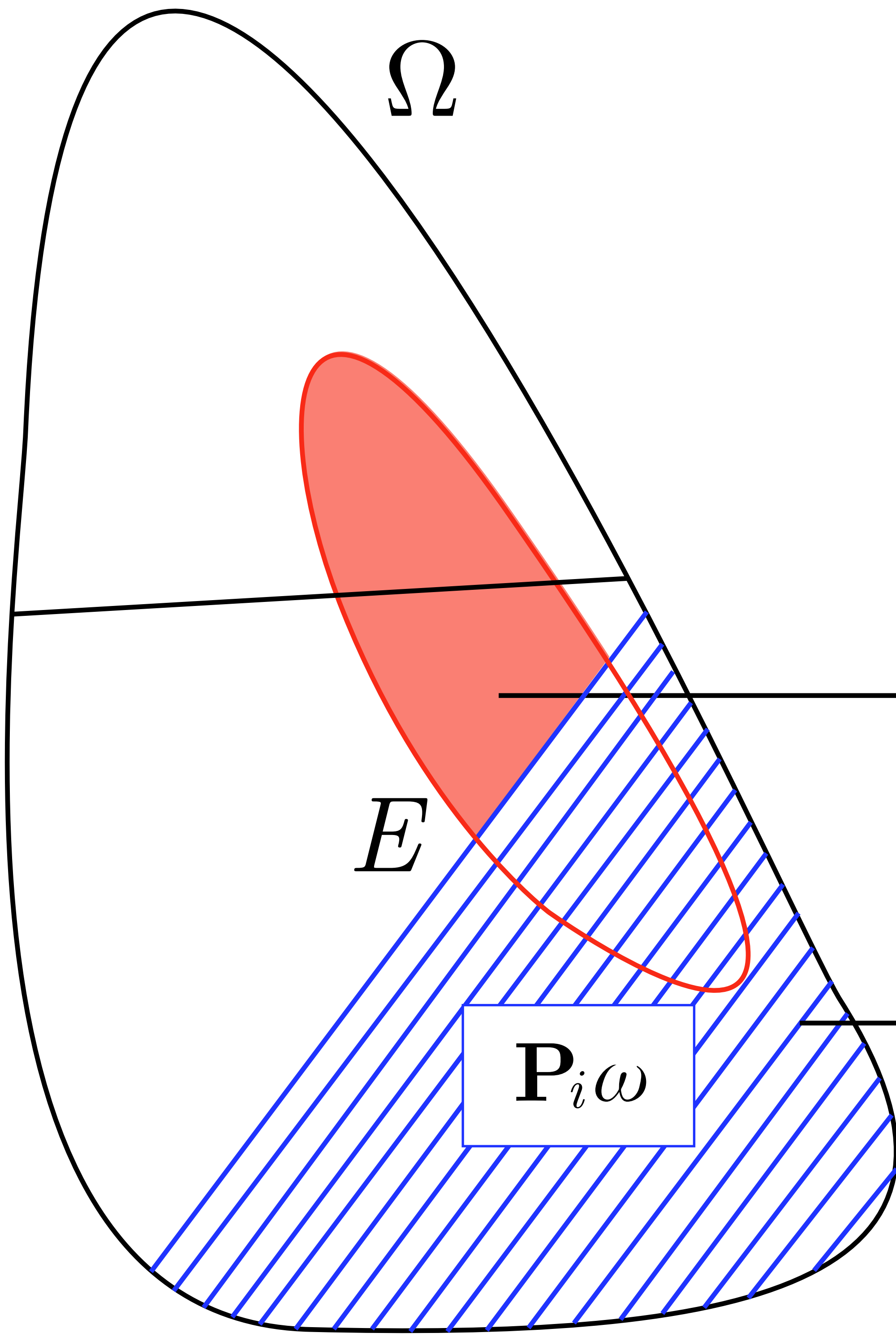
A player knows an event if:

$$\mathbf{P}_i\omega \subseteq E$$

$$\forall \omega \in \mathbf{P}_i\omega \rightarrow \omega \in E$$

Player i considers these states possible <u>when the state of the game is ω!</u>

Knowledge operator



Event $E \subseteq \Omega$

A player knows an event if:

