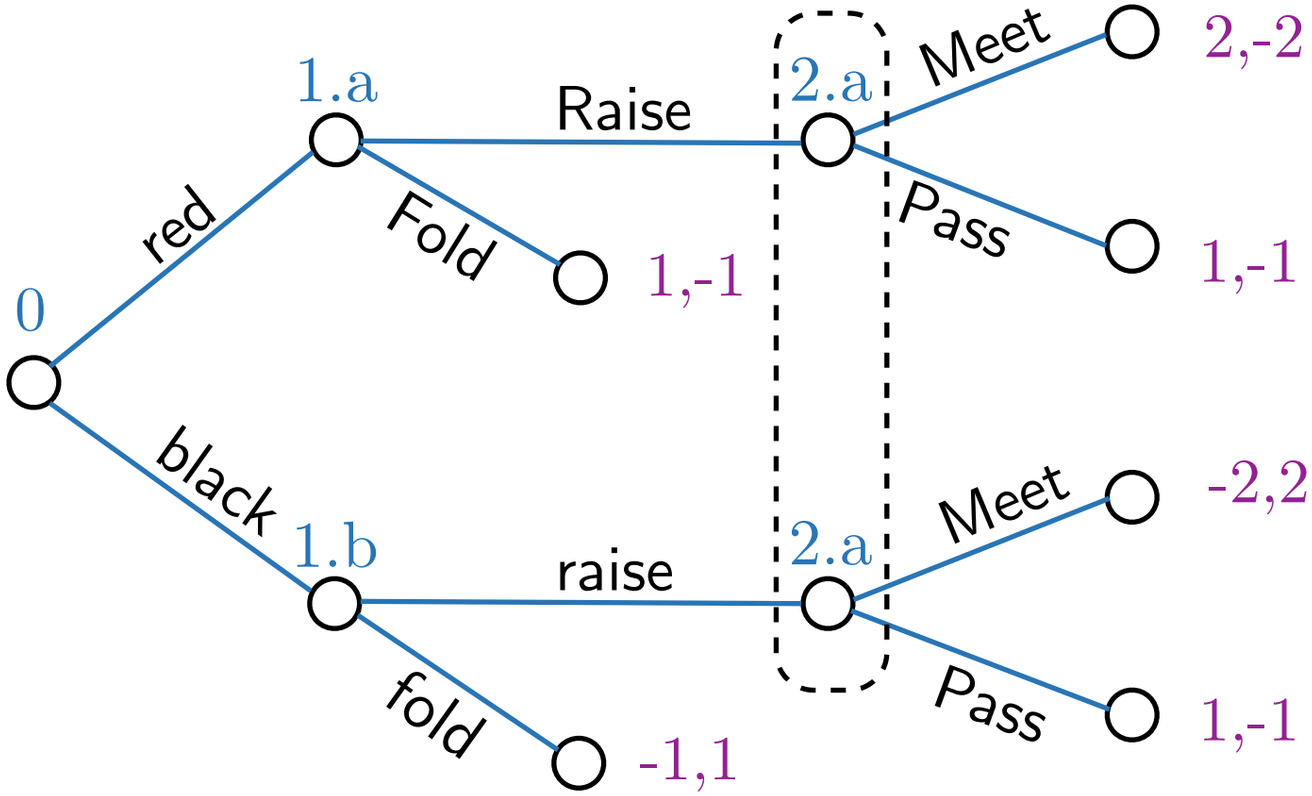
Lecture 5
- the fundamental theorem -

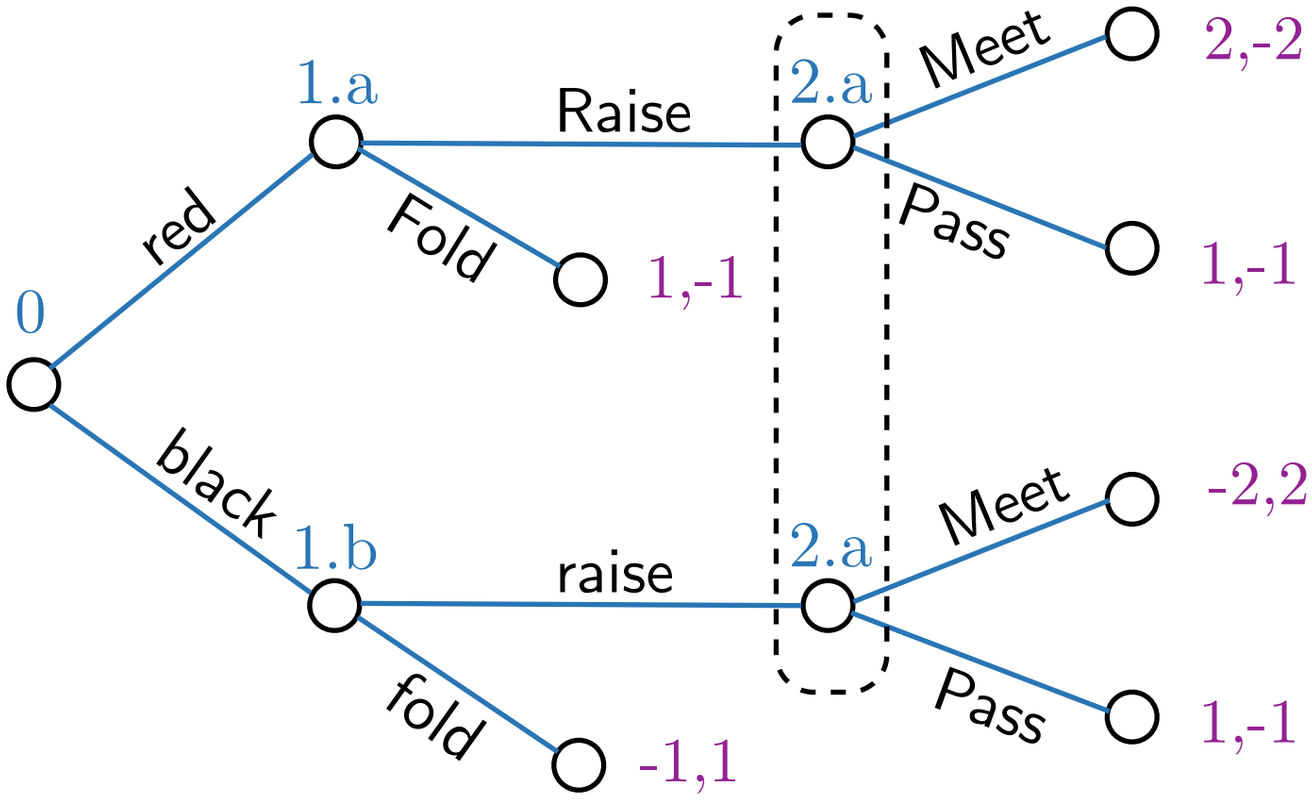
Review

- Representing a game
  - extensive form (strategies ≠ choices)



Review

- Representing a game
  - extensive form (strategies ≠ choices)
  - normal form (strategies → payoffs)



P_1 \ P_2	M	P
Rr	0,0	1,-1
Rf	0.5,-0.5	0,0
Fr	-0.5,0.5	1,-1
Ff	0,0	0,0

Review

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Extensive form

- set of players
- game tree
  - information set
  - choices
  - moves by nature
- set of payoffs
  - expected payoffs

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Extensive form

- set of players
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  - information set
  - choices
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- set of payoffs
  - expected payoffs

Normal form

- set of players
- set of strategies
  - for player i

    $s_1 = \mathrm{Rf} \in \mathrm{S}_1$

    $s_2 = \mathrm{M} \in \mathrm{S}_2$
  - game profile

    $s = (s_1,\ldots,s_n) \in \mathrm{S}$

    $s = (\mathrm{Rf},\mathrm{M})$
- lottery for player i

  $\pi_i\colon \mathrm{S} \to \mathbb{R}$

Review

- Solution concepts
  - strictly dominated strategy
  - iterated dominant strategy
  - Nash Equilibrium

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  - iterated dominant strategy
  - Nash Equilibrium

general rules: how rational parties are likely to behave

Example

- Lender lends $100
- Debtor can earn $110 in one year
- Is this a likely debtor default?
- Split profit and payback $105?

