- Lecture 14 -
A model of corruption

Review

- Modal logic of knowledge
- Formalize extensive form CKR
- CKR $\rightarrow$ backward induction

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$$\underbrace{\text{Formalize what it means to say an individual knows a fact about the world}}_{\text{extensive form game}}$$

Self-evident events

- If the agent knows E at each state $\omega \in \mathrm{E}$

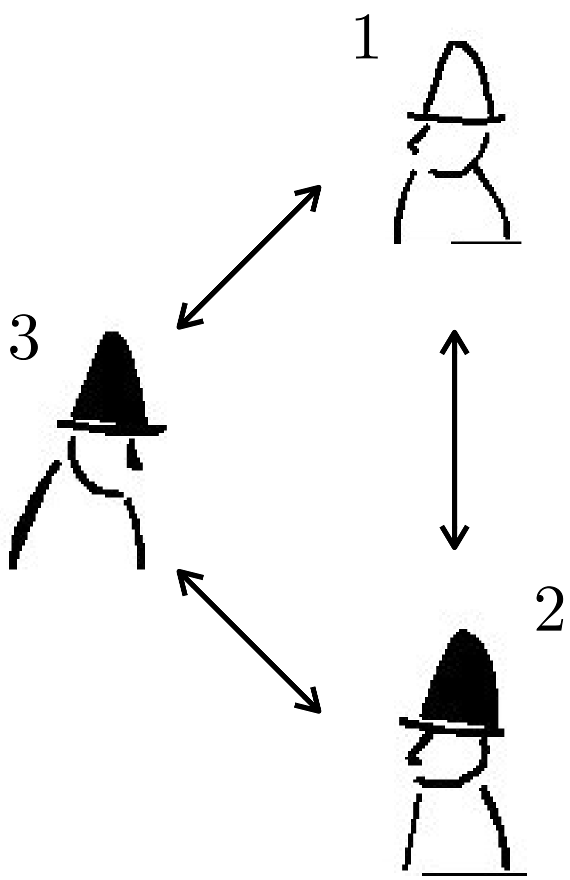
$$\mathbf{K}\mathrm{E} = \mathrm{E}$$

$$\mathbf{P}\omega \subseteq \mathrm{E} \text{ for every } \omega \in \mathrm{E}$$

- Minimal self-evident events
- Coincide with cells of the partition $\mathcal{P}$

Hat puzzle

- Red or white hats
- State of nature ω corresponds to the color of each person's hat
- What players could know at the outset?



States of the world

Player		ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8
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	2	r	r	w	w	r	r	w	w
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- $\Omega = \{\omega_1,\ \omega_2,\ \omega_3,\ \omega_4,\ \omega_5,\ \omega_6,\ \omega_7,\ \omega_8\}$
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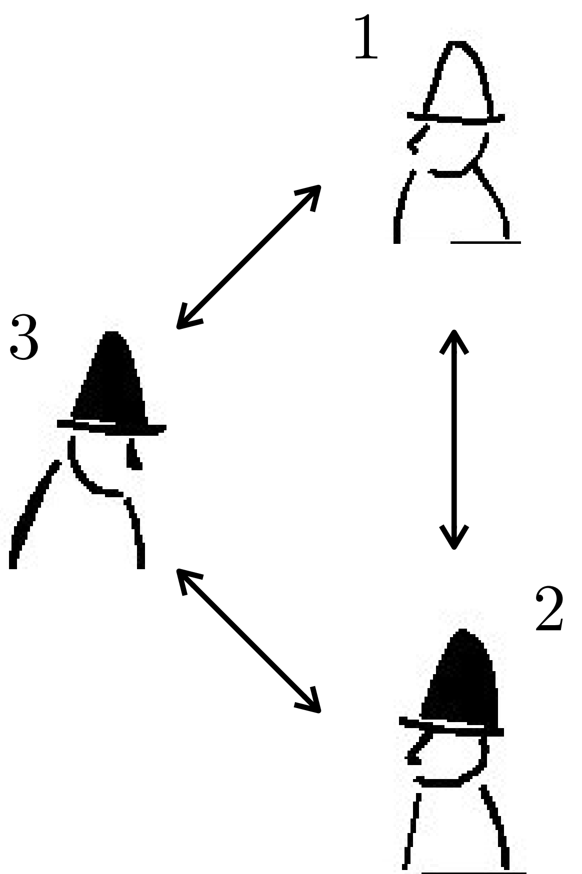

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- Actual state of the world is ω_1:
- Agent 1 considers the possible states $\mathbf{P}\omega_1 = \{\omega_1,\ \omega_5\}$
- Agent 1 knows that the actual state is:
  - $\omega_1 = \{\mathrm{r},\ \mathrm{r},\ \mathrm{r}\}$, or
  - $\omega_5 = \{\mathrm{w},\ \mathrm{r},\ \mathrm{r}\}$

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cannot distinguish between the two!

Knowledge

- Player i knows her hat color only if the color is the same in all states of nature ω which agent i regards as possible
- At state ω_1 does player 1 know her hat color is red?
- No! $\omega_1 = \{\mathrm{r}, \mathrm{r}, \mathrm{r}\}$ or $\omega_5 = \{\mathrm{w}, \mathrm{r}, \mathrm{r}\}$

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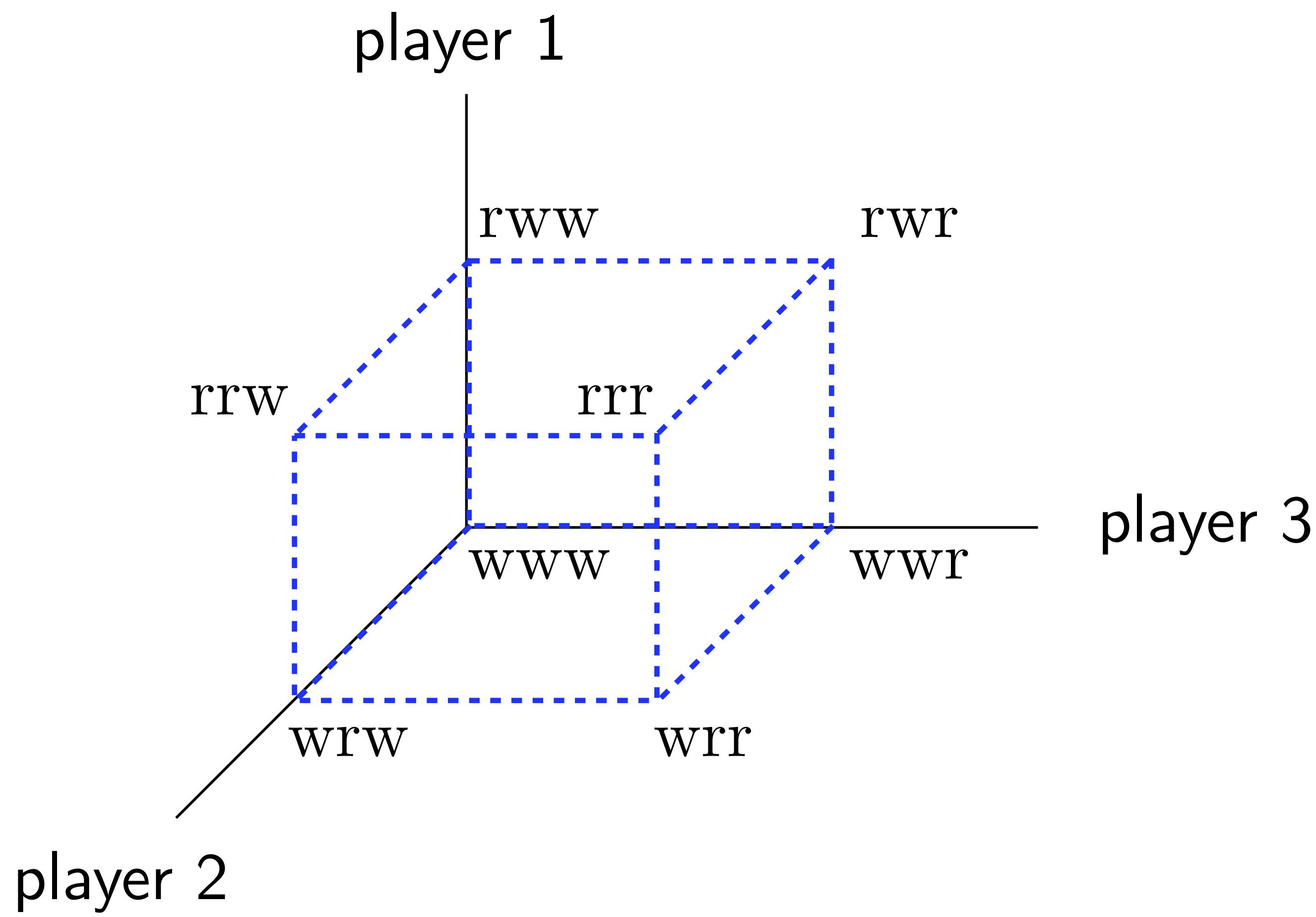
- Formally,
  - $\mathrm{E}=\{\omega_1,\ \omega_2,\ \omega_3,\ \omega_4\}$
  - $\mathbf{K}_1\mathrm{E} = \{\omega : \mathbf{P}_1\omega \subseteq \mathrm{E}\} \neq \mathrm{E}$
  - $\mathbf{P}_1\omega_1 = \{\omega_1,\ \omega_5\} \nsubseteq \mathrm{E}$ for some $\omega \in \mathrm{E}$

State space as the vertices

- Three coordinates: ith coordinate represents the hat color of ith player
- Represent r as 1 and w as 0



State space as the vertices

Connect two vertices with an edge if they lie in the same information cell in agent i's partition...