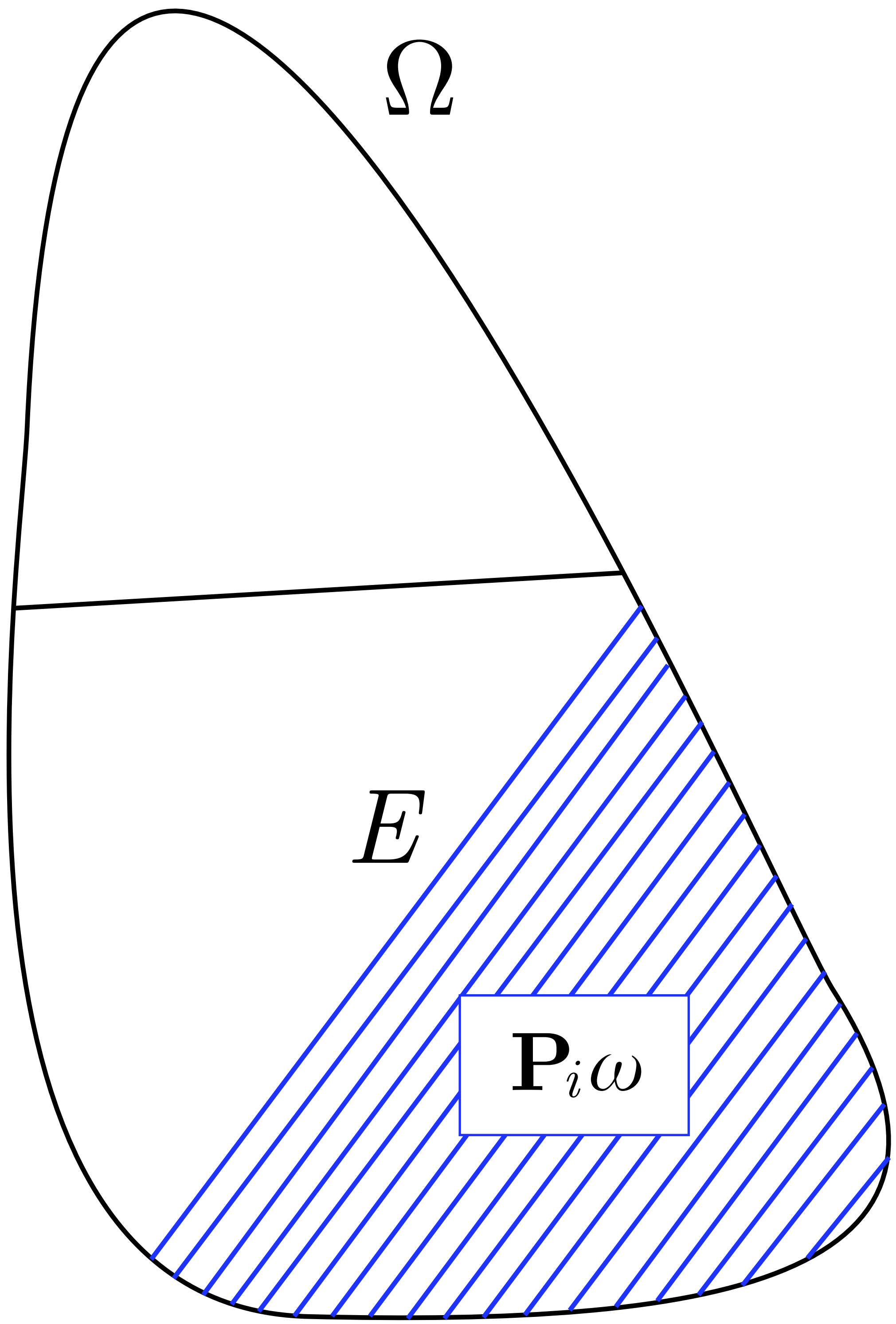
$$\mathbf{P}_i\omega \subseteq E$$

$$\forall \omega \in \mathbf{P}_i\omega \rightarrow \omega \in E$$

Player i does not consider these states possible!

Player i considers these states possible <u>when the state of the game is ω!</u>

Knowledge operator



Event $E \subseteq \Omega$

A player knows an event if:

$$\mathbf{P}_i\omega \subseteq E$$

$$\forall \omega \in \mathbf{P}_i\omega \rightarrow \omega \in E$$

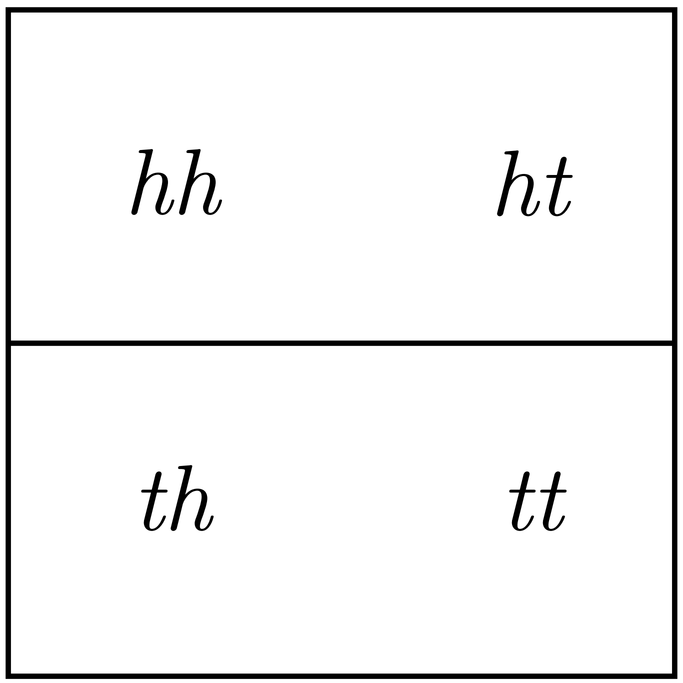
If an agent know in event in state ω, the event is true in ω

$$\mathbf{K}_i E = \{\omega \in \Omega : \mathbf{P}_i\omega \subseteq E\}$$

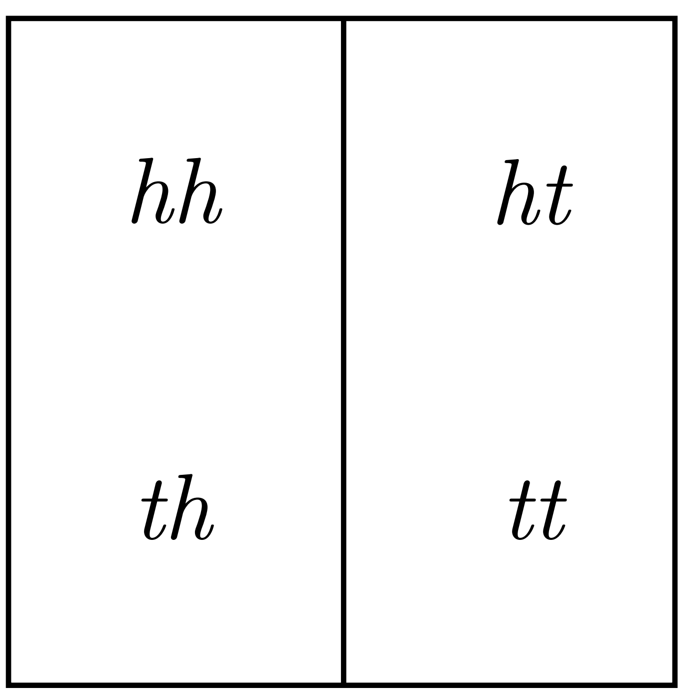
$\mathbf{K}_i$: Knowledge operator

Simple epistemic game

$\mathcal{P}_1$



$\mathcal{P}_2$

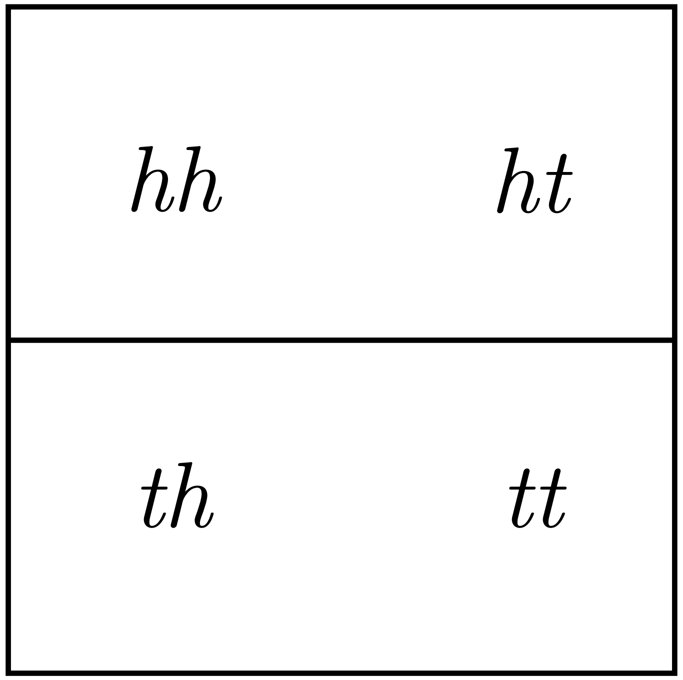


Neither observes the other's choice:

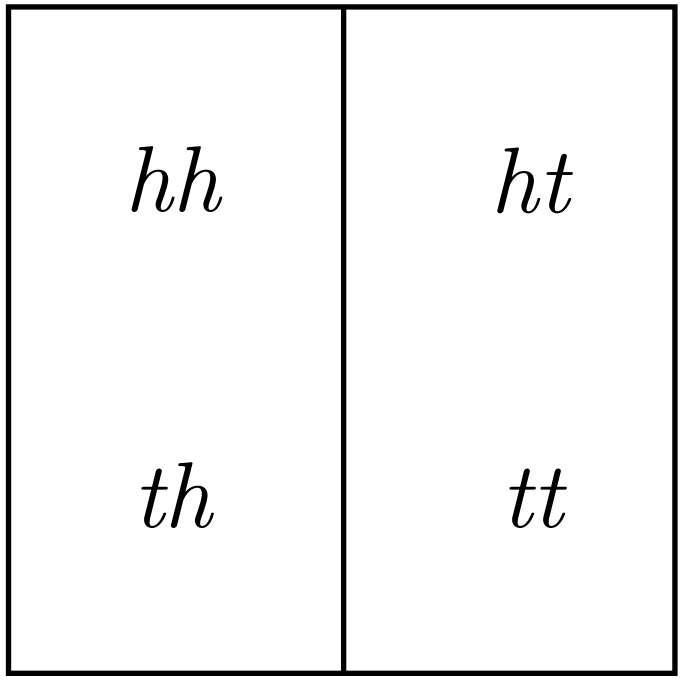
- $\mathbf{P}_1 hh = \mathbf{P}_1 ht = \{hh,\ ht\}$

Simple epistemic game

$\mathcal{P}_1$



$\mathcal{P}_2$

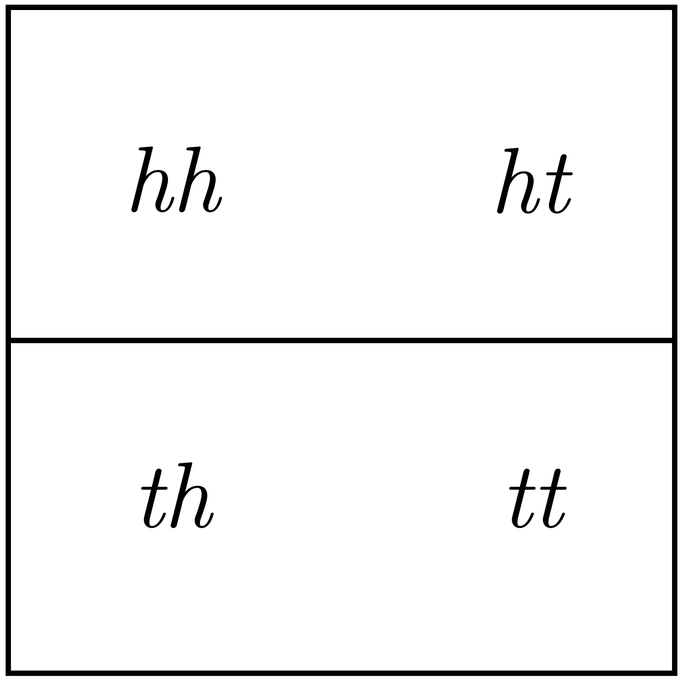


Neither observes the other's choice:

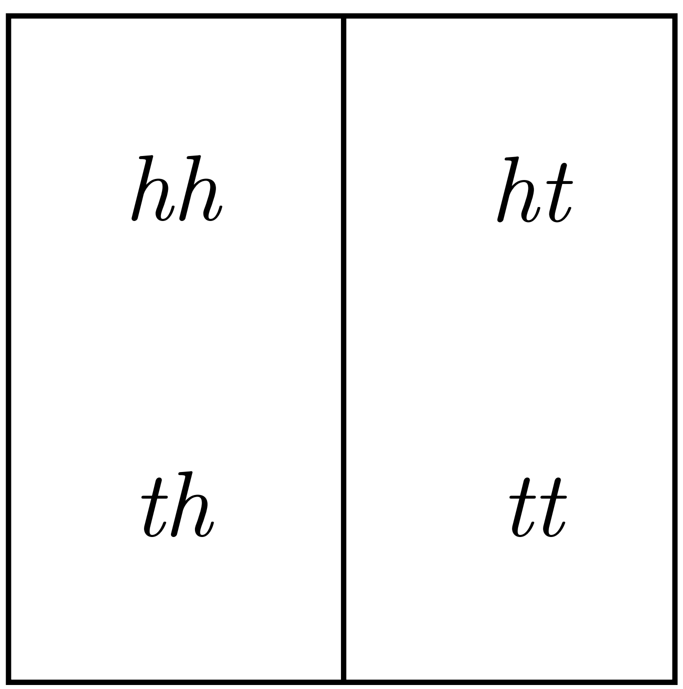
- $\mathbf{P}_1 hh = \mathbf{P}_1 ht = \{hh,\ ht\}$
- $\mathbf{P}_1 th = \mathbf{P}_1 tt = \{th,\ tt\}$

Simple epistemic game

$\mathcal{P}_1$



$\mathcal{P}_2$



Neither observes the other's choice:

- $\mathbf{P}_1 hh = \mathbf{P}_1 ht = \{hh,\ ht\}$
- $\mathbf{P}_1 th = \mathbf{P}_1 tt = \{th,\ tt\}$
- $\mathbf{P}_2 hh = \mathbf{P}_2 th = \{hh,\ th\}$
- $\mathbf{P}_2 ht = \mathbf{P}_1 tt = \{ht,\ tt\}$