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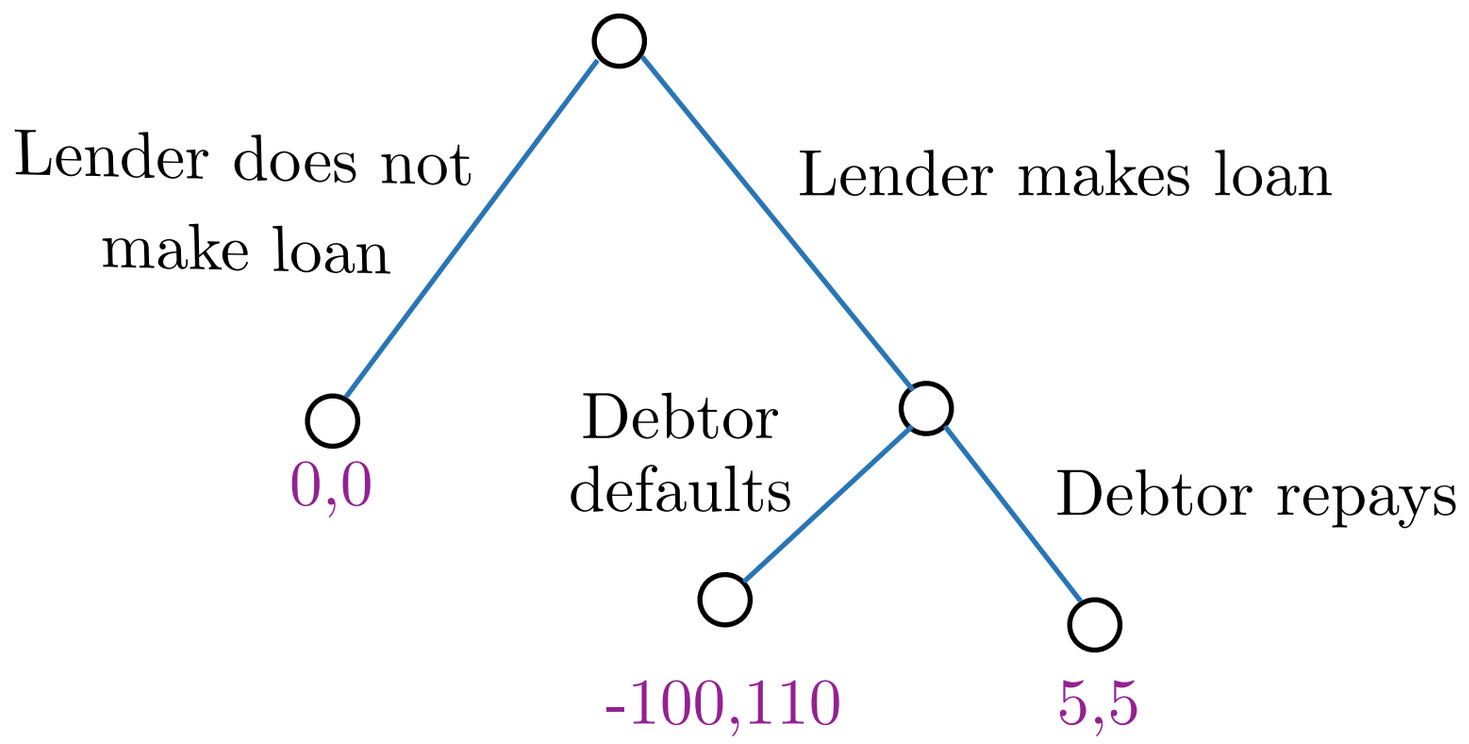
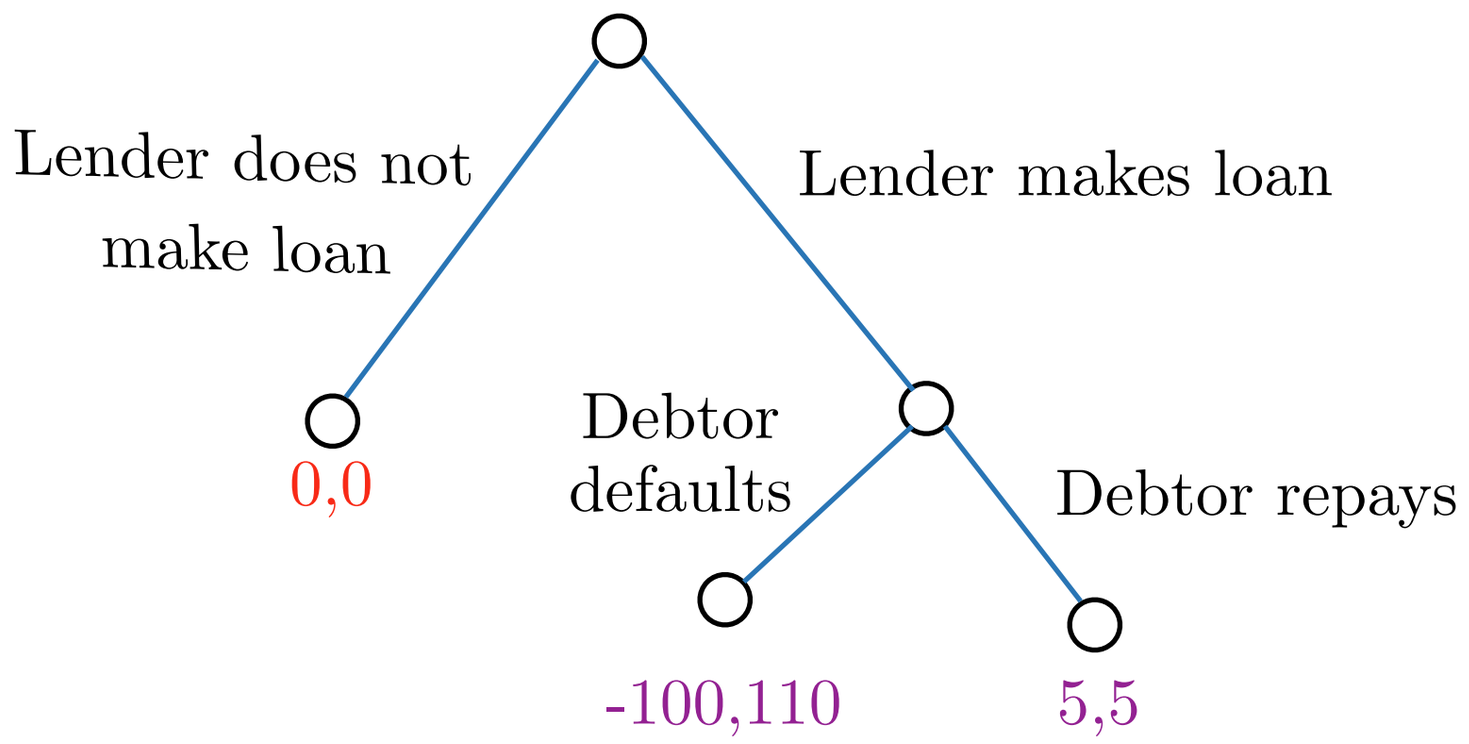


