

Lecture 11

- backward induction -

Review

- Common knowledge of rationality
- The beauty contest

Common knowledge

- Event that player i know the other players are rational

$$K_i^1 = [\forall j \neq i, \phi_i^\omega[\mathbf{s}_j(w) = s_j] > 0 \rightarrow s_j \in B_j(S_{-j})]$$

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- Event that player i knows that every player knows that every player is rational

$$K_i^2 = K_i^1 \cap [\forall j \neq i, \phi_i^\omega[\mathbf{s}_j(w) = s_j] > 0 \rightarrow s_j \in B_j(K_j^1)]$$

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- Knowledge of player i of degree r

$$K_i^r = K_i^{r-1} \cap [\forall j \neq i, \phi_i^\omega[\mathbf{s}_j(w) = s_j] > 0 \rightarrow s_j \in B_j(K_j^{r-1})]$$

CKR

$$K^r = \bigcap_{i=1}^n K_i^r$$

$$K^\infty = \bigcap_{r \geq 1} K^r$$

CKR

- Common (mutual) knowledge of degree r

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- Event of common knowledge of rationality

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CKR

- Common (mutual) knowledge of degree r

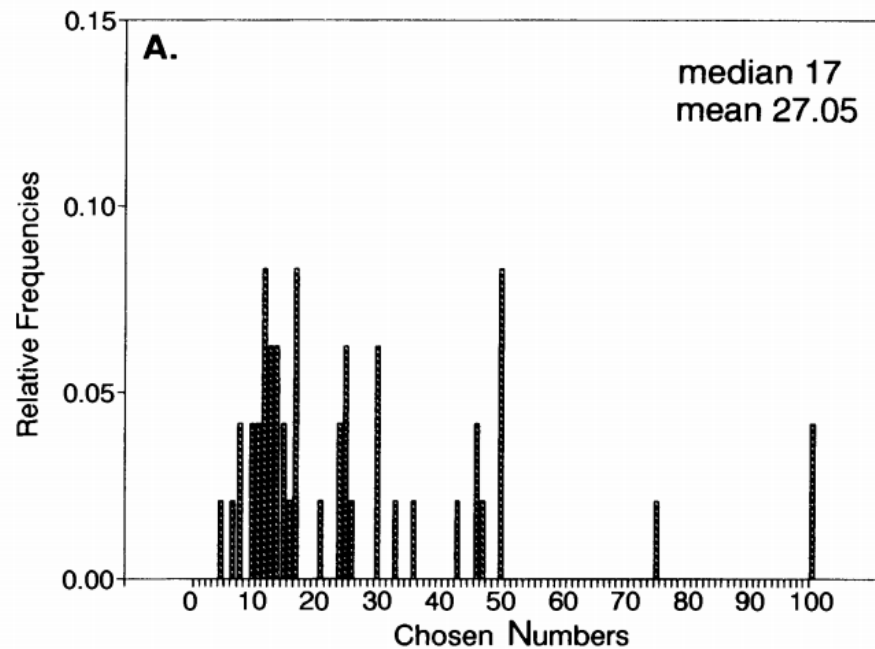
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- Event of common knowledge of rationality