Simple epistemic game

$\mathcal{P}_1$

hh	ht
th	tt

$\mathcal{P}_2$

hh	ht
th	tt

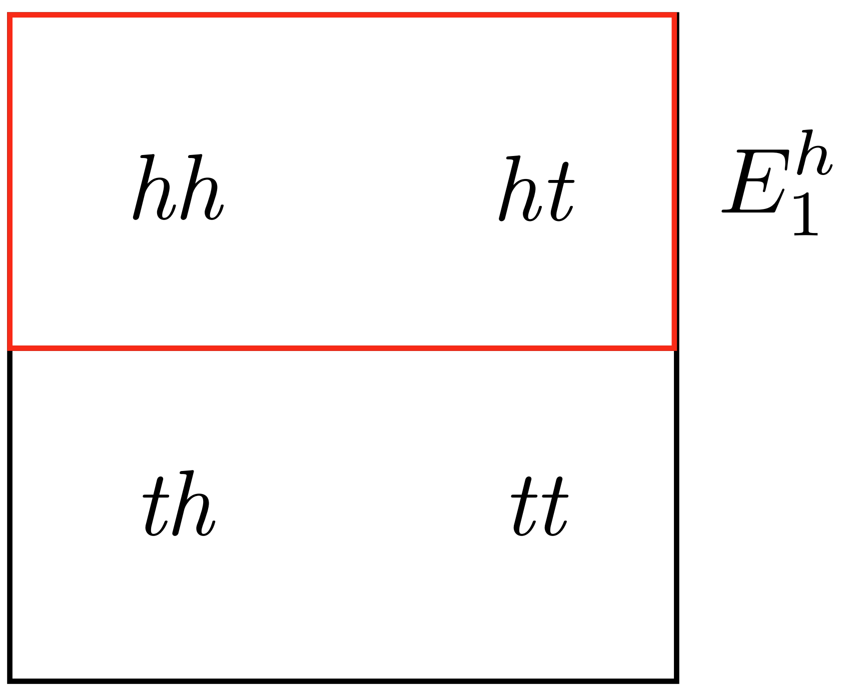
Neither observes the other's choice:

- $\mathbf{P}_1 hh = \mathbf{P}_1 ht = \{hh,\ ht\}$
- $\mathbf{P}_1 th = \mathbf{P}_1 tt = \{th,\ tt\}$
- $\mathbf{P}_2 hh = \mathbf{P}_2 th = \{hh,\ th\}$
- $\mathbf{P}_2 ht = \mathbf{P}_1 tt = \{ht,\ tt\}$

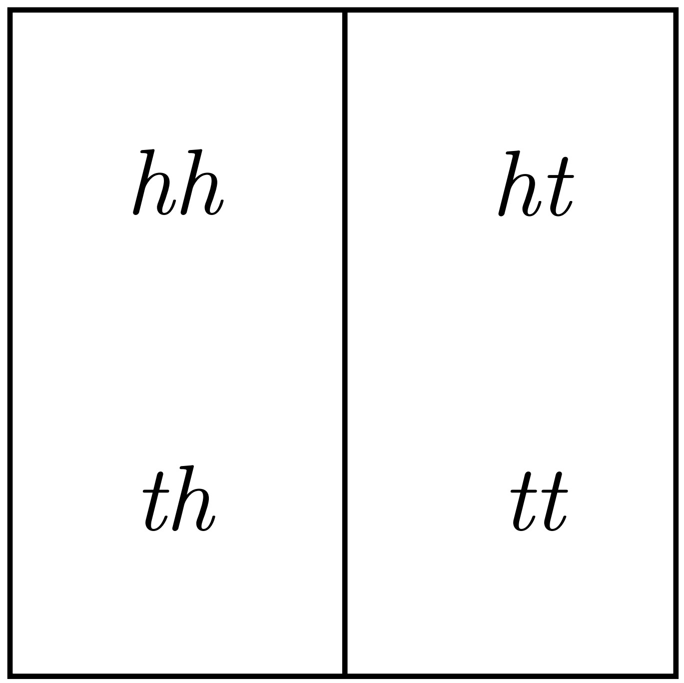
Defines $\mathbf{P}_i$!