Example

- Assume lender makes the loan
- Debtor has dominant strategy
- Lender prefers no profit than a loss os $100
- No loan

backwards induction

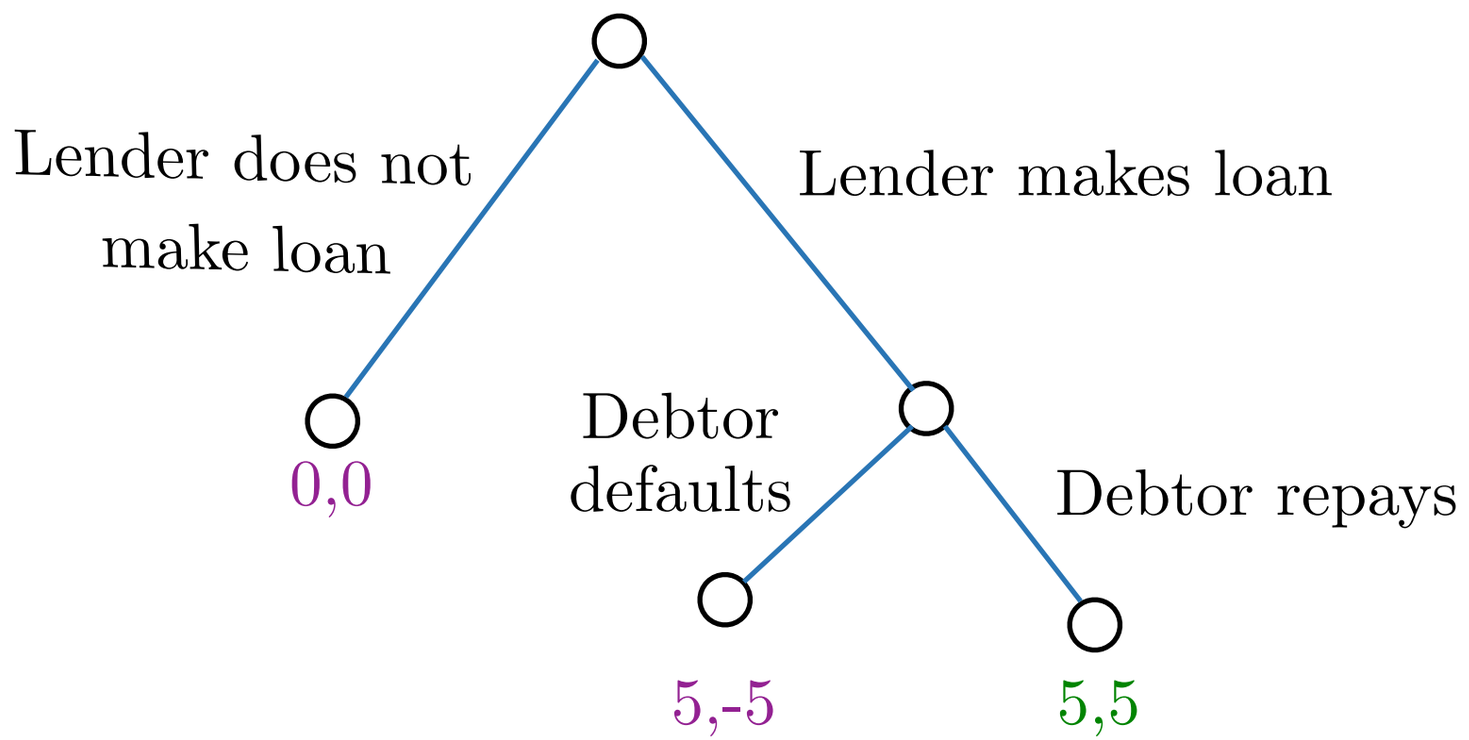


Example

- Contract:
  - No ability to keep the money
  - Litigation costs of $10 (debtor)
- Mechanism so that trade moves forward

Example

- Contract:
  - No ability to keep the money
  - Litigation costs of $10 (debtor)
- Mechanism so that trade moves forward
- Lender's self-interest to loan. Right incentive!



Example

		Player 2: x	Player 2: y
Player 1	a	10, 10	15, 5
	b	5, 15	12, 12

What is the Nash equilibrium of the above game?

- $a,\ x$
- $b,\ x$
- $a,\ y$
- $b,\ y$

Example

		Player 2	
		x	y
Player 1	a	10, 10	15, 5
	b	5, 15	12, 12

What is the Nash equilibrium of the above game?

- ✓ $a,\ x$
- ◉ $b,\ x$
- ◉ $a,\ y$
- ◉ $b,\ y$

Example

		Player 2		
		x	y	z
Player 1	a	6, 6	8, 20	0, 8
	b	10, 0	5, 5	2, 8
	c	8, 0	20, 0	4, 4

What is the Nash equilibrium of the above game?

- $b,\ y$
- $c,\ z$
- $a,\ x$
- $a,\ y$

Example

		Player 2		
		x	y	z
Player 1	a	6, 6	8, 20	0, 8
	b	10, 0	5, 5	2, 8
	c	8, 0	20, 0	4, 4

What is the Nash equilibrium of the above game?