States of the world

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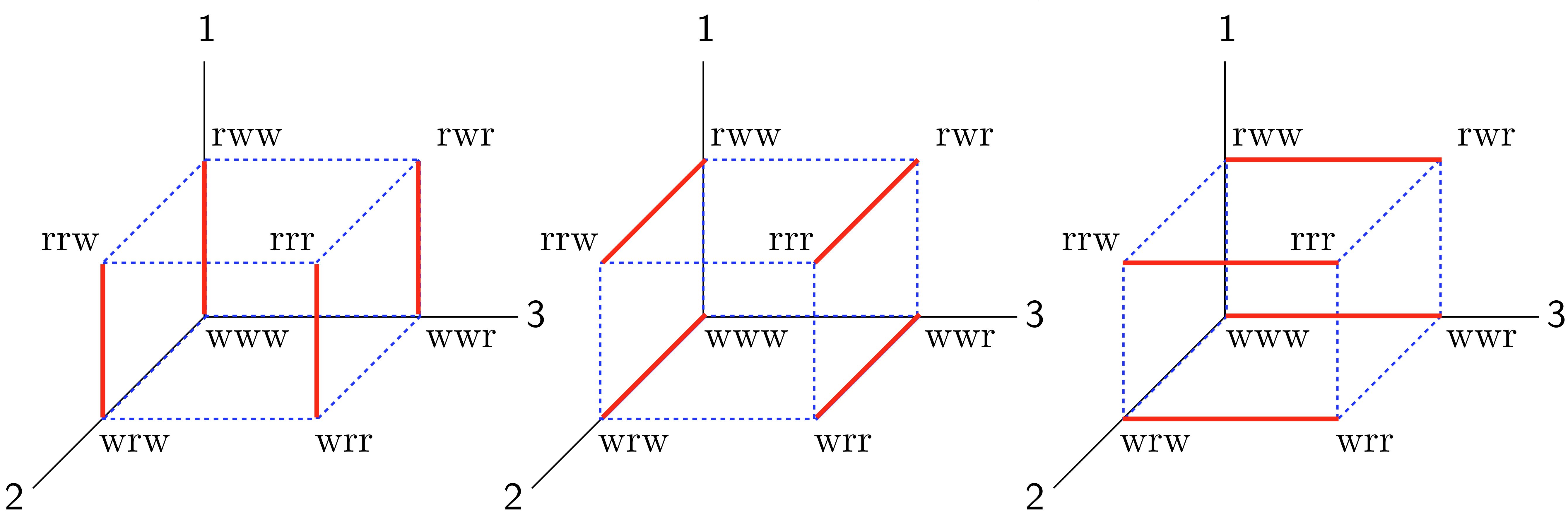
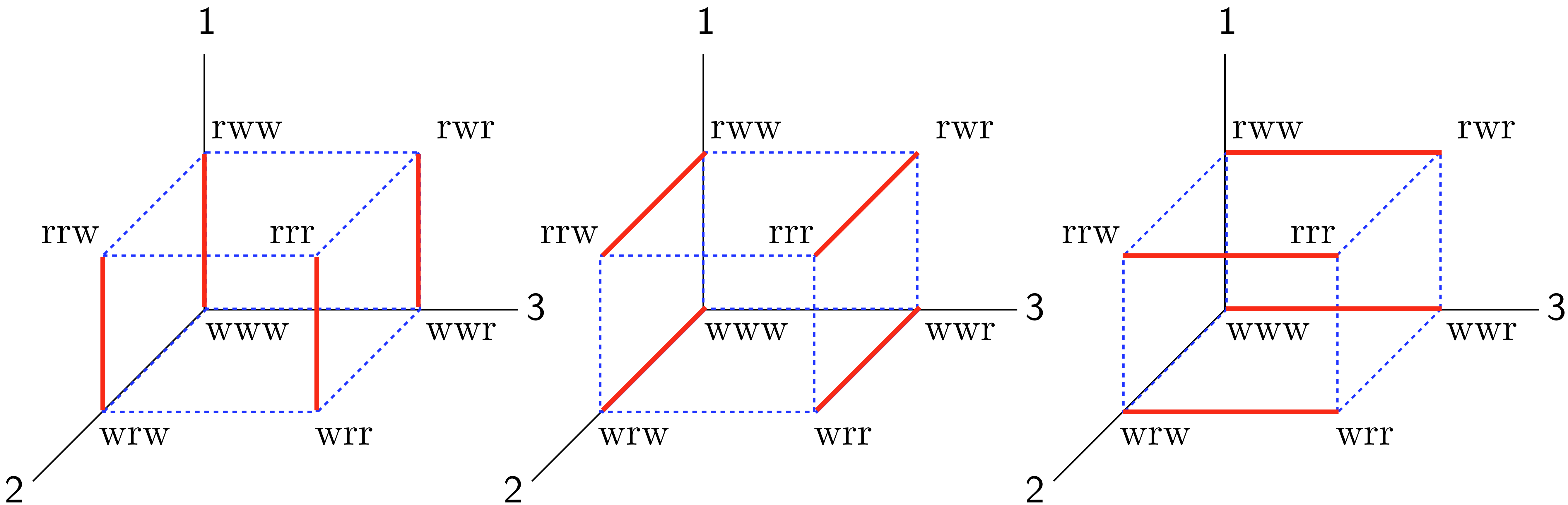
$\{\omega_1,\ \omega_5\} \in \mathcal{P}_1$



State space as the vertices

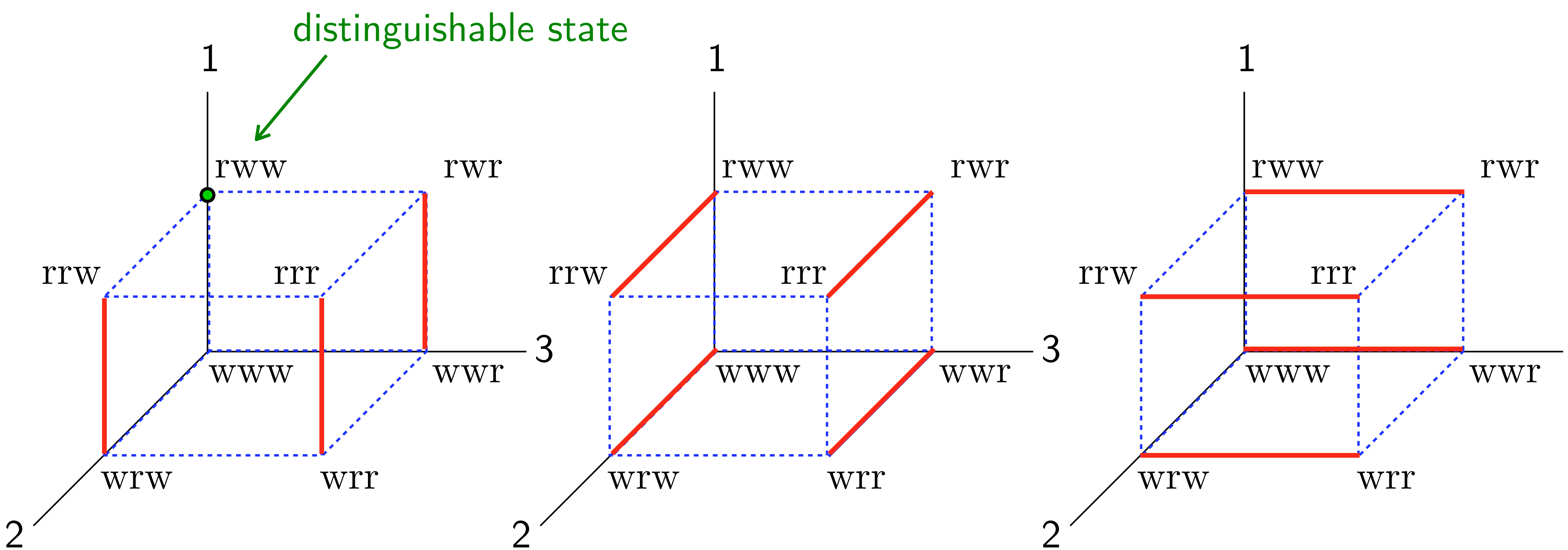
- A player i knows her hat color at a state if and only if the state is not connected by one of i's edges to another state in which i has a different hat color.
- Evolution of knowledge...?