Simple epistemic game

$\mathcal{P}_1$

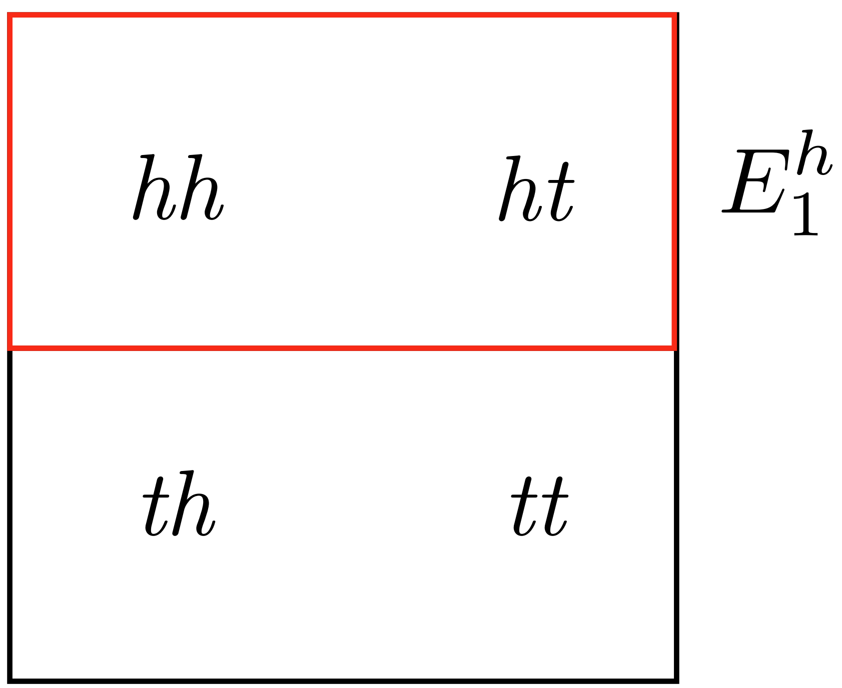


$\mathcal{P}_2$



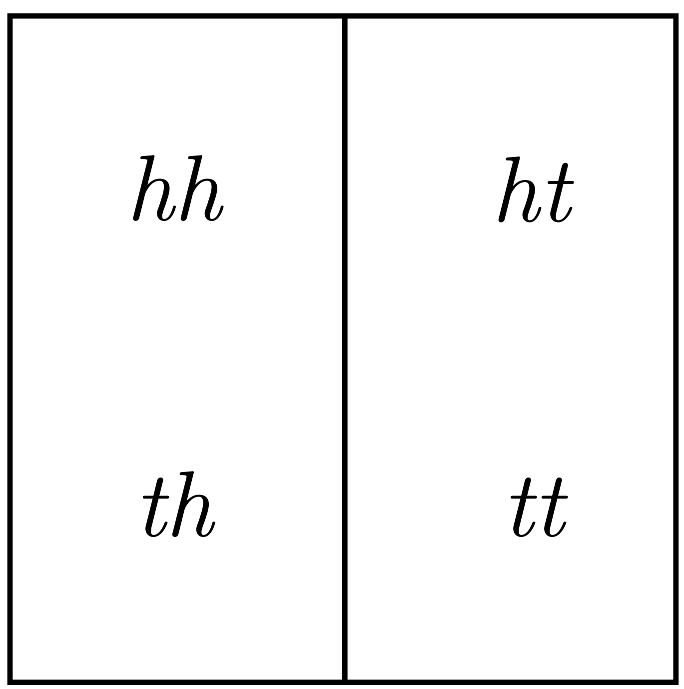
Simple epistemic game

$\mathcal{P}_1$



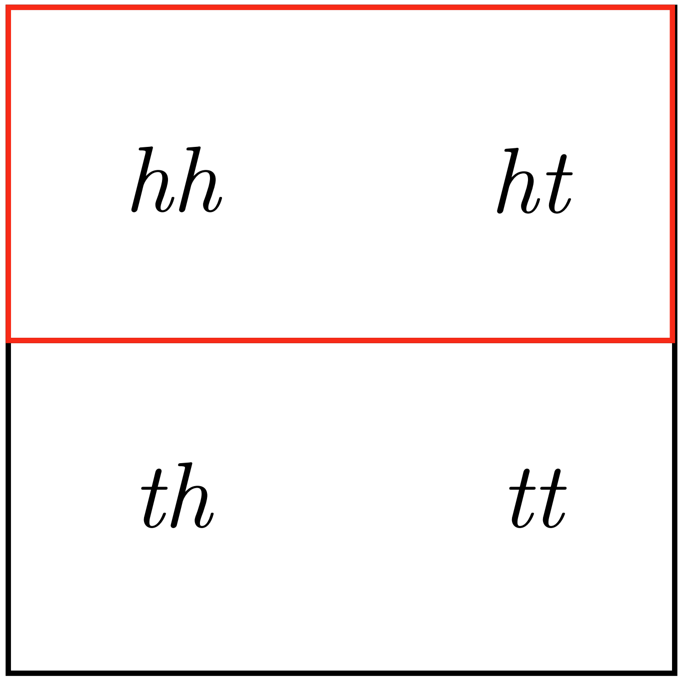
Event E_i^x : "Player i chooses x"

$\mathcal{P}_2$



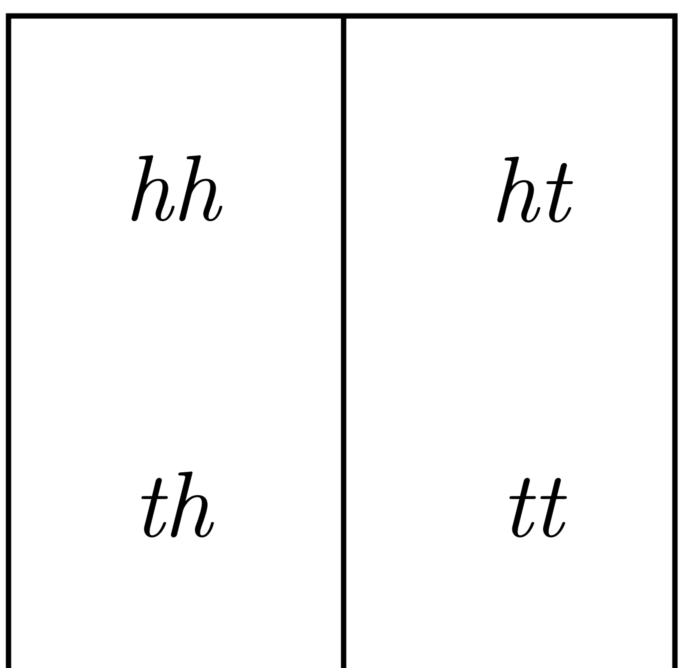
Simple epistemic game

$\mathcal{P}_1$



E_1^h

$\mathcal{P}_2$



Event E_i^x : "Player i chooses x"

- $\mathbf{P}_1 hh = \mathbf{P}_1 ht = \{hh,\, ht\}$
- Player 1 knows E_1^h

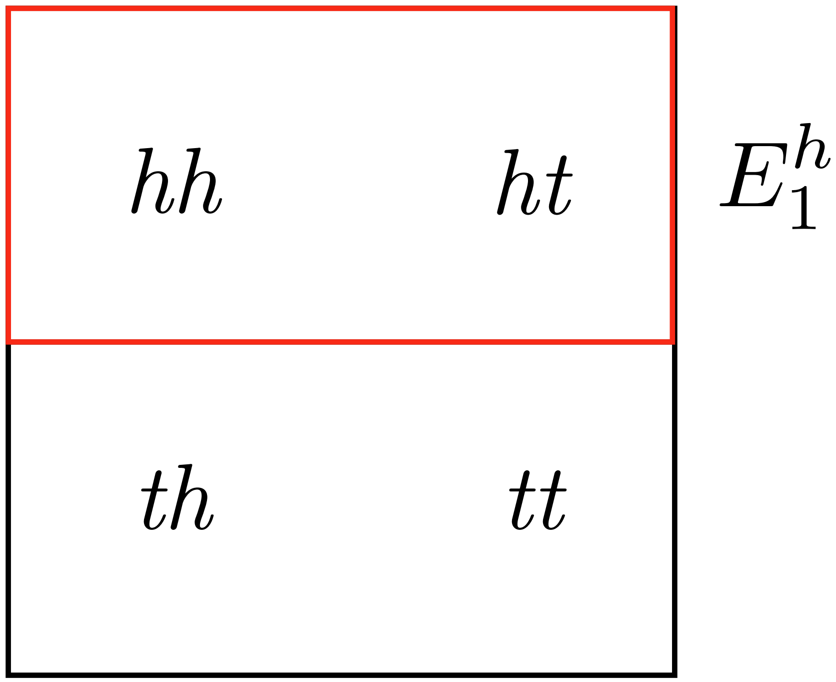
$$\mathbf{P}_{1hh} = \mathbf{P}_{1ht} \subseteq E_1^h$$

