

Lecture 6

- human behavior -

Review

- Mixed strategies
- Nash equilibrium
 - formal definition
 - existence

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 - formal definition
 - existence
- The fundamental theorem
 - properties
 - finding mixed-strategy equilibria

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- Mixed strategies
- Nash equilibrium
 - formal definition
 - existence
- The fundamental theorem
 - properties
 - finding mixed-strategy equilibria
- Paradigmatic games
 - capture principles
 - how might dilemmas be resolved?

Mixed strategies

- A player has pure strategies $s_1, \dots, s_k$
- Probability distribution over $s_1, \dots, s_k$:

$$\sigma_i = p_1 s_1 + \dots + p_k s_k$$

$$\sum_{i=1}^n p_i = 1$$

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- The probability that player i chooses s_i is p_i
- **Pure strategy:** If all p_i are zero except one
- **Strictly mixed:** If it is not a pure strategy
- **Completely mixed:** If $p_i > 0$ for all i

Nash Equilibrium of an n-player game

$$\sigma^* = (\sigma_1^*, \dots, \sigma_n^*)$$

$$\pi_i(\sigma^*) \geq \pi_i(\sigma_{-i}^*, \sigma_i)$$

$$(\sigma_{-i}^*, \sigma_i) = \begin{cases} (\tau_1, \sigma_2, \dots, \sigma_n) & \text{if } i = 1 \\ (\sigma_1, \dots, \sigma_{i-1}, \tau_i, \sigma_{i+1}, \dots, \sigma_n) & \text{if } 1 < i < n \\ (\tau_1, \dots, \sigma_{n-1}, \tau_n) & \text{if } i = n \end{cases}$$

Nash Equilibrium of an n-player game

- A profile of mixed-strategies $\sigma^* = (\sigma_1^*, \dots, \sigma_n^*)$, such that for every player $i = 1, \dots, n$, and every $\sigma_i \in \Delta S_i$

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- No player can do better given the choices of others
- No player has an incentive to deviate from its choice

Nash existence theorem

Theorem 2.1 - If each player in an n -player game has a finite number of pure strategies, then the game has a (not necessarily unique) Nash equilibrium in (possibly) mixed strategies

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- Proves the existence of equilibrium in a very general class of games
- Theory is useful!
- Paved the way for applications

Fundamental theorem of game theory

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$$\sigma_i = \dots + (p + p')s + \dots$$

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$$\pi_i(\sigma_{-i}^*, \sigma_i) > \pi_i(\sigma^*)$$

Solving for mixed-strategy Nash

		Player 2	
		y_1	y_2
Player 1	x_1	a_1, a_2	b_1, b_2
	x_2	c_1, c_2	d_1, d_2

- Mixed-strategy for P_2 : $\sigma_2 = \alpha y_1 + (1-\alpha) y_2$
 - Payoff to x_1 against σ_2 : $\alpha a_1 + (1-\alpha) b_1$
 - Payoff to x_2 against σ_2 : $\alpha c_1 + (1-\alpha) d_1$
- $\left. \begin{array}{l} \alpha a_1 + (1-\alpha) b_1 \\ \alpha c_1 + (1-\alpha) d_1 \end{array} \right\} P_1$

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$$\alpha = \frac{d_1 - b_1}{d_1 - b_1 + a_1 - c_1}$$

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Requirement that P_2 strategies (y_1 and y_2) be equal!

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- For a 2×2 game: 8 more possible!
- Which ones?

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- Then $\sigma = (\sigma_1, \sigma_2)$ is a Nash
- For a 2×2 game: 8 more possible!
- Which ones?
- Total of 9 possible configurations
- How about for an $n \times m$ game?

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Paradigmatic games

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- Prisoner's dilemma
 - better off not choosing dominant strategies

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2, 0	0, 0

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- Battle of the sexes
 - rather together than apart, but different preferences

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 - opposite preferences

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1, -1	-1, 1
-1, 1	1, -1

Paradigmatic games

- Prisoner's dilemma
 - better off not choosing dominant strategies
- Battle of the sexes
 - rather together than apart, but different preferences
- Matching pennies
 - opposite preferences
- Chicken
 - first player gains advantage

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Today

- Game theory and human behavior
- Conditions for altruism
 - the ultimatum game
- Norms of cooperation
 - the public goods game
- Implications to policy-making

Decision theory

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General framework:

- Patterns in individual behavior

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Key questions:

- Consistency in preference ordering?
- Consistency over time?
- Payoffs for individual decisions?

Strategic games

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General framework:

- Anybody's strategy choice affects everybody's payoff

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Key questions:

- how do individual take into account the behavior of others?

Behavioral game theory

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General framework:

- Design laboratory experiments
- Discuss external validation of experiments
- Evaluate conceptual, procedural, and analytical tools to capture social interaction

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- What are the payoffs?