"At least one red hat in the room"

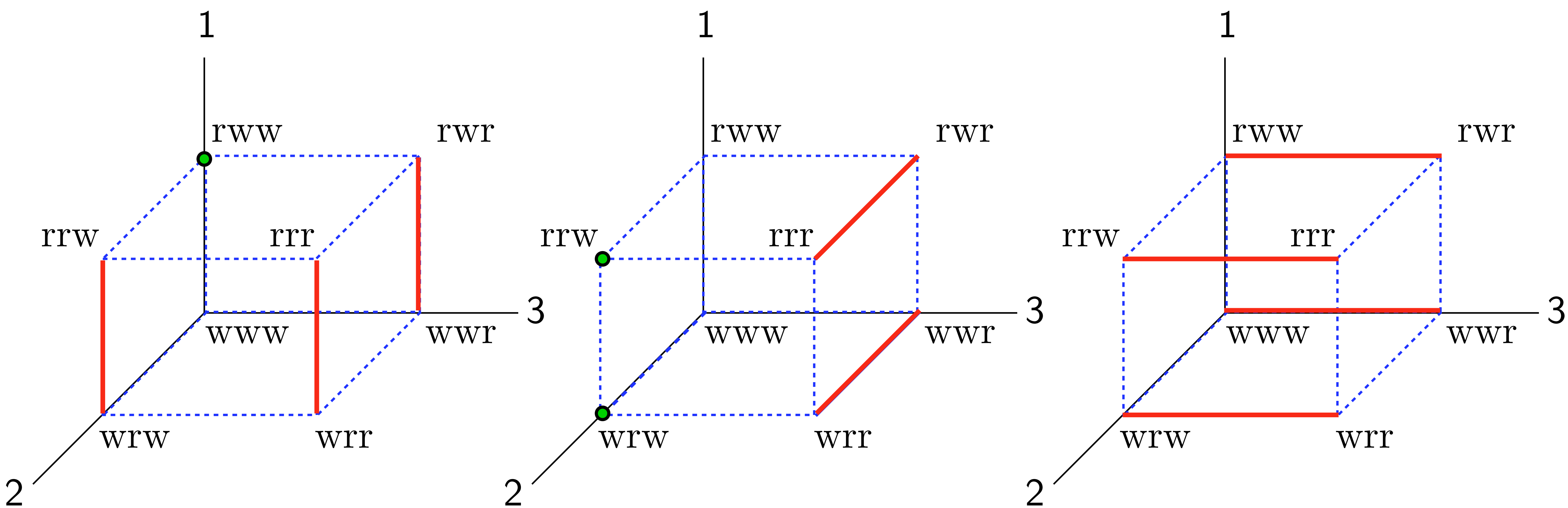
Player 1

- Drop all the edges leading out of the state www
- P_1: "I do now know my hat color"
- P_1 at an undistinguishable state
- The state ω cannot be $\{\text{www}, \text{rww}\} = \{\omega_4, \omega_8\}$



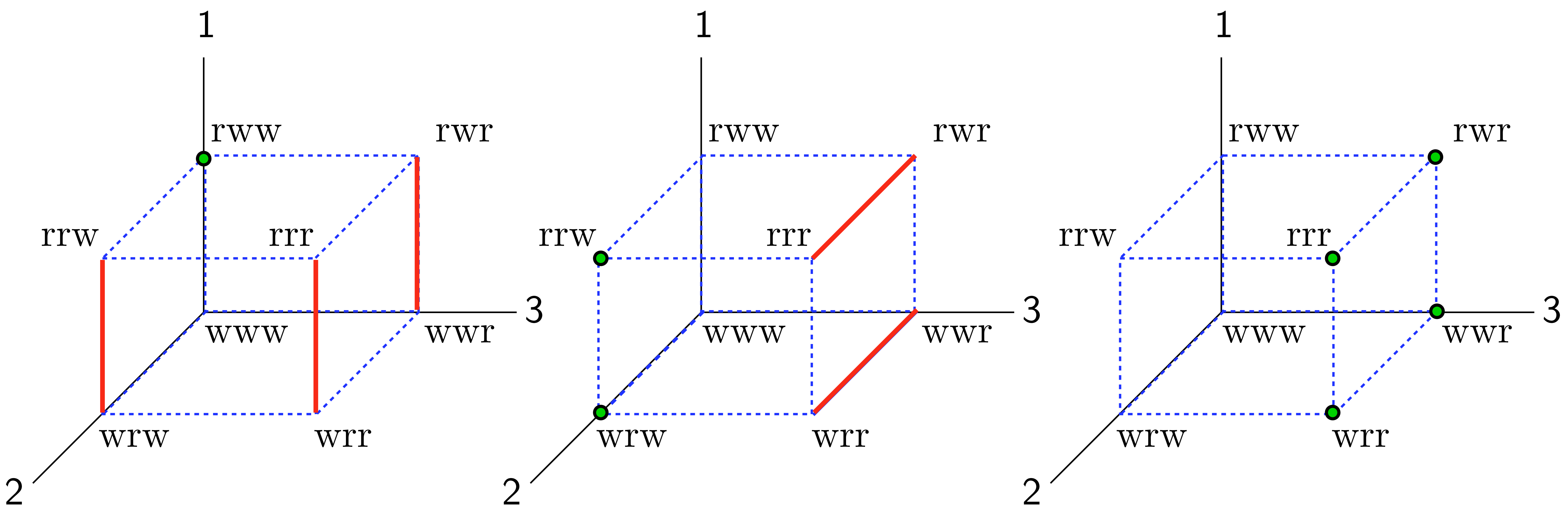
Player 2

- drop all the edges leading out of the state www
- P_2: "I do now know my hat color" (undistinguishable state)
- Then $\omega \notin \{\mathrm{www},\ \mathrm{rww},\ \mathrm{rrw},\ \mathrm{wrw}\} = \{\omega_2, \omega_4, \omega_6, \omega_8\}$



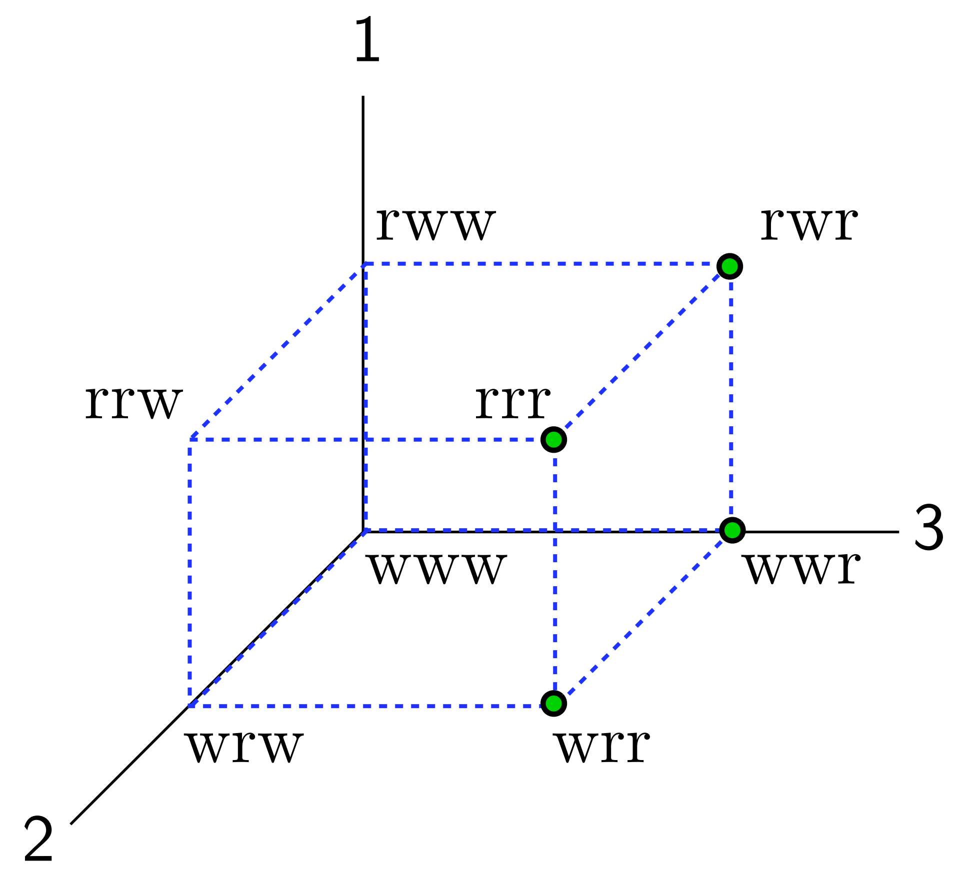
Player 3

- drop all the edges leading out of the state www
- P_2: “I do now know my hat color” (undistinguishable state)
- Then $\omega \notin \{\mathrm{www}, \mathrm{rww}, \mathrm{rrw}, \mathrm{wrw}\} = \{\omega_2, \omega_4, \omega_6, \omega_8\}$
- So $\omega \in \{\omega_1, \omega_3, \omega_5, \omega_7\}$



Knowledge

- $E=\{\omega_1,\ \omega_3,\ \omega_5,\ \omega_7\}$
- $\mathbf{K}_3 E = \{\omega_1,\ \omega_3,\ \omega_5,\ \omega_7\} = E$
- $\mathbf{P}_3\omega \subseteq E$ for every $\omega \in E$
- Player 3 knows her hat color!
- Self-evident event



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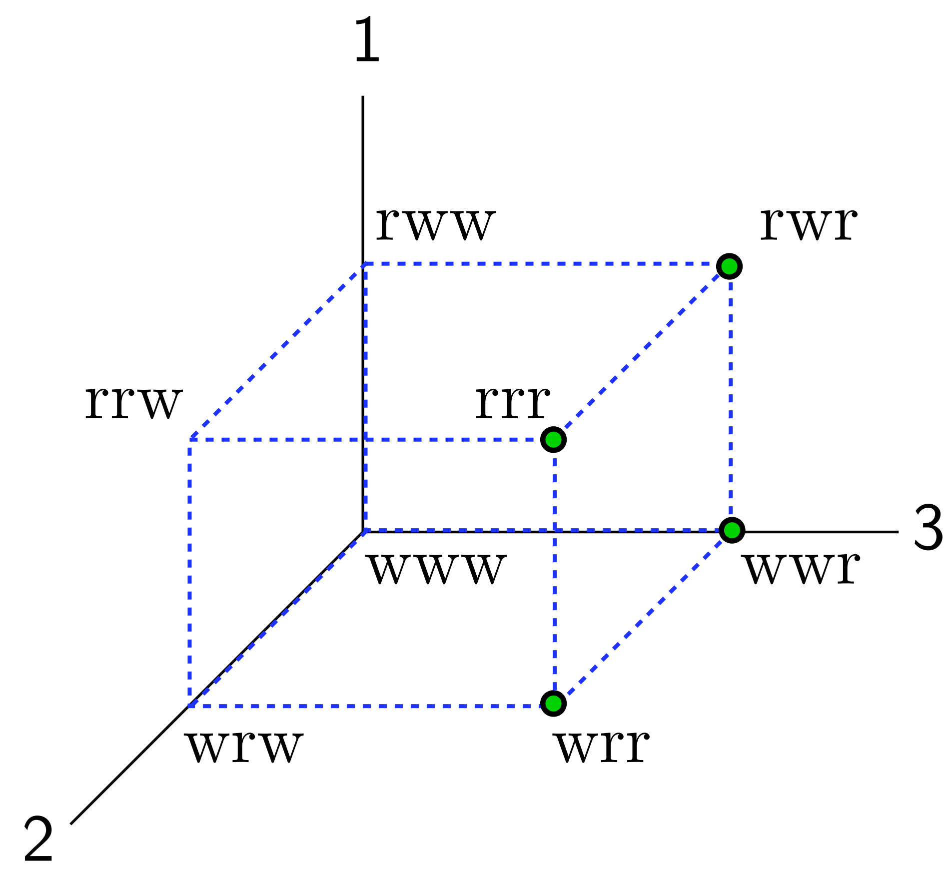

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Sequence of the players' responses matters!