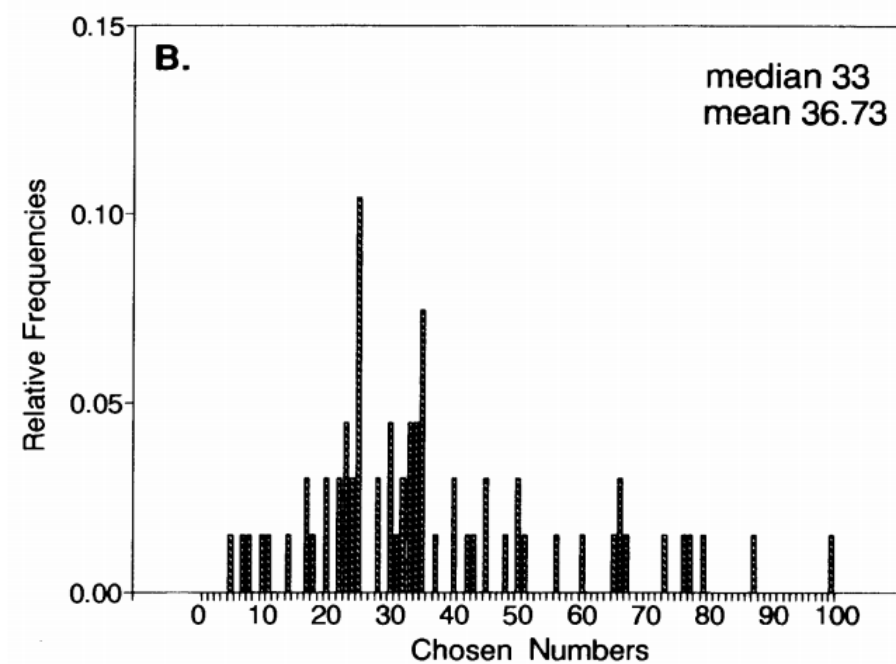
$$K^\infty = \bigcap_{r \geq 1} K^r$$

- CKR cannot simply be assumed
- Failure of CKR **does not** imply irrationality
- Players may still be Bayesian rational

Experimental results



$$p = 1/2$$



$$p = 2/3$$

- Depth of reasoning: $2 < r < 3$
- Nash? All players announce zero!
- Only rationalizable strategy!

Rosemarie Nagel (1995) Unraveling in
Guessing Games: An Experimental Study

Today

- Backward induction (extensive form game)
 - technique of iterated elimination
 - weakly dominated strategies
- Subgame perfect Nash equilibria
 - Nash in every subgame
- Perfect Bayesian Nash equilibria
 - Best course of action given the player's beliefs

Today

- Backward induction (extensive form game)
 - technique of iterated elimination
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 - Best course of action given the player's beliefs
- CKR $\rightarrow$ backward induction

Backward induction

Market preemption

- Market power in a small region
- DuPont cost leader in the industry (titanium dioxide)
- Plans: increase production (far beyond current needs)
- Federal Trade Commission v. DuPont

FEDERAL TRADE COMMISSION ACT, § 5, 15 U.S.C. § 45

Sec. 45. Unfair methods of competition unlawful; prevention by Commission

(a) Declaration of unlawfulness; power to prohibit unfair practices; inapplicability to foreign trade

(1) Unfair methods of competition in or affecting commerce, and unfair or deceptive acts or practices in or affecting commerce, are hereby declared unlawful.

(2) The Commission is hereby empowered and directed to prevent persons, partnerships, or corporations, except banks, savings and loan institutions described in section 57a(f)(3) of this title, Federal credit unions described in section 57a(f)(4) of this title, common carriers subject to the Acts to regulate commerce, air carriers and foreign air carriers subject to part A of subtitle VII of title 49, and persons, partnerships, or corporations insofar as they are subject to the Packers and Stockyards Act, 1921, as amended (7 U.S.C. 181 et seq.), except as provided in section 406(b) of said Act (7 U.S.C. 227(b)), from using unfair methods of competition in or affecting commerce and unfair or deceptive acts or practices in or affecting commerce.

(3) This subsection shall not apply to unfair methods of competition involving commerce with foreign nations (other than import commerce) unless -

(A) such methods of competition have a direct, substantial, and reasonably foreseeable effect -

(i) on commerce which is not commerce with foreign nations, or on import commerce with foreign nations; or

(ii) on export commerce with foreign nations, of a person engaged in such commerce in the United States; and

(B) such effect gives rise to a claim under the provisions of this subsection, other than this paragraph.

If this subsection applies to such methods of competition only because of the operation of subparagraph (A)(ii), this subsection shall apply to such conduct only for injury to export business in the United States.

DuPoint act improperly?