- ◉ $b,\ y$
- ✓ $c,\ z$
- ◉ $a,\ x$
- ◉ $a,\ y$

Example

		Player 2	
		x	y
Player 1	a	0, 0	0, 1
	b	2, 0	0, 0

What is/are the equilibrium/equilibria of the above game?

- $b,\ x$ only
- $a,\ y$ and $b,\ x$
- $a,\ y$; $b,\ x$; and $b,\ y$
- none

Example

		Player 2	
		x	y
Player 1	a	0, 0	0, 1
	b	2, 0	0, 0

What is/are the equilibrium/equilibria of the above game?

- $b,\ x$ only
- ✓ $a,\ y$ and $b,\ x$
- $a,\ y$; $b,\ x$; and $b,\ y$
- none

Example

		Player 2		
		Rock	Paper	Scissors
Player 1	Rock	0, 0	-1, 1	1, -1
	Paper	1, -1	0, 0	-1, 1
	Scissors	-1, 1	1, -1	0, 0

How many equilibria does the above game have?

- 9
- 3
- 1
- 0

Example

		Player 2		
		Rock	Paper	Scissors
Player 1	Rock	0, 0	-1, 1	1, -1
	Paper	1, -1	0, 0	-1, 1
	Scissors	-1, 1	1, -1	0, 0

How many equilibria does the above game have?

- ◉ 9
- ◉ 3
- ◉ 1
- ✓ 0

Today

- Mixed strategies
- Nash equilibria
    - formal definition
    - existence
- The fundamental theorem
    - properties
    - finding mixed-strategy equilibria
- Paradigmatic games

Nash may not be the overall best strategy

- Levee costs 2
- Flood brings damages:
  3 (both build), 4 (neither one), or 10 (only of them)

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		Landlord 2: ~~Don't build~~	Landlord 2: Build
Landlord 1	~~Don't build~~	-4, -4	-10, -2
	Build	-2, -10	-5, -5

Levee collection action game

Nash may not be the overall best strategy

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- Flood brings damages:
  3 (both build), 4 (neither one), or 10 (only of them)

Landlord 1 \ Landlord 2	~~Don't build~~	Build
~~Don't build~~	-4, -4	-10, -2
Build	-2, -10	-5, -5

Levee collection action game

Both parties better off building the levee!

- WARNING -

Nash may not be the best strategy

Coping with multiple Nash

- Maintaining levee costs 4
  (only if both maintain levee will there be no flood)
- Flood brings damages of 6 (for both)

Coping with multiple Nash

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Landlord 1 \ Landlord 2	Maintain	Don't maintain
Maintain	-4, -4	-10, -6
Don't maintain	-6, -10	-6, -6

Levee coordination game

Coping with multiple Nash

- Maintaining levee costs 4
  (only if both maintain levee will there be no flood)
- Flood brings damages of 6 (for both)

Landlord 1 \ Landlord 2	Maintain	Don't maintain
Maintain	-4, -4	-10, -6
Don't maintain	-6, -10	-6, -6

Levee coordination game

We cannot be confident what the parties will do

- WARNING -

Nash may not be unique

Legal regimes

- Multiple Nash may provide a justification to change payoffs
- A landowner is made responsible if the other maintains the levee and he/she does not

Landlord 1 \ Landlord 2	Maintain	Don't maintain
Maintain	-4, -4	-10, -6
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Legal regimes

- Multiple Nash may provide a justification to change payoffs
- A landowner is made responsible if the other maintains the levee and he/she does not

		Landlord 2: Maintain	Landlord 2: Don't maintain
Landlord 1	Maintain	-4, -4	-10, -6
	Don't maintain	-6, -10	-6, -6

		Landlord 2: Maintain	Landlord 2: Don't maintain
Landlord 1	Maintain	-4, -4	-4, -12
	Don't maintain	-12, -4	-6, -6

Levee coordination game
with legal duty to maintain

Mixed strategies

Example

- Landlord 1 decides to sometimes maintain the levee
- With probability 1/2

		Landlord 2: Maintain	Landlord 2: Don't maintain
Landlord 1: Maintain		-4, -4	-10, -6
Landlord 1: Maintain with prob. 1/2		0.5 (-4) + 0.5 (-6), 0.5 (-4) + 0.5 (-10) = -5, -7	0.5 (-10) + 0.5 (-6), 0.5 (-6) + 0.5 (-6) = -8, -6
Landlord 1: Don't maintain		-6, -10	-6, -6