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- Discuss external validation of experiments
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Key questions:

- What are the rules?
- How does information flow?
- What are the payoffs?
- Replicated in different laboratories?

Behavioral game theory

- Individuals subject to strategic interaction
- Various game settings
 - diverse payoffs
 - information conditions
 - decision constraints
- Consistence in preference in games
+ performance error?

Behavioral game theory

- Individuals subject to strategic interaction
 - Various game settings
 - diverse payoffs
 - information conditions
 - decision constraints
 - Consistence in preference in games + performance error?
- deduce underlying preferences in behavior?

Can experiments offer new
insights?

Self-regarding v. other-regarding

- Maximizes his own payoff
- Care about other behavior and payoff of other players
insofar as their behavior impacts my own payoff
- The ultimatum game
- The public goods game

The ultimatum game

The ultimatum game

- Two players are given \$10
- P_A may offer P_B \$1-10
- If P_B rejects both receive nothing. If not..
- Best response self-regarding P_B : **Accept!**
- A self-regarding P_A would offer?

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- Experiments:
 - P_A frequently offers 50%
 - P_B frequently reject offers below 30%
- Across different cultures and varying amounts of money?

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- Across different cultures and varying amounts of money?
- **Strong reciprocity + inequality aversion!**

PERCENTAGE OF OFFERS BELOW 0.2 AND BETWEEN 0.4 AND 0.5
IN THE ULTIMATUM GAME

Study (Payment method)	Number of observations	Stake size (country)	Percentage of offers with $s < 0.2$	Percentage of offers with $0.4 \leq s \leq 0.5$
Cameron [1995] (All Ss Paid)	35	Rp 40.000 (Indonesia)	0	66
Cameron [1995] (all Ss paid)	37	Rp 200.000 (Indonesia)	5	57
FHSS [1994] (all Ss paid)	67	\$5 and \$10 (USA)	0	82
Güth et al. [1982] (all Ss paid)	79	DM 4–10 (Germany)	8	61
Hoffman, McCabe, and Smith [1996] (All Ss paid)	24	\$10 (USA)	0	83
Hoffman, McCabe, and Smith [1996] (all Ss paid)	27	\$100 (USA)	4	74
Kahneman, Knetsch, and Thaler [1986] (20% of Ss paid)	115	\$10 (USA)	?	75 ^a
Roth et al. [1991] (random pay- ment method)	116 ^b	approx. \$10 (USA, Slovenia, Israel, Japan)	3	70
Slonim and Roth [1997] (random pay- ment method)	240 ^c	SK 60 (Slovakia)	0.4 ^d	75
Slonim and Roth [1997] (random pay- ment method)	250 ^c	SK 1500 (Slovakia)	8 ^d	69
Aggregate result of all studies ^e	875		3.8	71

a. percentage of equal splits, b. only observations of the final period, c. observations of all ten periods,
d. percentage of offers below 0.25, e. without Kahneman, Knetsch, and Thaler [1986].

Altruistic punishment

- Strong reciprocity
 - harm for harm
- Inequality aversion
 - weak urge to reduce inequality when on top
 - strong urge to reduce inequality when on the bottom
- Model of inequality aversion
 - the ultimatum game
 - the public goods game

Objections to experiments/data

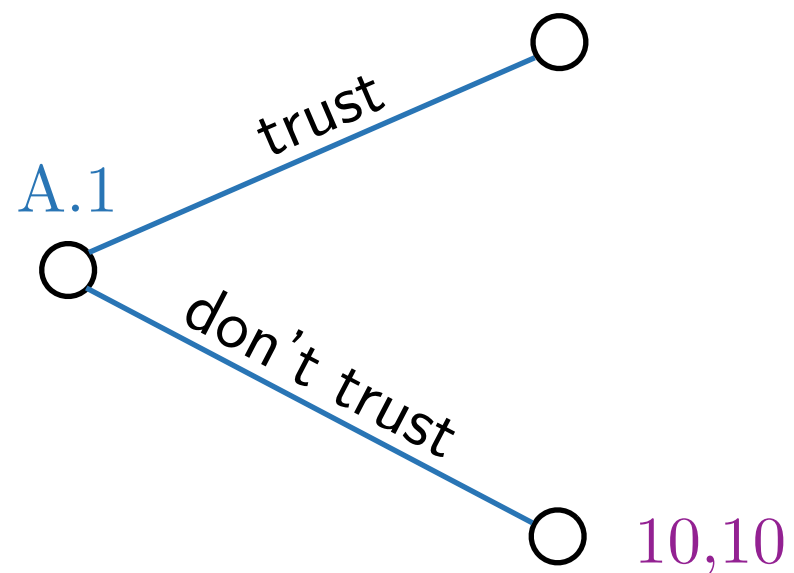
- Controlled circumstance (external validity?)
- Games are unusual (how to behave?)
 - real interactions are repeated rather than one-shot
 - among acquaintances rather than being anonymous
 - role of social contact networks?