Key questions

- Bayesian rationality for extensive form games?
- Common knowledge of rationality (CKR)?

Bayesian rationality

- If every node v at which player i chooses and for every strategy $t_i \in S_i$

$$R_i \subseteq \neg \mathbf{K}_i \{\omega \in \Omega \mid \pi_i^v(s/t_i) > \pi_i^v(s)\}$$

- Player i does not know that there is a better strategy than $s_i(\omega)$ at state ω

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- Weaker than for normal form game:
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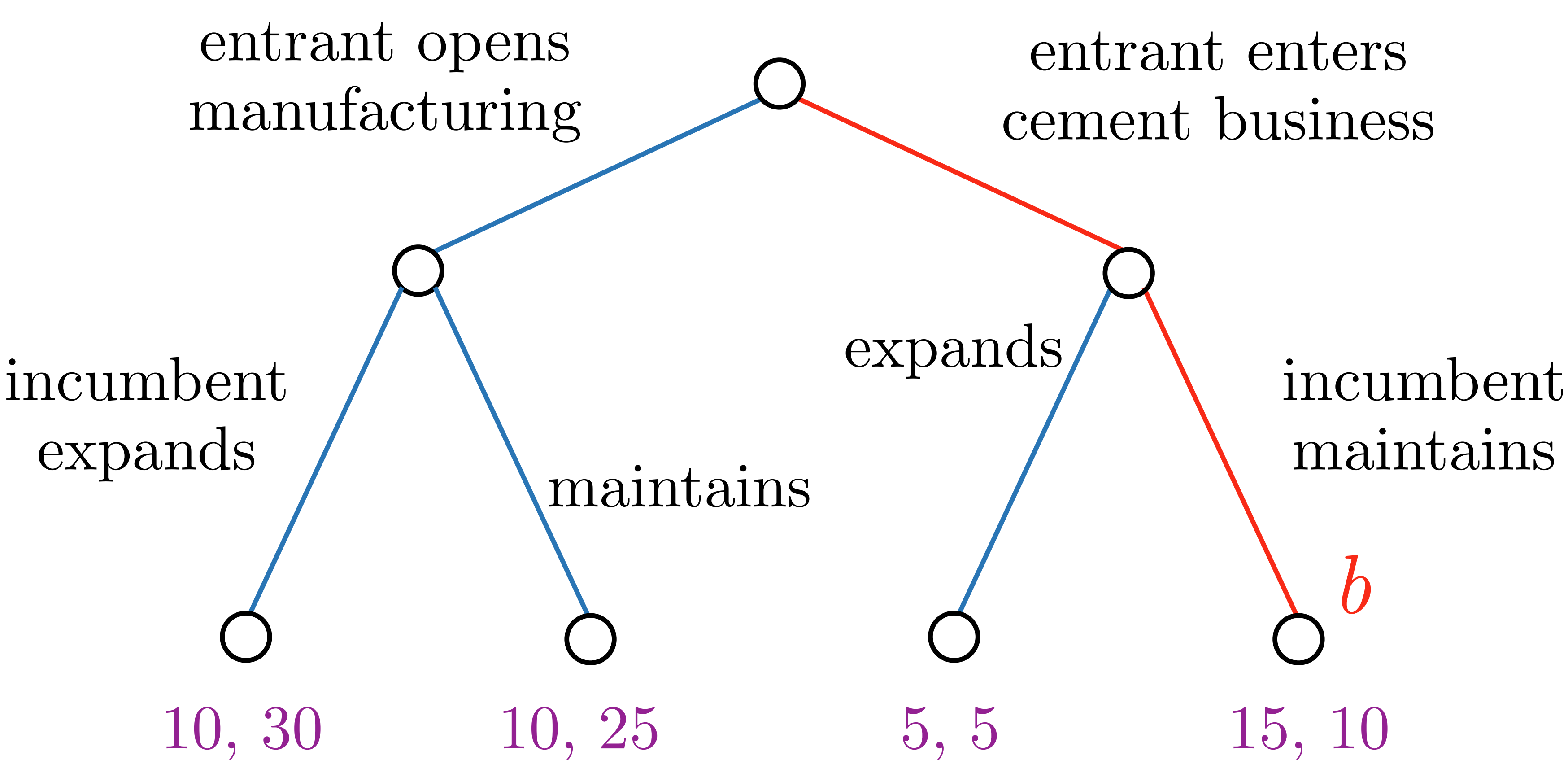
- CKR: The event R_i is common to all players

Backward induction

- Let b be the unique backward induction path
- The event that b^v is chosen

$$I^v = \{\omega \in \Omega \mid s^v(\omega) = b^v\}$$

- The event that backward induction is chosen: $I = \cap_v I^v$



Review

Theorem 5.1 - $\mathrm{CKR} \subseteq \mathrm{I}$.

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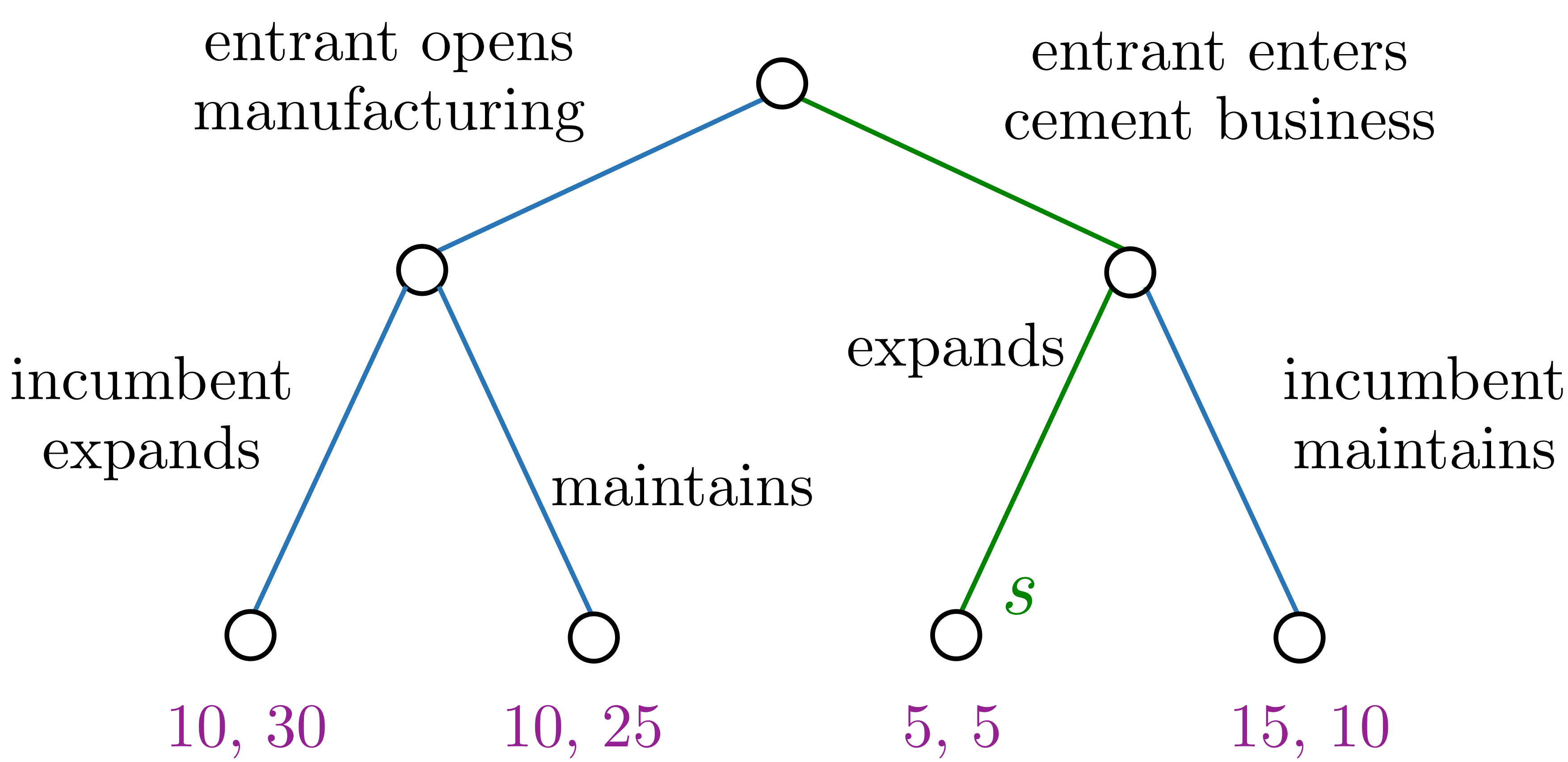