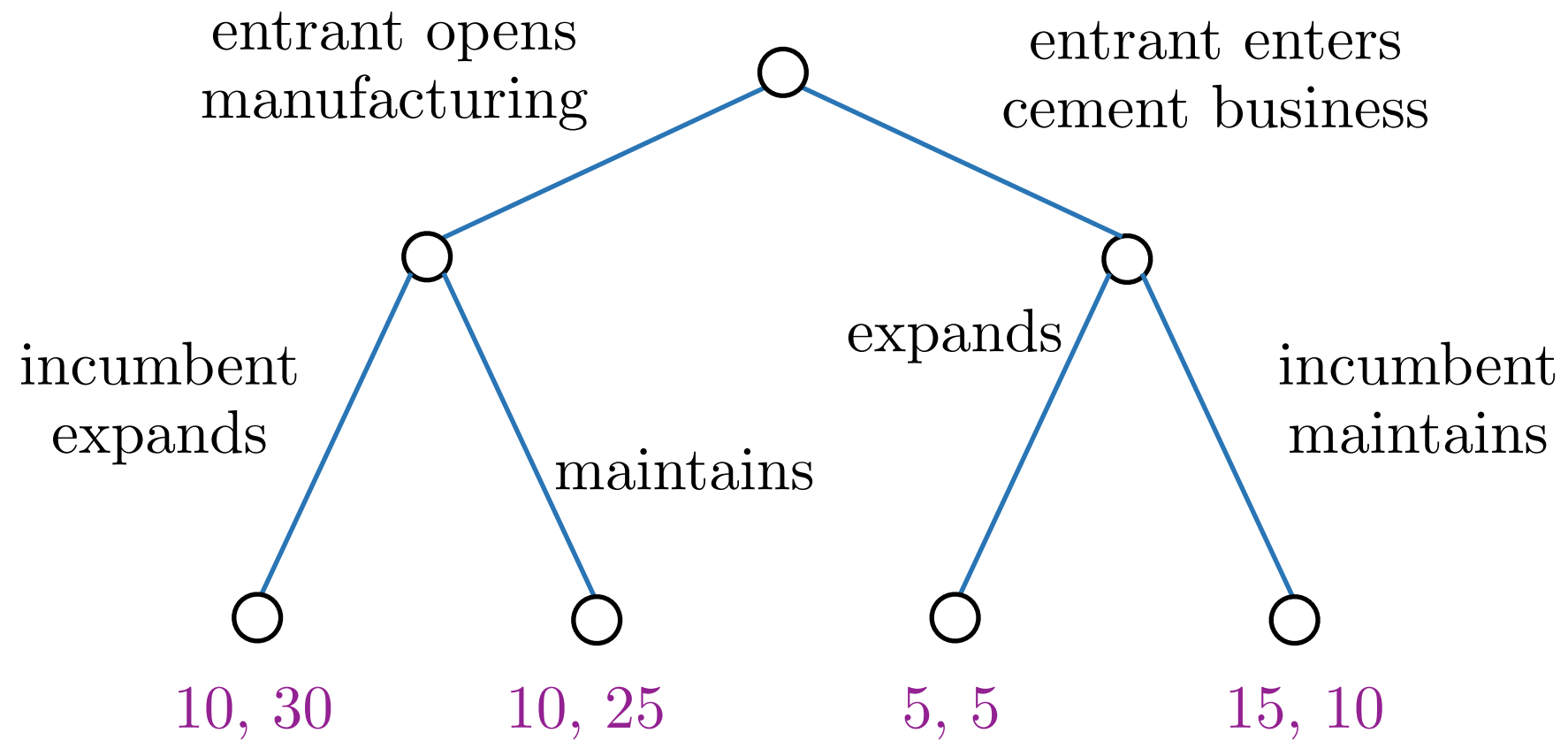
A model

- Incumbent (expand or not?)
- Entrant (build second cement plant or not?)
- **Game:** the benefits each enjoys from either course of action depends on what the other does