The Neural Basis of Altruistic Punishment

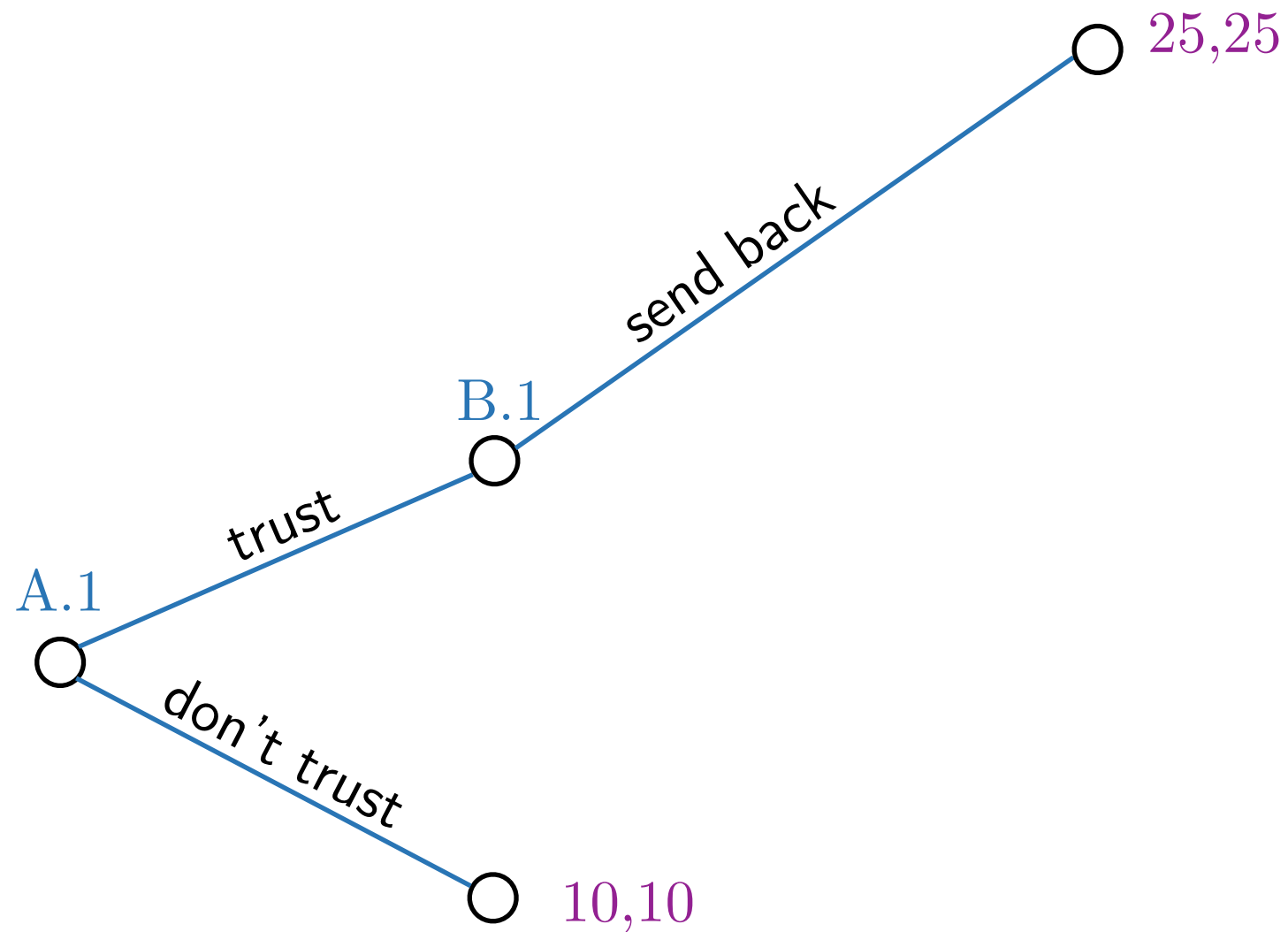
**Dominique J.-F. de Quervain,^{1*†} Urs Fischbacher,^{2*}
Valerie Treyer,³ Melanie Schellhammer,² Ulrich Schnyder,⁴
Alfred Buck,³ Ernst Fehr^{2,5†}**

Many people voluntarily incur costs to punish violations of social norms. Evolutionary models and empirical evidence indicate that such altruistic punishment has been a decisive force in the evolution of human cooperation. We used $H_2^{15}O$ positron emission tomography to examine the neural basis for altruistic punishment of defectors in an economic exchange. Subjects could punish defection either symbolically or effectively. Symbolic punishment did not reduce the defector's economic payoff, whereas effective punishment did reduce the payoff. We scanned the subjects' brains while they learned about the defector's abuse of trust and determined the punishment. Effective punishment, as compared with symbolic punishment, activated the dorsal striatum, which has been implicated in the processing of rewards that accrue as a result of goal-directed actions. Moreover, subjects with stronger activations in the dorsal striatum were willing to incur greater costs in order to punish. Our findings support the hypothesis that people derive satisfaction from punishing norm violations and that the activation in the dorsal striatum reflects the anticipated satisfaction from punishing defectors.