Levee coordination game

Example

- Landlord 1 decides to sometimes maintain the levee
- With probability 1/2

		Landlord 2: Maintain	Landlord 2: Don't maintain
Landlord 1: Maintain		-4, -4	-10, -6
Landlord 1: Maintain with prob. 1/2		0.5 (-4) + 0.5 (-6), 0.5 (-4) + 0.5 (-10) = -5, -7	0.5 (-10) + 0.5 (-6), 0.5 (-6) + 0.5 (-6) = -8, -6
Landlord 1: Don't maintain		-6, -10	-6, -6

Levee coordination game

Best response to mixed strategy is not to maintain the levee

Can we find a mixed Nash equilibrium?

- Landlord 2: expected payoff for not maintaing the levee for any p_1 is -6
- p_1 (-4) + (1-p_1) (-10) = -6 $\rightarrow$ p_1=$2/3$
- Landlord 1 is indifferent between M and D
  $\rightarrow$ Any mixed strategy is a best response

		Landlord 2: Maintain	Landlord 2: M with prob. p_2	Landlord 2: Don't maintain
Landlord 1	Maintain	-4, -4	-6, ·	-10, -6
	M with prob. p_1	·, -6	-6, -6	·, -6
	Don't maintain	-6, -10	-6, ·	-6, -6

Levee coordination game

Paradigmatic games

Tragedy of the commons

Should players choose a dominant strategy?

- Prisoner's dilemma: Both parties have dominant strategies

  - Both parties are better off not choosing a dominant strategy

P_2

P_1		Silent	Confess
	Silent	-2, -2	-10, 0
	Confess	0, -10	-6, -6

Prisoner's dilemma

Tragedy of the commons

Should players choose a dominant strategy?

- Prisoner's dilemma: Both parties have dominant strategies

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		P_2: Silent	P_2: Confess
P_1	Silent	-2, -2	-10, 0
	Confess	0, -10	-6, -6

Prisoner's dilemma

		P_2: Don't build	P_2: Build
P_1	Don't build	-4, -4	-10, -2
	Build	-2, -10	-5, -5

Levee collection action game

Tragedy of the commons

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Prisoner's dilemma

P_2

P_1	Don't build	Build
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Levee collection action game

same structure as prisoner's dilemma

Emergence of standards

Standards in an industry with different firms: Choose between adopting own strategy or a common standard?

- Battle of the sexes: Both want to coordinate game, but they want different outcomes

		Spouse 2: Fight	Spouse 2: Opera
Spouse 1	Fight	8, 4	3, 3
	Opera	2, 2	4, 8

Battle of the sexes

Weigh what outcome is more efficient!

Law enforcement

Criminals have powerful incentives to act unpredictably

- Matching pennies: Both parties monitor one another

		Player 2: Heads	Player 2: Tails
Player 1	Heads	1, -1	-1, 1
	Tails	-1, 1	1, -1

Matching pennies

There are no pure Nash equilibria

Negotiation

Acquire an edge by being able to commit oneself to a particular strategy

- Chicken: First player gains advantage if he/she can make an offer and the other player immediately takes it

Union

Employer \ Union	Bargain	Commit
Bargain	5, 5	3, 7
Commit	7, 3	2, 2

Collective bargaining

- If both parties commit themselves, there is a strike!

- Nash equilibria involve combinations of opposite moves

- May adopt mixed Nash equilibrium

Remarks

- Paradigmatic games serve as benchmarks
  - Forces that are at work are readily apparent
  - Easy to understand effect of different legal rules
  - How might such dilemmas be resolved?

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- Paradigmatic games serve as benchmarks
  - Forces that are at work are readily apparent
  - Easy to understand effect of different legal rules
  - How might such dilemmas be resolved?
- Use paradigms with caution
  - Capture problem, then look for appropriate paradigm, not the other way around

Next class

- Game theory an human behavior
- Conditions for altruism
  - the ultimatum game
- Norms of cooperation
  - the public goods game
- Implications to policy-making