A model

- Incumbent (expand or not?)
- Entrant (build second cement plant or not?)
- **Game:** the benefits each enjoys from either course of action depends on what the other does
- **Timing is crucial!**

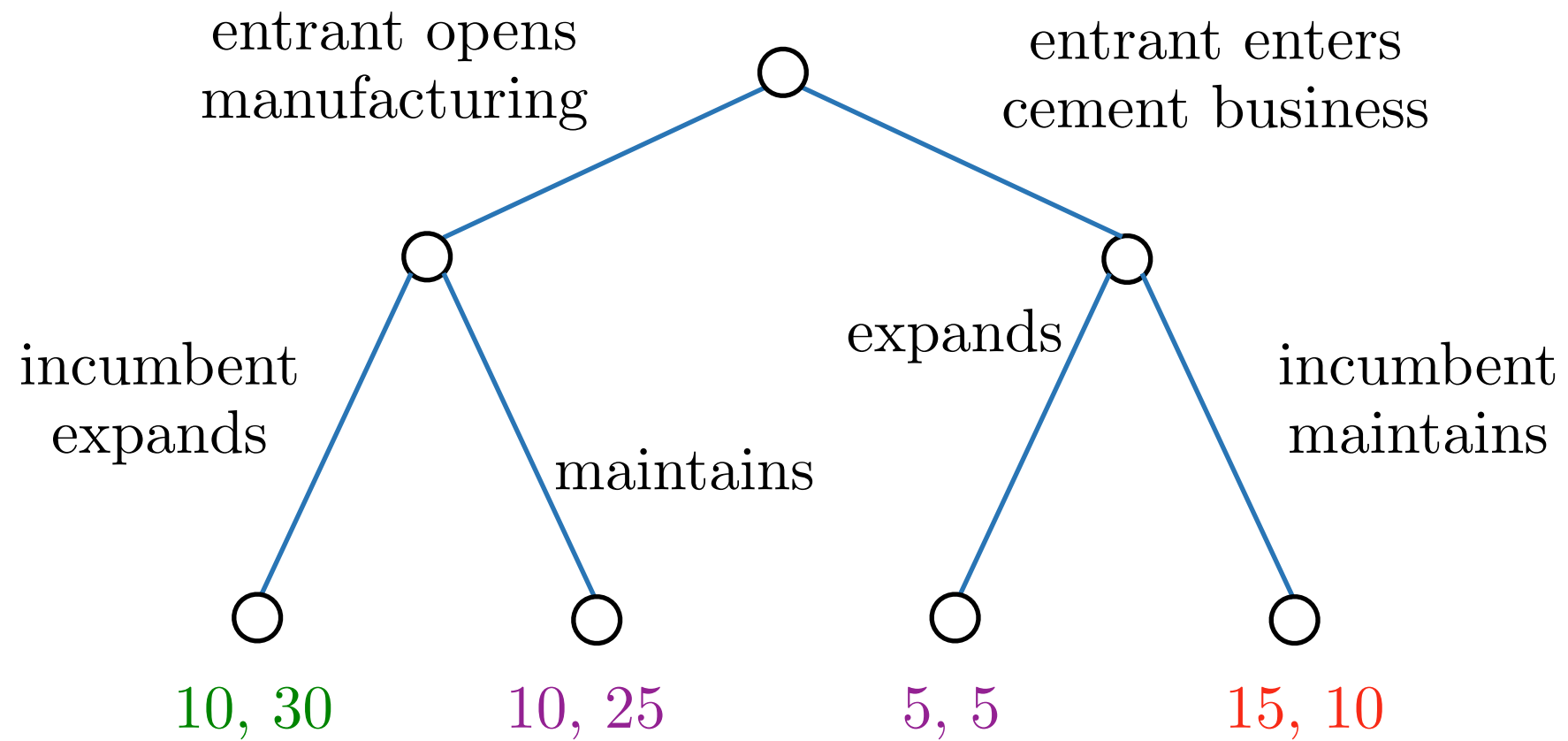
Entrant moves first



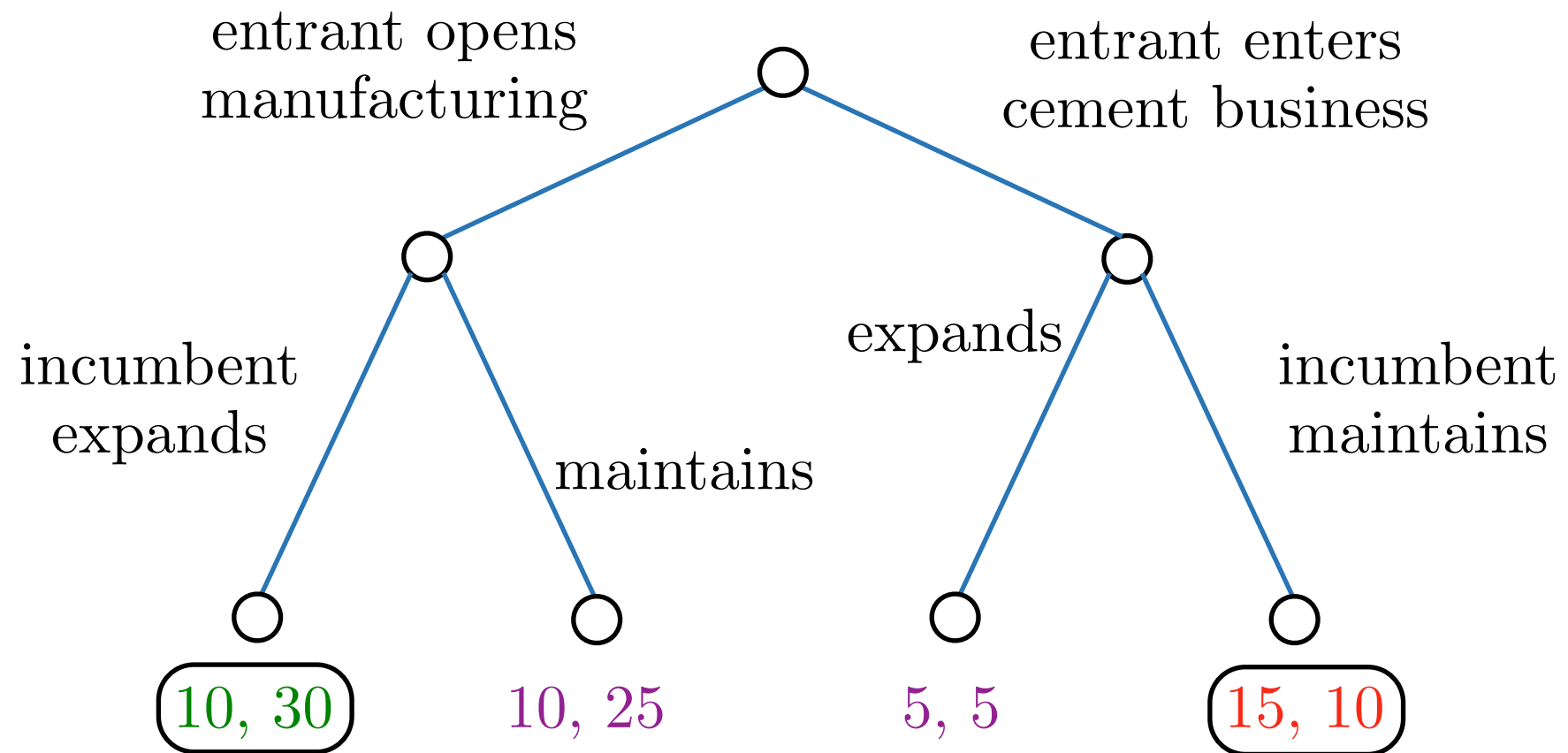
Backward induction

- Reason backwards in time
- Determine a sequence of optimal actions
- Best action for every possible information set