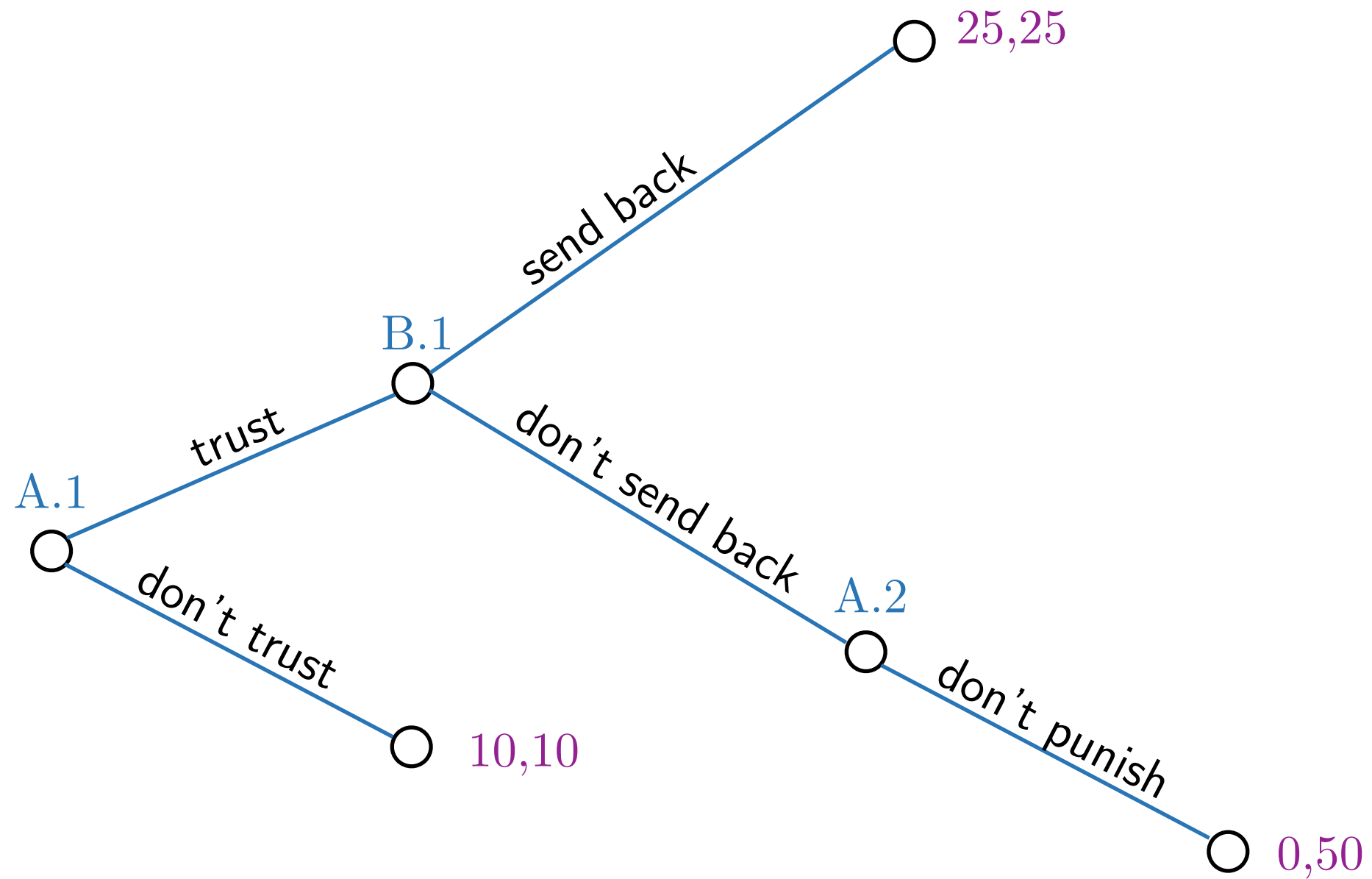
The trust game



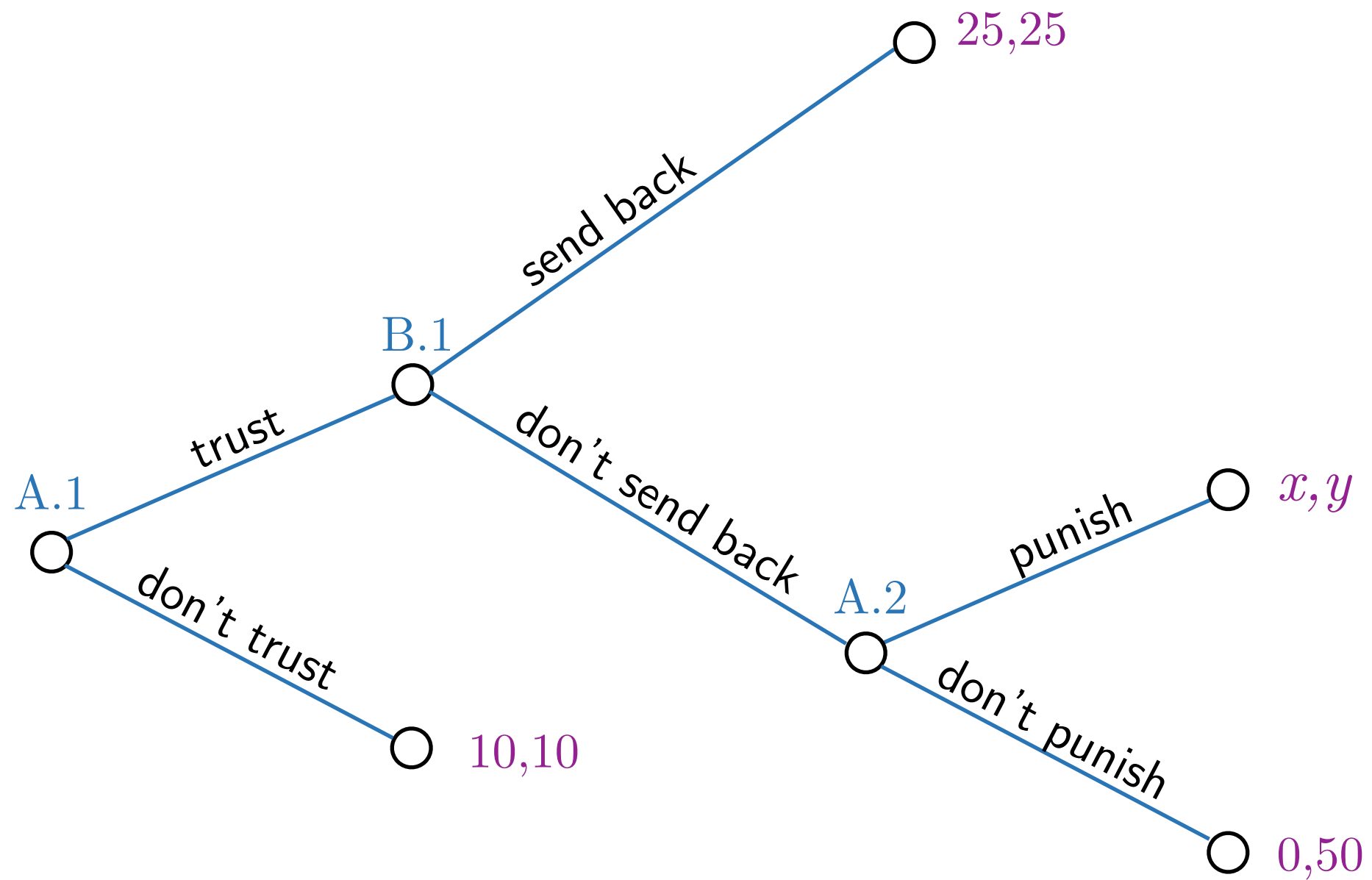
The trust game



The trust game



The trust game



Payoffs

Cost of punishment $p > 0$ (determined by P_A)

- IC - intentional and costly, $x = -p$, $y = 50 - 2p$
- IF - intentional and free, $x = 0$, $y = 50 - 2p$
- IS - intentional and symbolic, $x = 0$, $y = 50$
- NC - non-intentional and costly, $x = -p$, $y = 50 - 2p$