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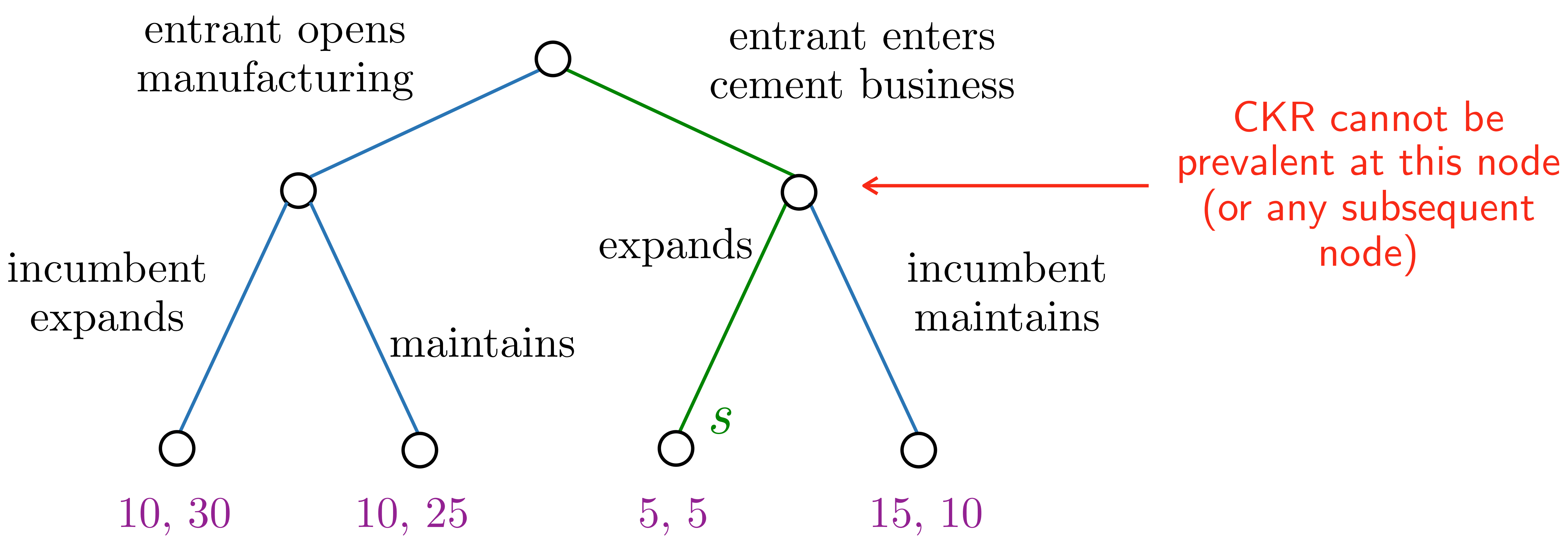

Today

- A model of reputation
- Honesty and corruption
- Social norms

World Development, Vol. 10, No. 8, pp. 677-687, 1982.

Underdevelopment and the Economics of Corruption: A Game Theory Approach

JOHN MACRAE*

Summary. - Previous attempts to treat corruption arc surveyed critically. A game theory approach is preferred on the grounds that it can most effectively explain the basis for decisions of reasonable men to be corrupt. A simple model is presented showing how bribery might be a dominant strategy. A prisoner's dilemma type of situation emerges with the added complication that the judge and jailer may be corrupt. Other conclusions arc that one official will not accept bribes from more than one firm. It is impossible to predict which firm will win the contract. No obvious solution emerges and legal remedies are discounted. This paper then reviews the principal general equilibrium effects and concludes that their likely effect on economic development is negative.

Methodology

- Economic analysis of corruption
- Game theory useful?
- Assumptions
  - regarding rival behavior
- Implications (testable?)
  - policy proposal
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1. Value of analytical treatment?
2. Quantifiable variables make sense?
3. Foster critical thinking skills?
4. Guide experimental design?

Focus

- Bribing by oligopolistic firms
- Contract tendering

Corrupt behavior

- Arrangement between two parties (exchange)
- Influence the allocation of resources
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- Counterpart - political patronage, tutelage, etc.
- Underlying problems
  - queues
  - greed for monopoly profits
  - inefficiencies

Key questions

- Role of assumptions about behavior of others play?
  - demander (official)
  - supplier (firm)
- Do all firms have an interest in being corrupt? (d)
- Is there a limit to the amount of corruption? (d+s)
- Role of legal sanctions? (d+s)
- Who ultimately pays for the cost of the arrangement?

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What is the joint bases for corrupt decisions?

Prices for arrangements

- Level of arrangement (grand v. petty)
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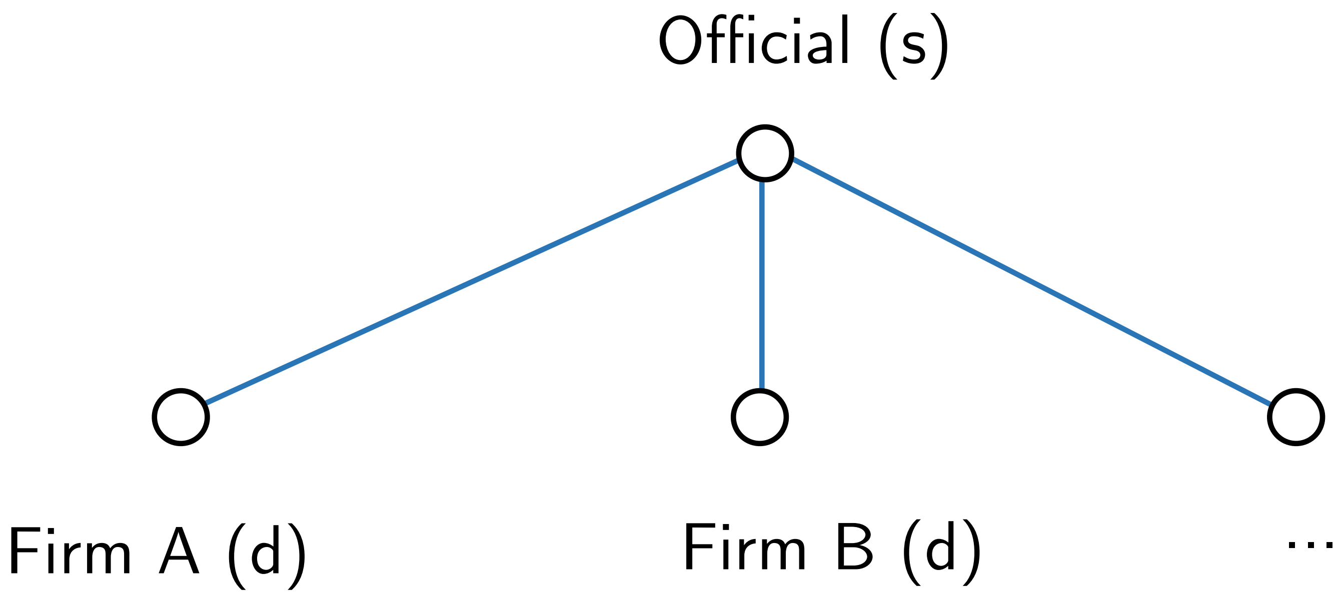
Behavior determined by marginal utilities!

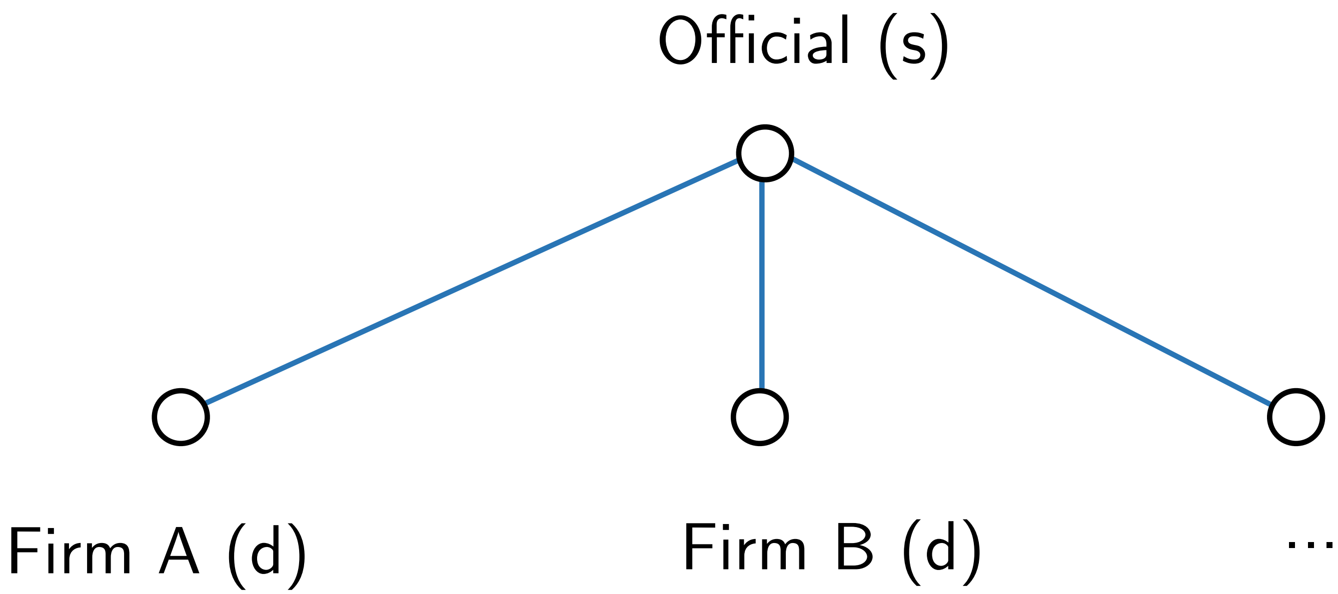
A simple model of corruption



"Special characteristics cancel out"