$$E_1^h = \mathbf{K}_1 E_1^h$$

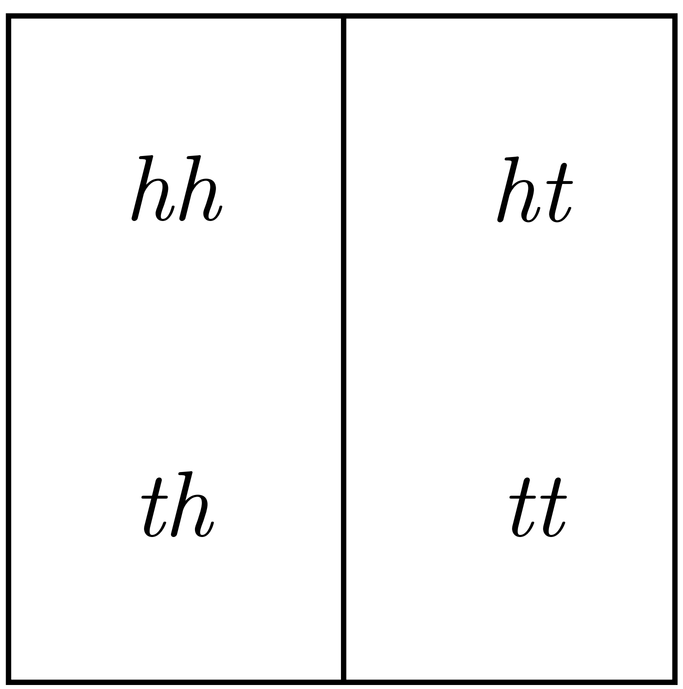
$$\mathbf{K}_1 E_1^h = \{\omega \in \Omega : \mathbf{P}_i \omega \subseteq E_1^h\}$$

Simple epistemic game

$\mathcal{P}_1$



$\mathcal{P}_2$



Event E_i^x : "Player i chooses x"

- $\mathbf{P}_1 hh = \mathbf{P}_1 ht = \{hh,\ ht\}$
- Player 1 knows E_1^h

$$\mathbf{P}_{1hh} = \mathbf{P}_{1ht} \subseteq E_1^h$$

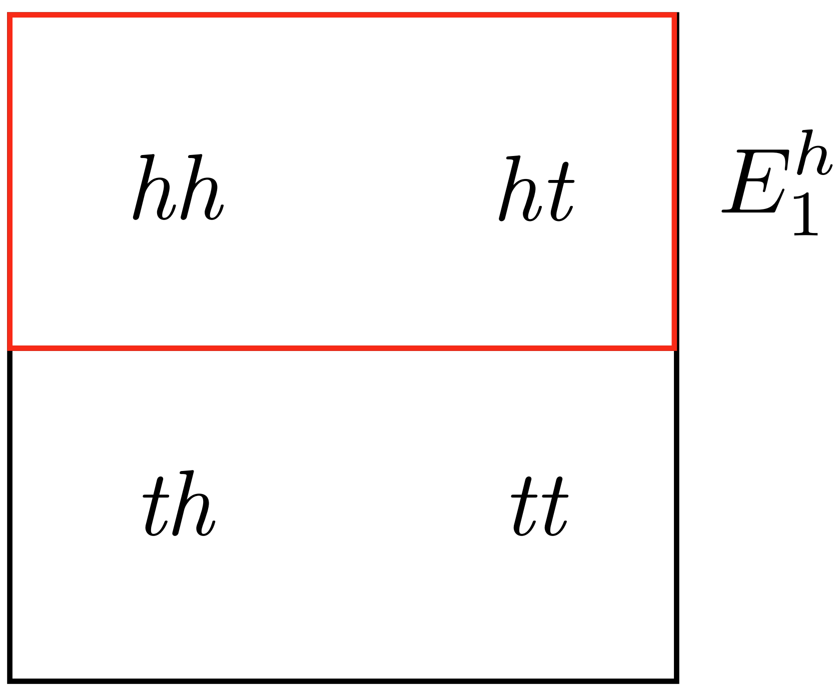
$$E_1^h = \mathbf{K}_1 E_1^h$$

$$\mathbf{K}_1 E_1^h = \{\omega \in \Omega : \mathbf{P}_i \omega \subseteq E_1^h\}$$

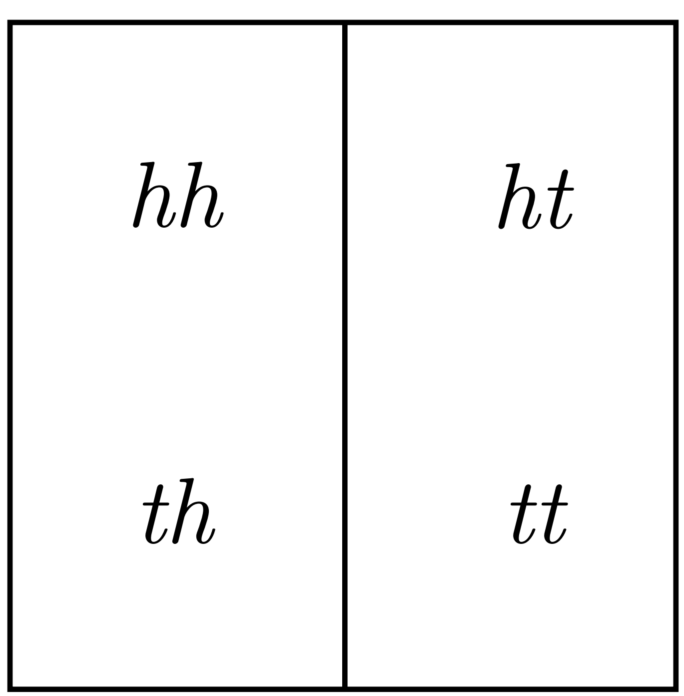
- Similarly, $\mathbf{K}_1 E_1^t, \mathbf{K}_2 E_2^h, \mathbf{K}_2 E_2^t$