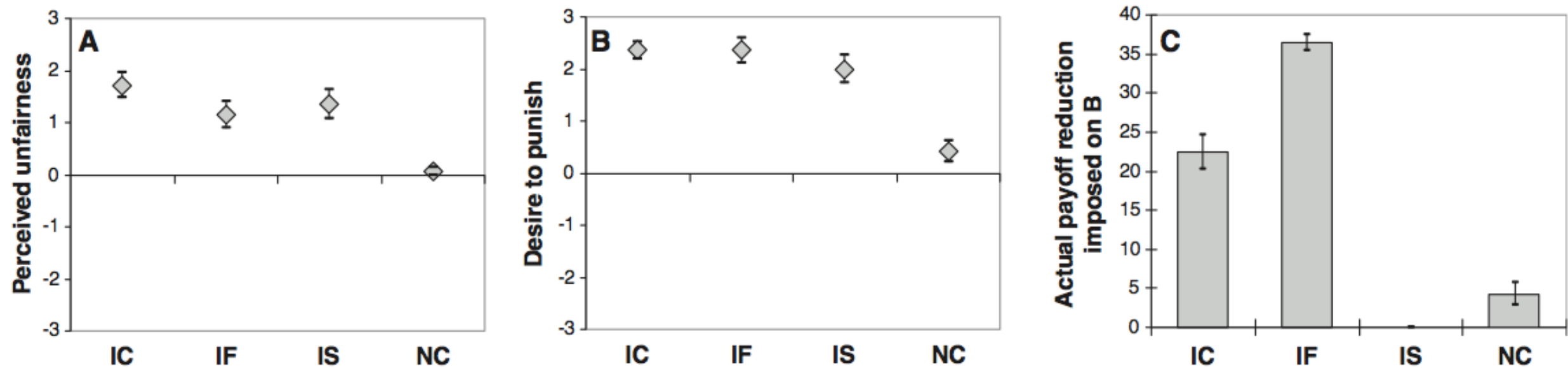
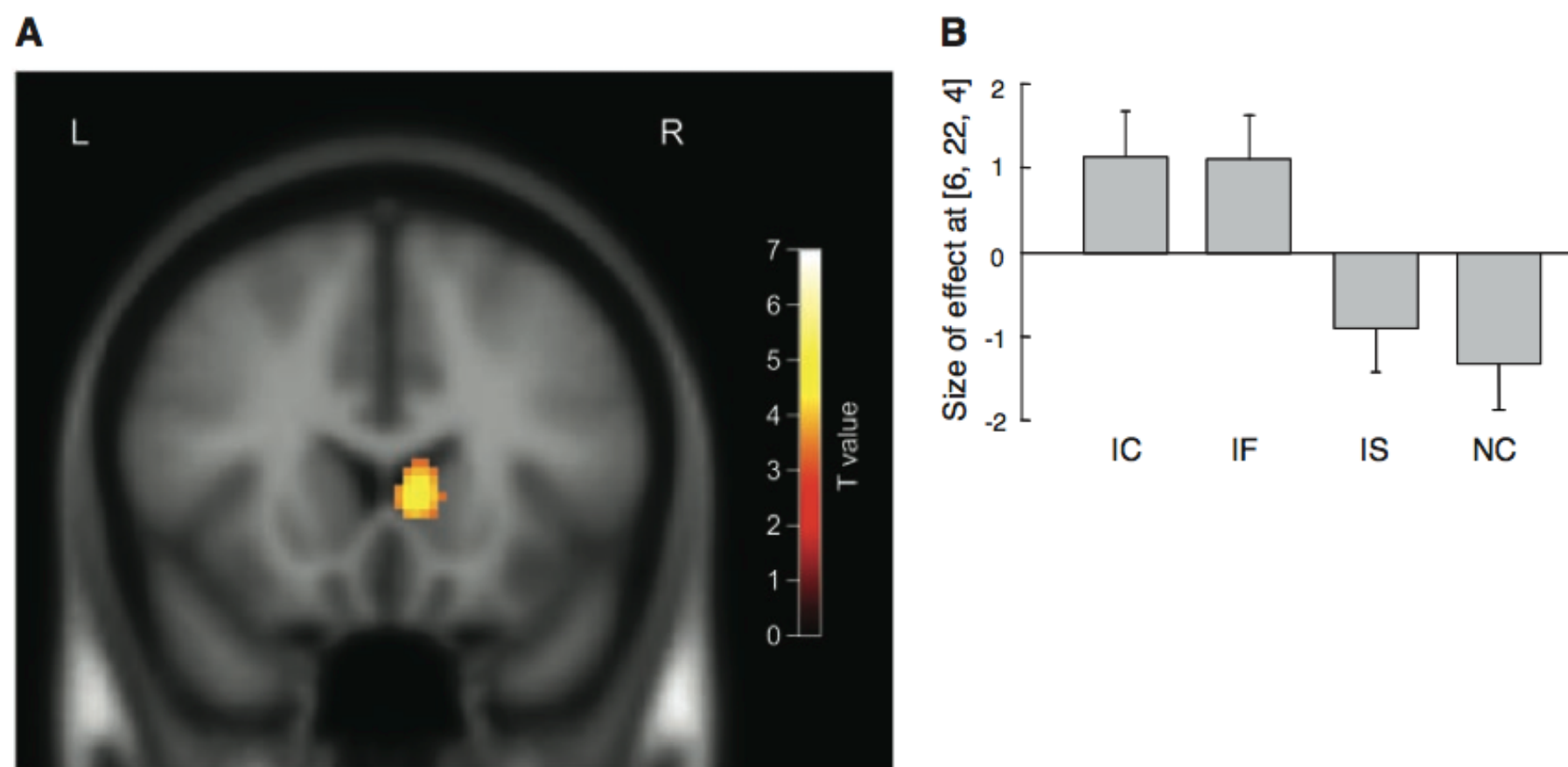