- N firms in total, project worth P

A simple model of corruption



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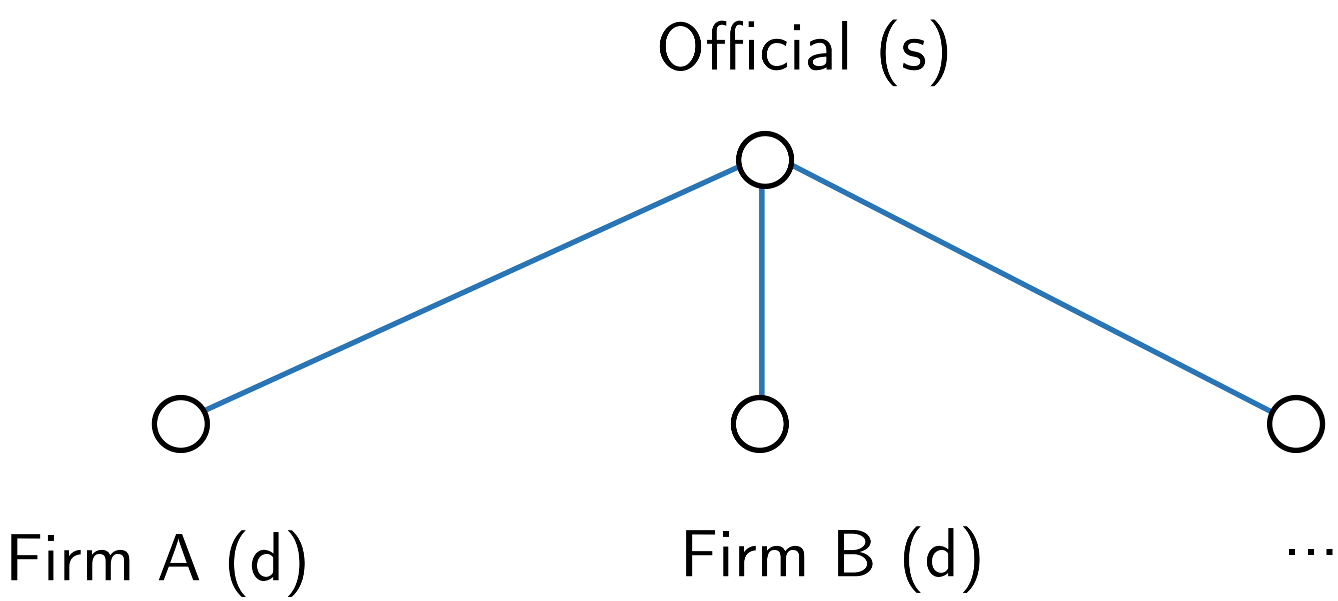
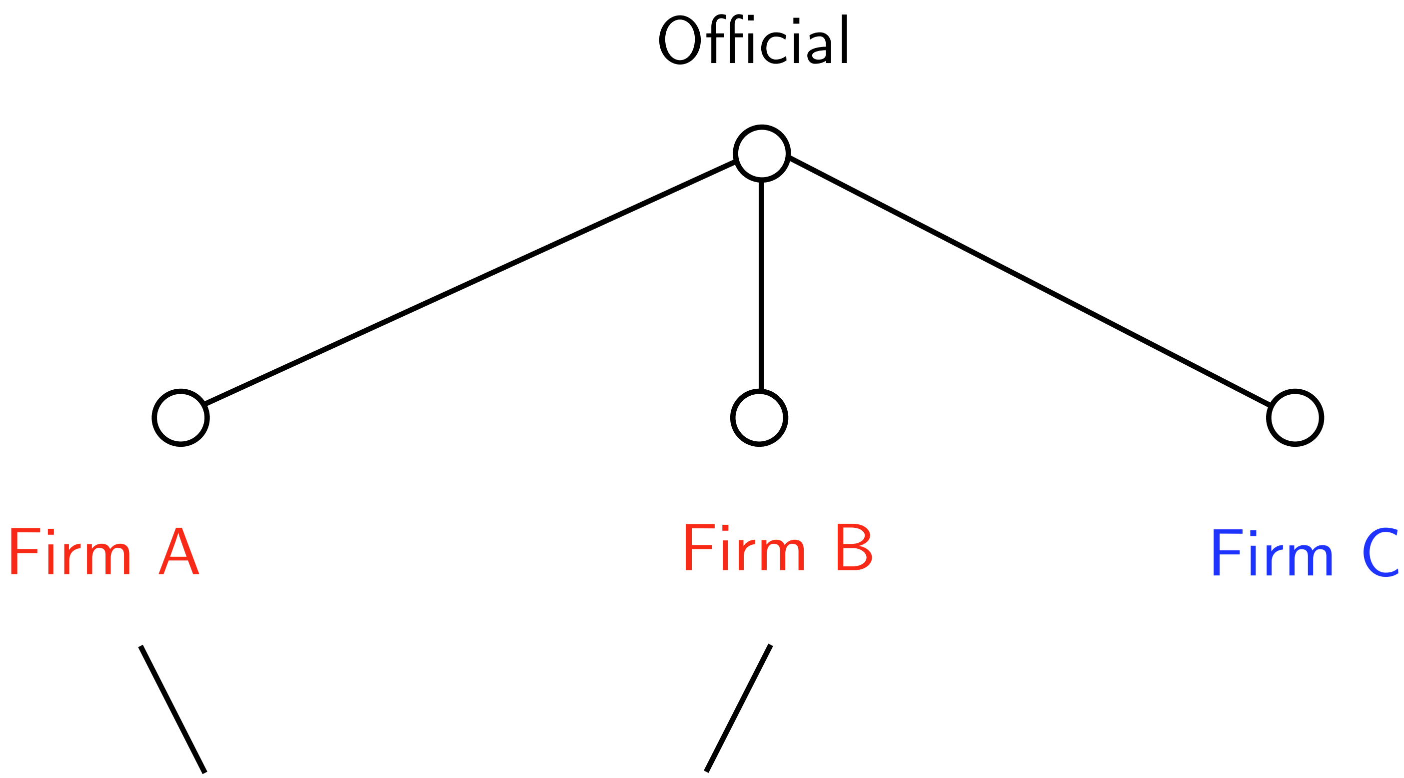


"Special characteristics cancel out"

- N firms in total, project worth P
- Corruption-free - $p = 1/N$ getting the contract
- Corruption arrangement - enhance the probability p of A
- Capacity to enhance outcome - $0 \leq a \leq 1$
- Successful bribe - $p' = p + a(1 - p)$

...if others are corrupt too...

A simple model of corruption



$$p'_b = p + a\frac{(1-p)}{2}$$

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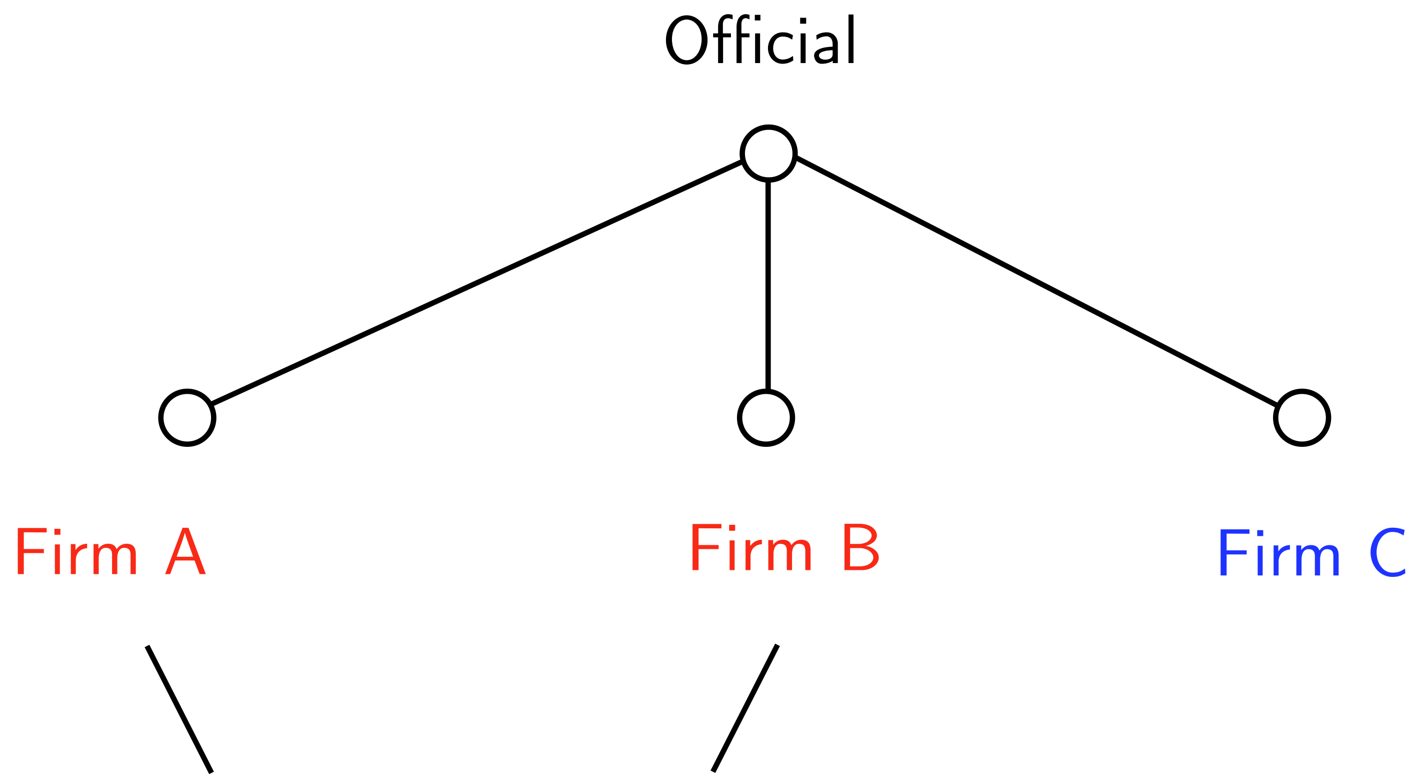