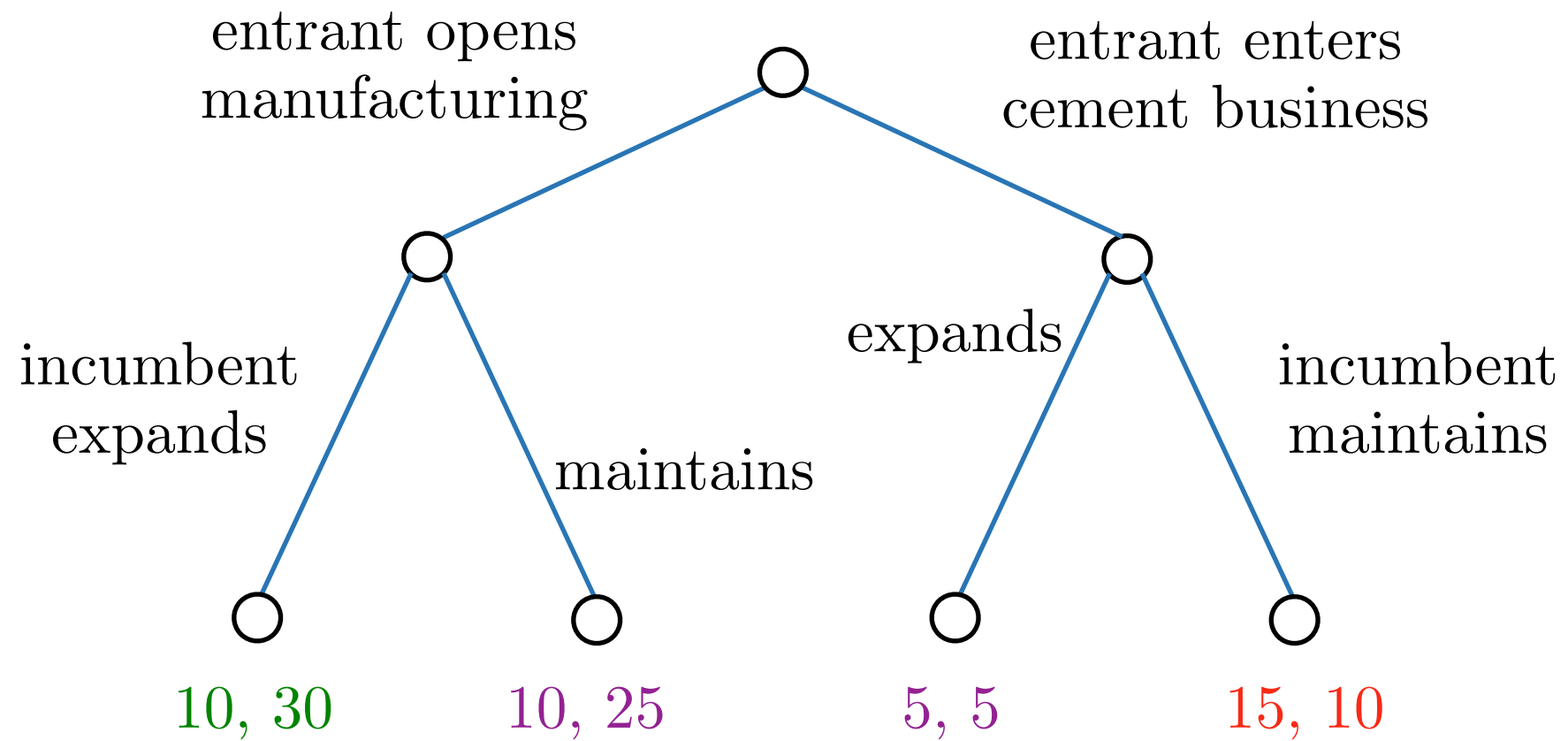
Entrant moves first