Fig. 1. Player A's feelings about player B and actual payoff reduction imposed on B. **(A)** Player A's perceived unfairness if B kept all the money. During the 10-min interval between PET scans, player A indicated on a seven-point Likert scale (from -3 to +3) whether he perceived B's action in the previous trial as fair or unfair. Maximal fairness is indicated by -3, maximal unfairness by +3. The figure shows the mean perception across subjects $\pm$ SE. **(B)** Player A's desire to punish B if the latter kept all the

money. During the 10-min interval between PET scans, A indicated on a seven-point Likert scale (from -3 to +3) the strength of his desire to reward or to punish B. The maximal desire to reward is indicated by -3, the maximal desire to punish by +3. We show the mean desire to reward/punish $\pm$ SE. **(C)** Actual payoff reduction imposed on B if the latter keeps all the money. The figure shows the mean payoff reduction A imposed on B $\pm$ SE. In the IS condition, the economic payoff of B could not be reduced.

Does punishment activate
reward-related brain circuits?

Fig. 2. (A) Activation in the caudate nucleus in conditions in which subjects indicated a strong desire to punish and could effectively do so (IC and IF) relative to conditions in which there is no effective punishment or the desire to punish is absent (IS and NC). **(B)** Effect sizes at the peak of blood-flow increase in the caudate nucleus. Bars indicate caudate activity in each condition relative to the mean brain activation $\pm$ SD.



Stronger caudate activation
punish more strongly?

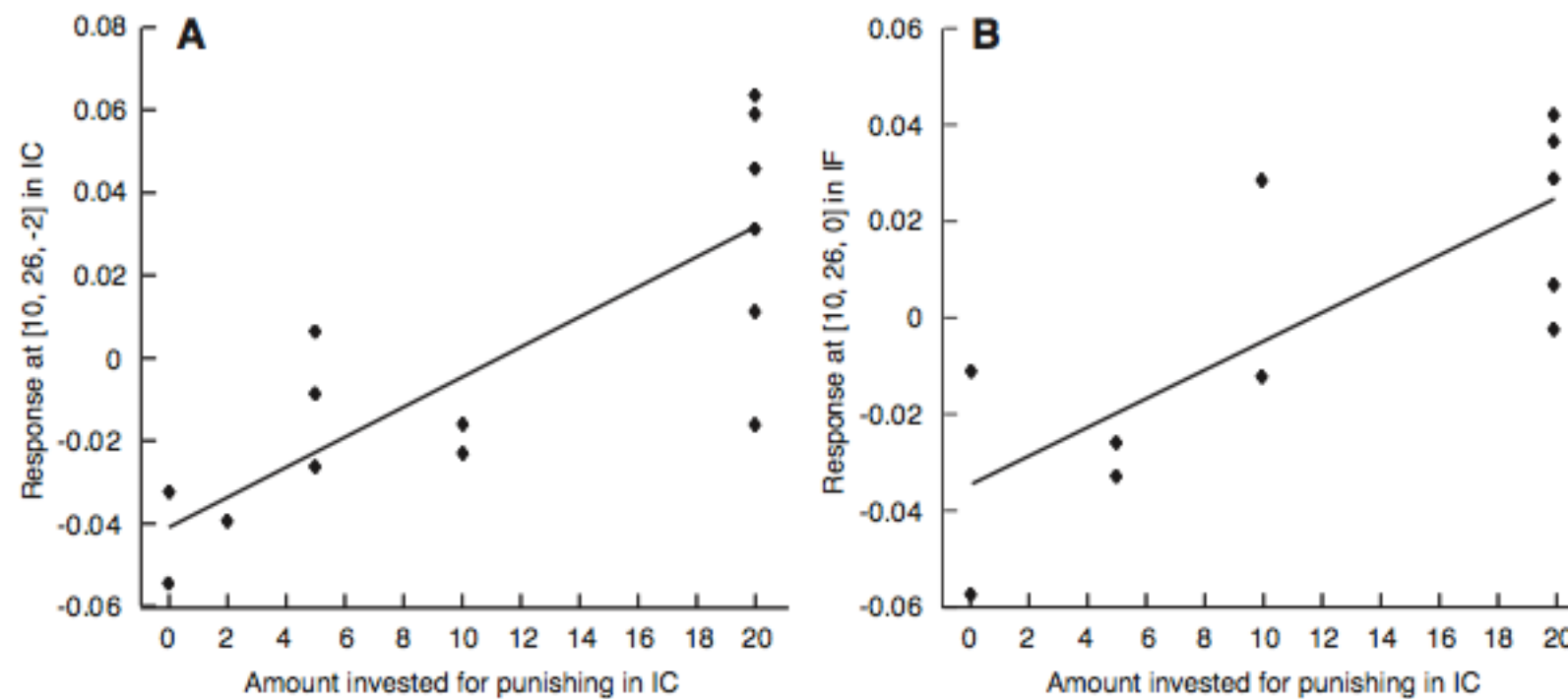
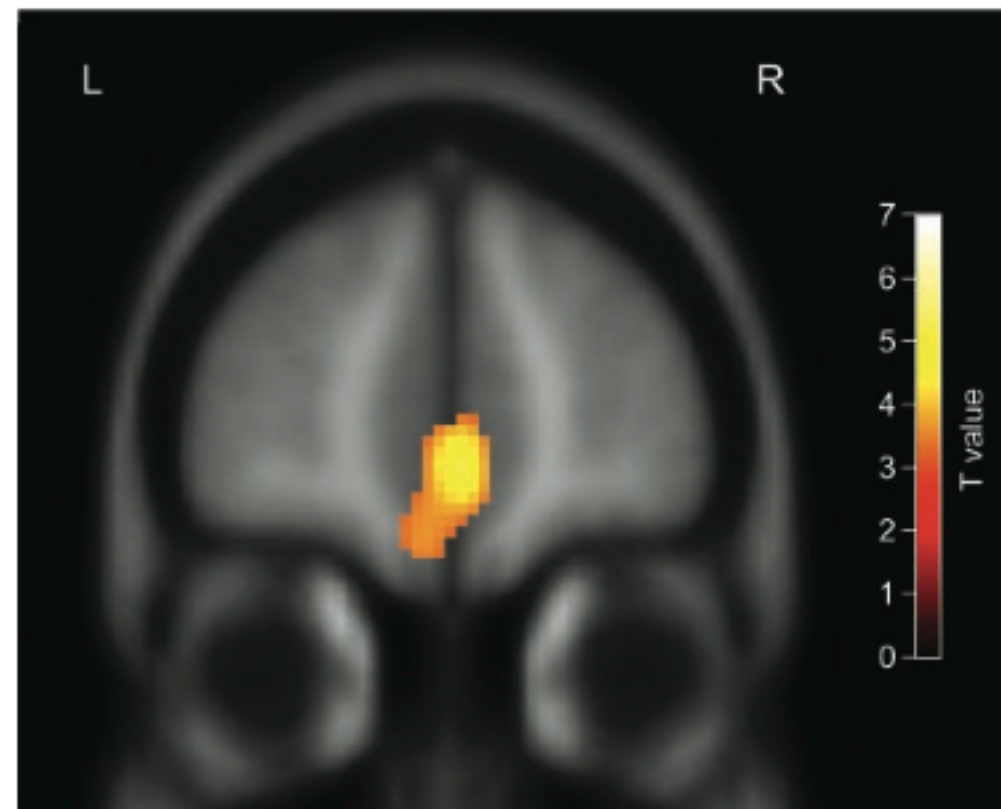