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↑

maximum premium
official can offer

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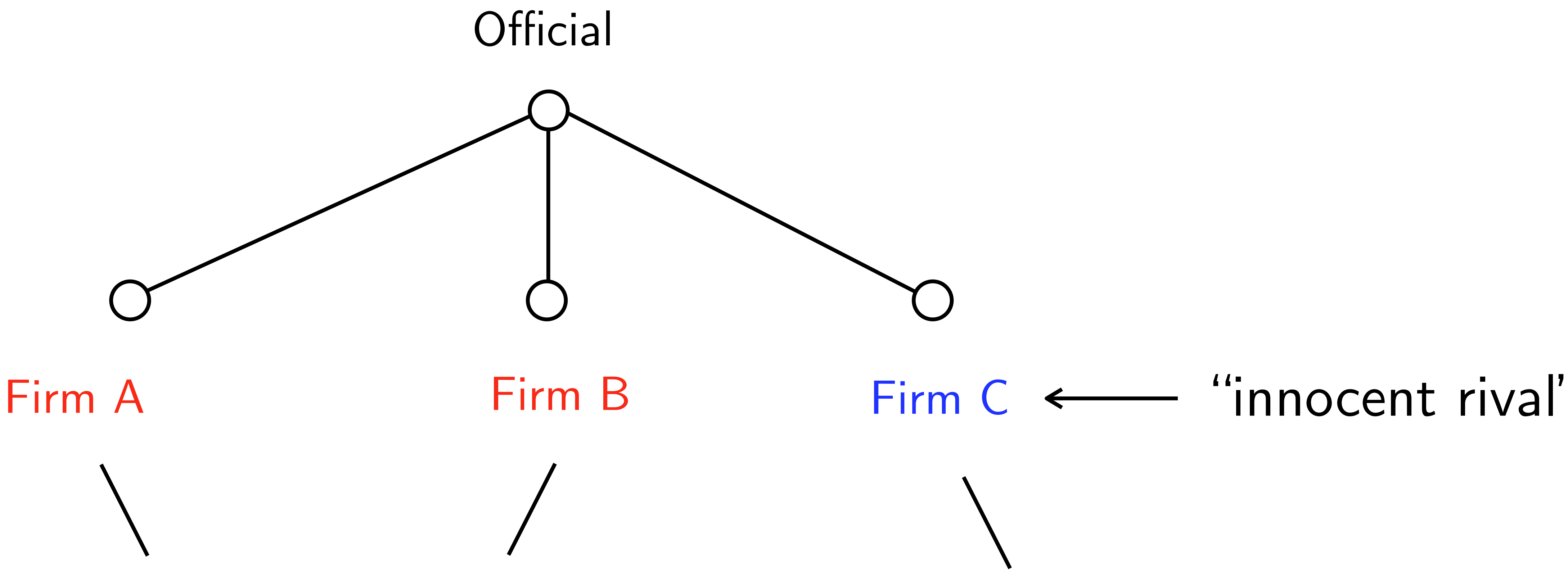
A simple model of corruption



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maximum premium official can offer

$$p'_b = p + \frac{a(1-p)}{N-1}$$

$$p'' = 1 - (N-1)p'_b$$

$$= \frac{1}{N} - \frac{a(N-1)}{N}$$

$$p'' > 0, \quad a < \frac{1}{N-1}$$

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- EY_2: Expected payoff if non-corrupt
  - d : cost of being honest
  - e.g., delays and difficulties in formalities
  - In general, $d \neq c$

Legal sanctions

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- Proportionality factor v
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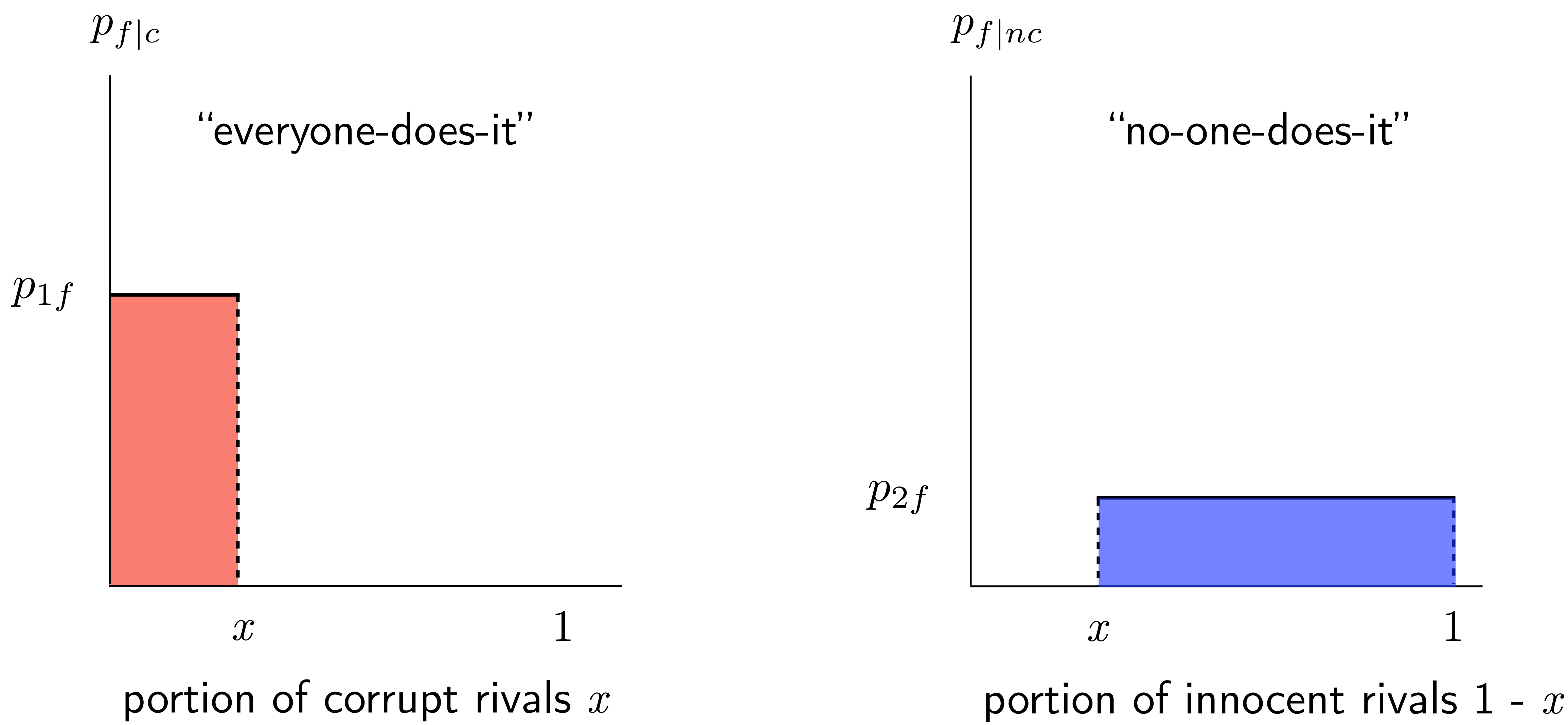
p_f given that all rivals are also involved in bribing