Simple epistemic game

$\mathcal{P}_1$



$\mathcal{P}_2$



Event E_i^x : "Player i chooses x"

- $\mathbf{P}_1 hh = \mathbf{P}_1 ht = \{hh,\ ht\}$
- Player 1 knows E_1^h

$$\mathbf{P}_{1hh} = \mathbf{P}_{1ht} \subseteq E_1^h$$

$$E_1^h = \mathbf{K}_1 E_1^h$$

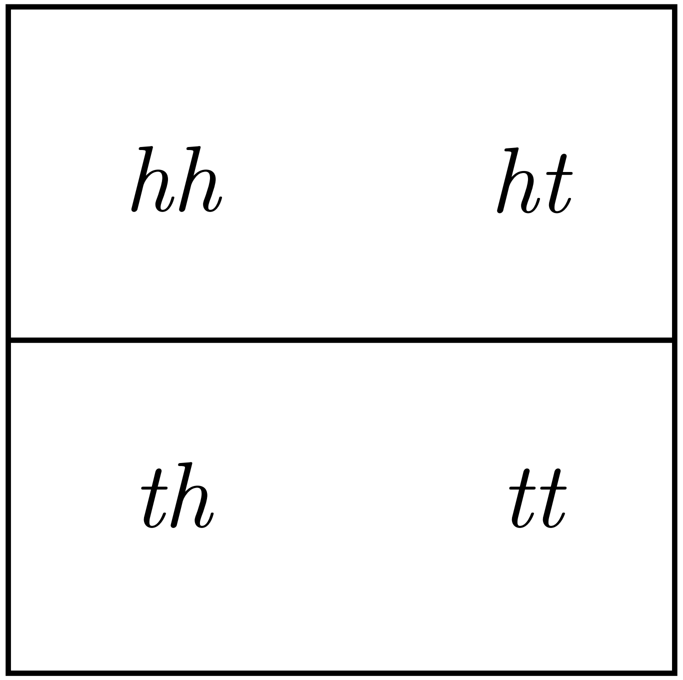
$$\mathbf{K}_1 E_1^h = \{\omega \in \Omega : \mathbf{P}_i \omega \subseteq E_1^h\}$$

- Similarly, $\mathbf{K}_1 E_1^t, \mathbf{K}_2 E_2^h, \mathbf{K}_2 E_2^t$

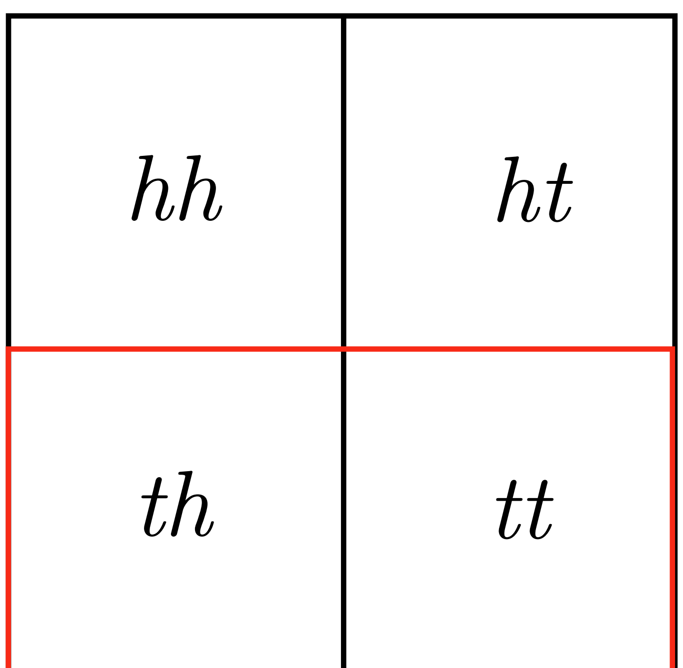
Defines $\mathbf{K}_i$!

Simple epistemic game

$\mathcal{P}_1$



$\mathcal{P}_2$



E_1^t

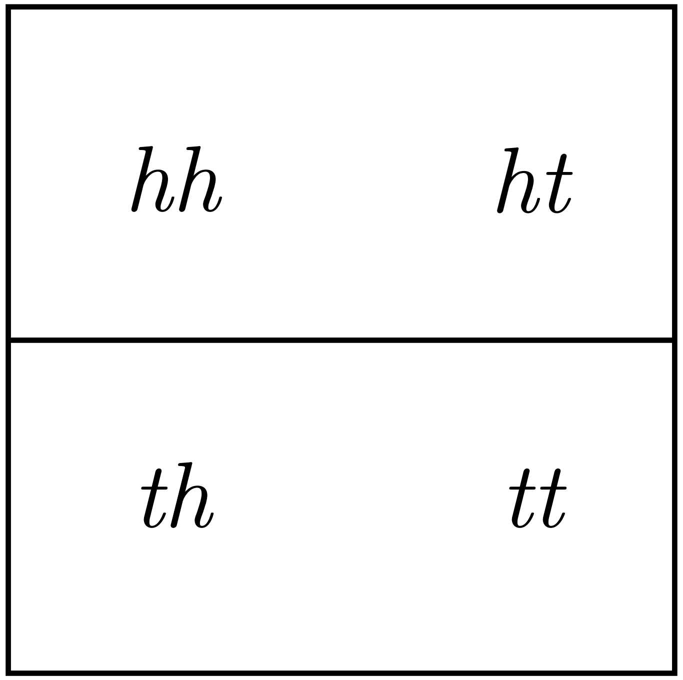
Event E_i^x : "Player i chooses x"

- $\mathbf{P}_1 hh = \mathbf{P}_1 ht = \{hh, ht\}$
- Player 2 does not know E_1^t