Fig. 3. (A) Positive correlation between caudate activation at coordinates [10, 26, -2] and the amount of money spent on punishment in the IC condition. Subjects with higher caudate activation in the IC condition spent more money on punishment in this condition. (B) Positive correlation between caudate activation at coordinates [10, 26, 0] in the IF condition in those subjects that punished maximally and the amount of money spent on punishment by these subjects in the IC condition. Subjects with higher caudate activation at the same (maximal) level of punishment in the IF condition spent more money on punishment in the IC condition.

Fig. 4. The role of the prefrontal cortex in integrating the benefits and costs of punishing. Activation of the ventromedial prefrontal cortex and the medial orbitofrontal cortex in the condition where subjects have a strong desire to sanction but where sanctioning is costly for the punisher (IC) relative to the condition where there is also a strong desire to sanction but sanctioning is costless for the punisher (IF).



Implications for policy making

Experimental game theory and policy making

Experimental game theory and policy making

- Libertarian v. paternalism
- Cost-benefit analysis (externalities?)
- High consumer information costs

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- E.g., trans fats (NYC, California)
 - ban products?
 - require providing more information?
 - consumer informs himself?

Experimental game theory and policy making

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- E.g., trans fats (NYC, California)
 - ban products?
 - require providing more information?
 - consumer informs himself?
- Can experiments evaluate cost/externalities?
- Increase confidence what informed consumers would do?
(if they knew the facts?)

Class Projects

- Project 1 (paper review, report + presentation)
 - choosing a topic of your interest
 - picking 2-3 papers in the area
 - understand, evaluate, extend research
- Project 2 (experiment, report + presentation)
 - game-theoretic framework
 - contrast with actual data
 - measure externalities?

Next class

- Epistemic games
- Incorporate beliefs into games
- An epistemic battle of the sexes