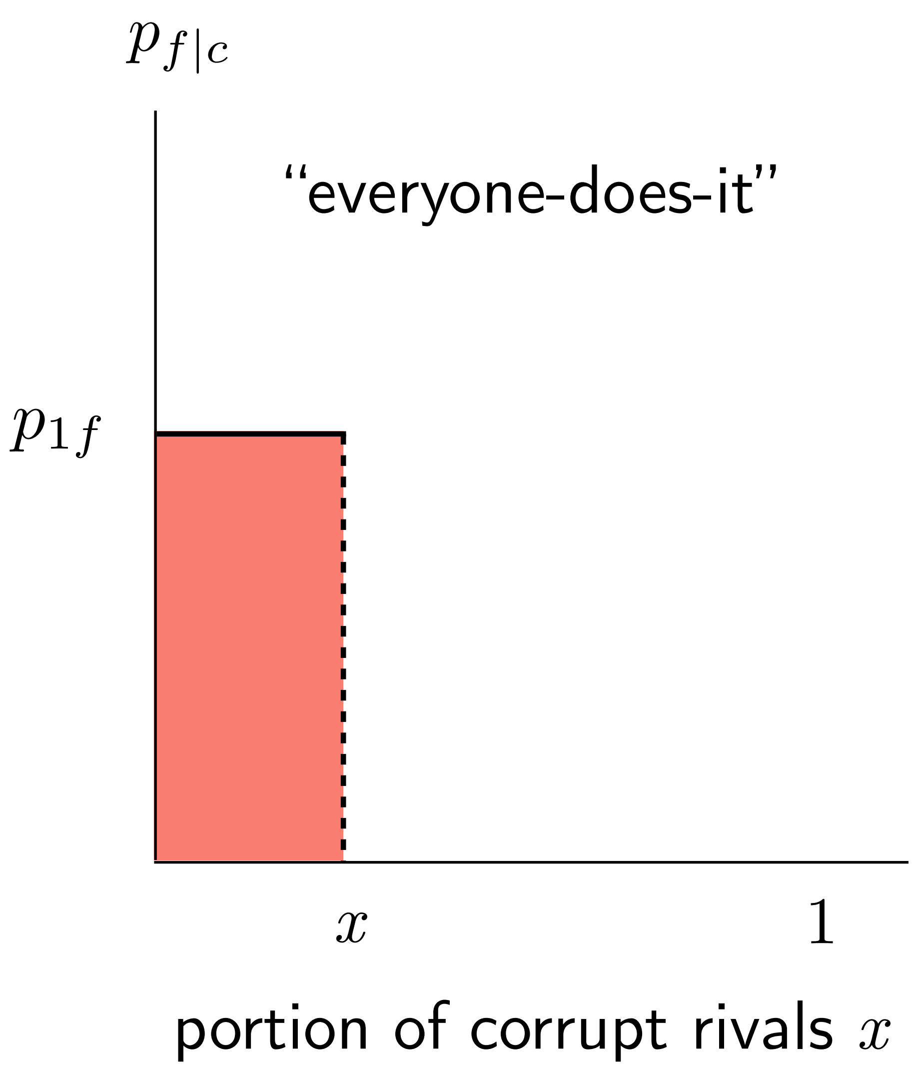
p_f given innocent rivals

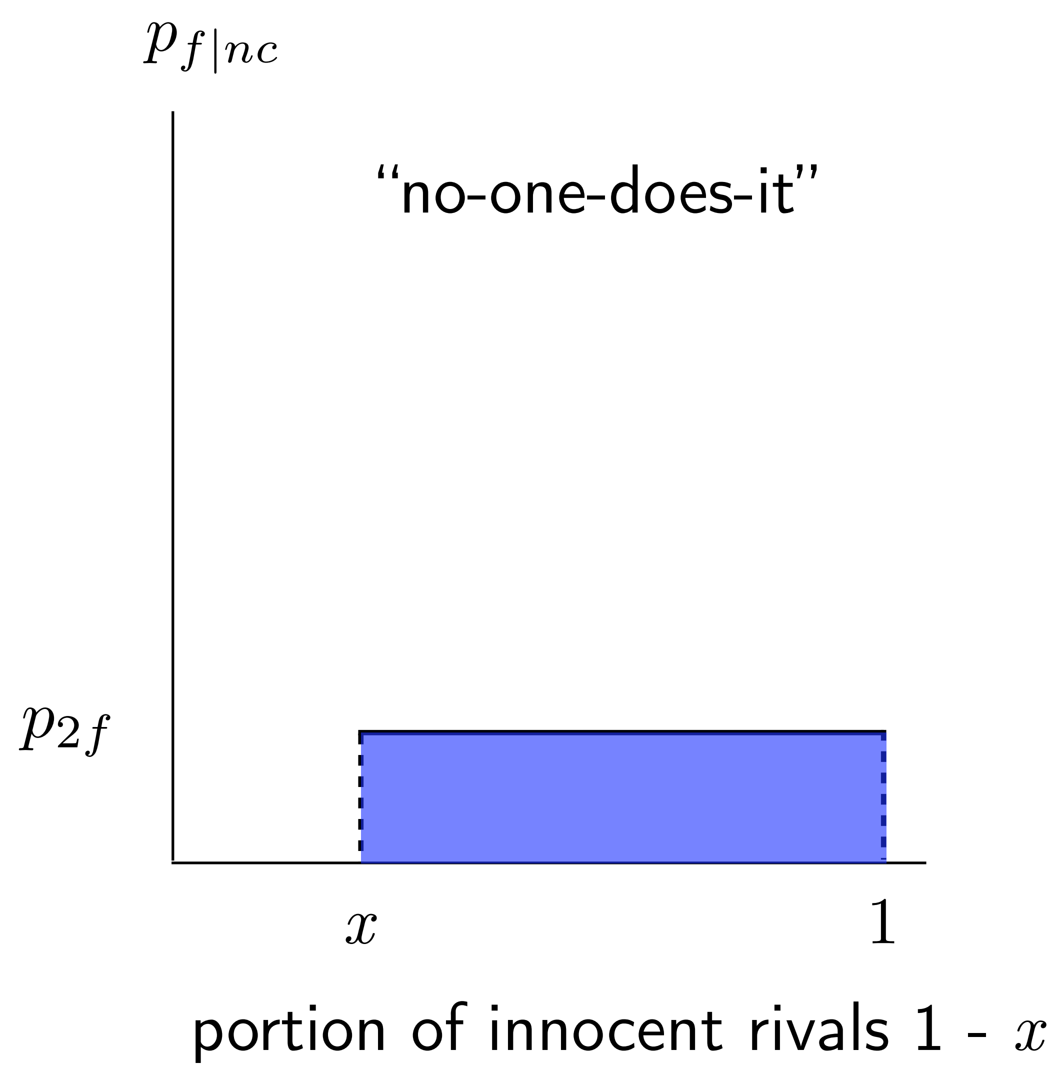
Legal sanctions



Legal sanctions

$$p_f F = p_{1f} x v P + p_{2f}(1 - x) v P$$





Expected net incomes

$$EY_1 = pxP + p'(1-x)P - p_f F - c \quad \text{: corrupt}$$

$$EY_2 = p''xP + p(1-x)P - d \quad \text{: non-corrupt}$$

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Expected net incomes

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$$\hat{Y} = [Y_1',\ Y_2'] \;\; Y_i' = EY_i + c$$

$$\hat{X} = [x,\ 1-x]$$

$$A = \begin{bmatrix} (p - vp_{1f})P & p''P \\ (p' - vp_{2f})P & pP \end{bmatrix}$$

$$\hat{Y} = \hat{X}A$$

Example

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$$p = \frac{1}{N} = \frac{1}{6}$$

$$p' = p + a(1-p) = \frac{1}{4}$$

$$p'' = \frac{1}{N} - \frac{a(N-1)}{N} = \frac{5}{60}$$

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- Uniform probabilities:

$$vp_{1f} = \frac{1}{20},\ vp_{2f} = \frac{1}{20}$$

Probability of being fined equal when all rivals are corrupt than rivals are innocent

Example

$$A = \begin{bmatrix} (p - vp_{1f})P & p''P \\ (p' - vp_{2f})P & pP \end{bmatrix}$$

$$= \begin{bmatrix} \frac{10}{6} - \frac{10}{20} & \frac{50}{60} \\ \frac{10}{4} - \frac{10}{20} & \frac{10}{6} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{7}{6} & \frac{5}{6} \\ \frac{12}{6} & \frac{10}{6} \end{bmatrix}$$

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$$A = \begin{bmatrix} (p - vp_{1f})P & p''P \\ (p' - vp_{2f})P & pP \end{bmatrix}$$

$$= \begin{bmatrix} \frac{10}{6} - \frac{10}{20} & \frac{50}{60} \\ \frac{10}{4} - \frac{10}{20} & \frac{10}{6} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{7}{6} & \frac{5}{6} \\ \frac{12}{6} & \frac{10}{6} \end{bmatrix}$$

$$\hat{Y} = [Y_1', \ Y_2'] = [x, \ 1 - x] \begin{bmatrix} \frac{7}{6} & \frac{5}{6} \\ \frac{12}{6} & \frac{10}{6} \end{bmatrix}$$

Case scenario

- 2 x non-corrupt, 2 x corrupt
- Firms A and B are undecided rivals
- What will A choose?

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		Firm B	
		c	nc
Firm A	c	$\frac{3}{2}, \frac{3}{2}$	$\frac{5}{3}, \frac{7}{6}$
	nc	$\frac{7}{6}, \frac{5}{3}$	$\frac{4}{3}, \frac{4}{3}$

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Firm B

Firm A	c	nc
c	$\frac{3}{2}, \frac{3}{2}$	$\frac{5}{3}, \frac{7}{6}$
nc	$\frac{7}{6}, \frac{5}{3}$	$\frac{4}{3}, \frac{4}{3}$

Corruption is a dominant strategy!

Resemble paradigmatic game?

Prisoner's dilemma

		P_2: defect	P_2: coop.
P_1	defect	$P,\ P$	$T,\ S$
	coop.	$S,\ T$	$R,\ R$

$$T > R > P > S$$

		Firm B: c	Firm B: nc
Firm A	c	$\frac{3}{2},\ \frac{3}{2}$	$\frac{5}{3},\ \frac{7}{6}$
	nc	$\frac{7}{6},\ \frac{5}{3}$	$\frac{4}{3},\ \frac{4}{3}$

$$T > P > R > S$$

Prisoner's dilemma

P₁ \ P₂	defect	coop.
defect	$P,\ P$	$T,\ S$
coop.	$S,\ T$	$R,\ R$

$$T > R > P > S$$

Firm A \ Firm B	c	nc
c	$\frac{3}{2},\ \frac{3}{2}$	$\frac{5}{3},\ \frac{7}{6}$
nc	$\frac{7}{6},\ \frac{5}{3}$	$\frac{4}{3},\ \frac{4}{3}$

$$T > {\color{red}P > R} > S$$

- Better to defect when the other cooperates
- Corruption yields the highest outcome for both

Remarks

$$A = \begin{bmatrix} (p - vp_{1f})P & p''P \\ (p' - vp_{2f})P & pP \end{bmatrix}$$

- Corruption dominant unless vp_{1f} quite high
- Problem $p > p'', \; p' > p''$
- Differentiate vp_{1f} from vp_{2f}
- Testable implications
- Model validation
- Role of engineering?

Next class

- Unification of the behavioral sciences
- Role of game theory?
- Behavioral game theory?
- Research directions