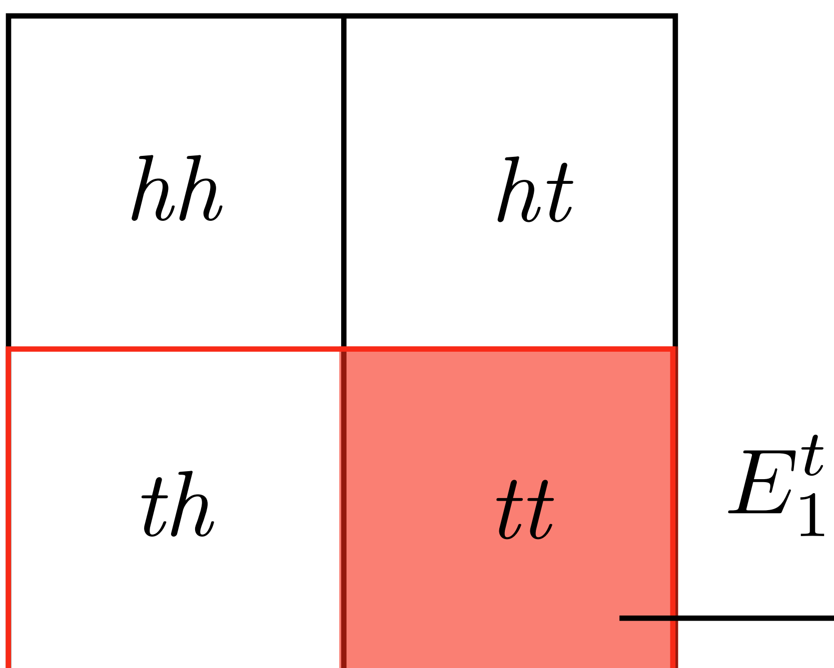
$$\mathbf{P}_{2hh} \not\subseteq E_1^h$$

Simple epistemic game

$\mathcal{P}_1$



$\mathcal{P}_2$



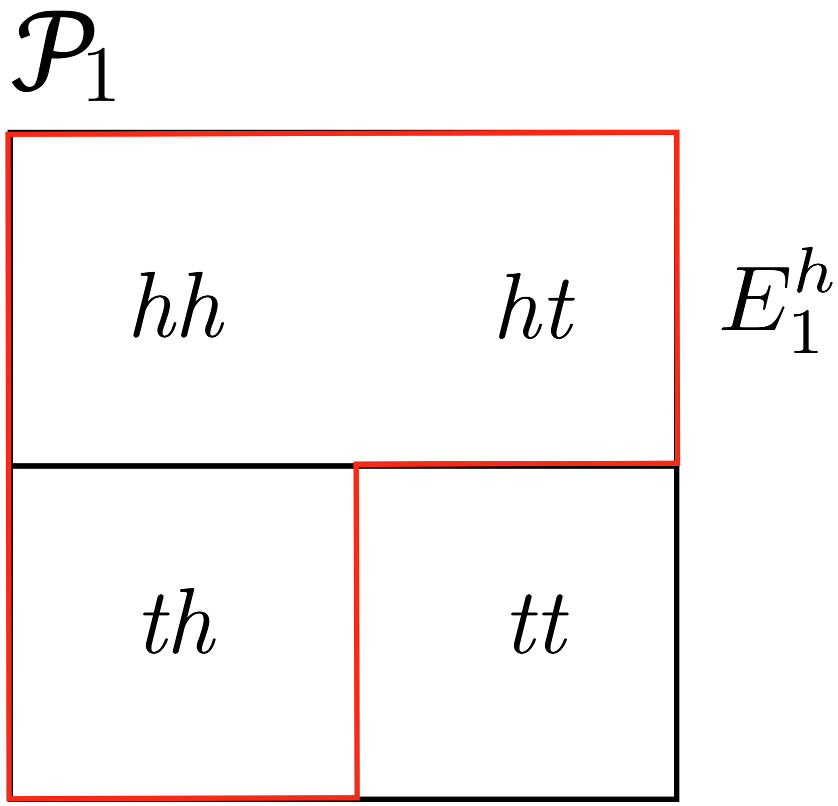
Event E_i^x : "Player i chooses x"

- $\mathbf{P}_1 hh = \mathbf{P}_1 ht = \{hh,\ ht\}$
- Player 2 does not know E_1^t

$$\mathbf{P}_{2hh} \nsubseteq E_1^h$$

Player 1 does not consider these states possible!

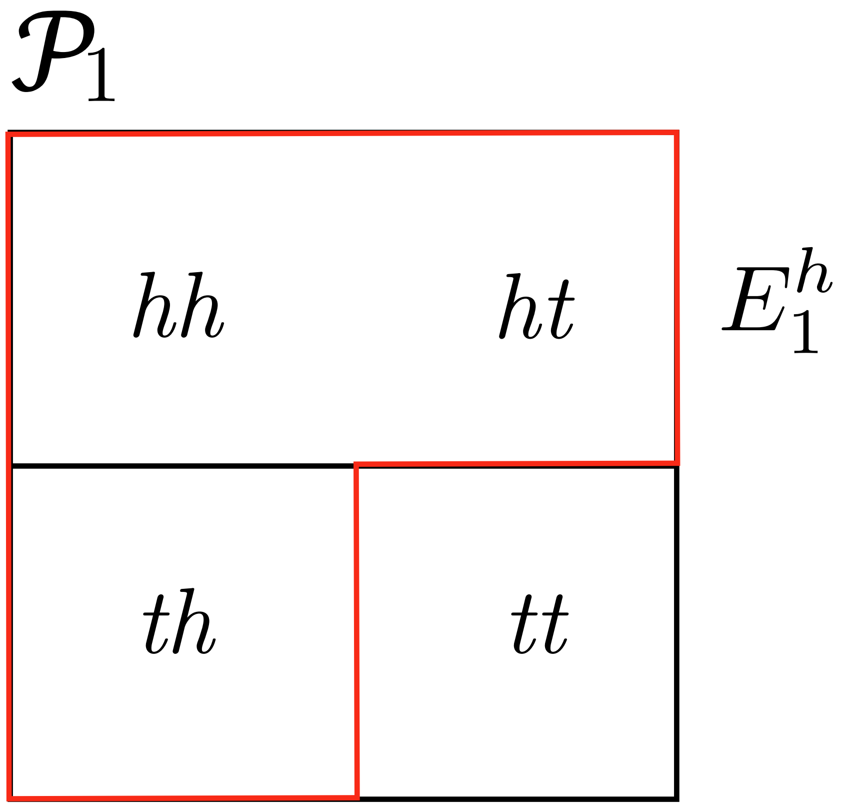
Simple epistemic game



Event E_i^x : "One of them chooses x"

Does player 1 know E_1^h?

Simple epistemic game



Event E_i^x : "One of them chooses x"

Does player 1 know E_1^h?

$$\mathbf{P}_{1th} \not\subseteq E_1^h$$

Today

- An epistemic battle of the sexes

Then

- Rationalizable strategies
- Common knowledge of rationality
- Extensive form Rationalizability