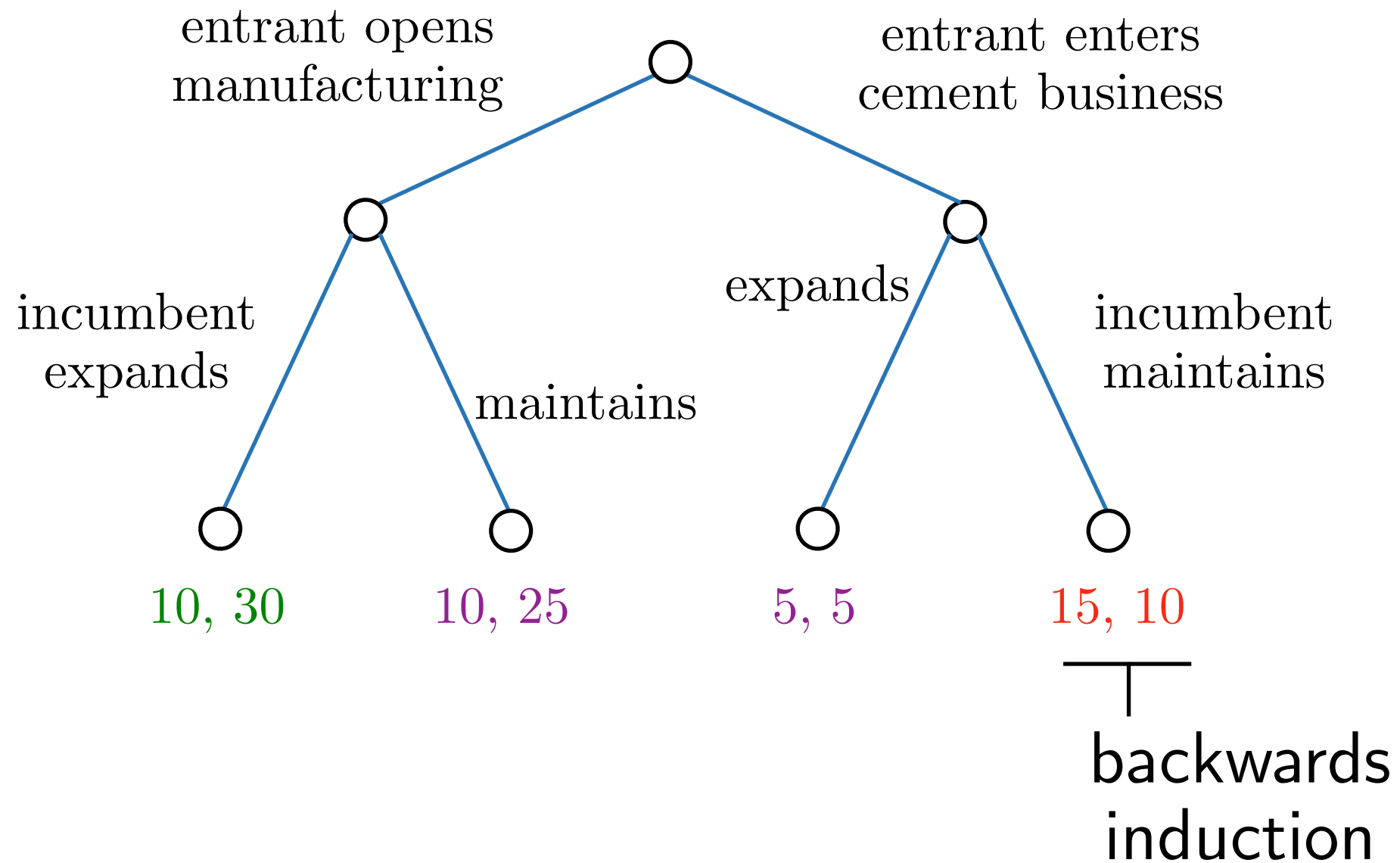
Entrant moves first



Entrant moves first

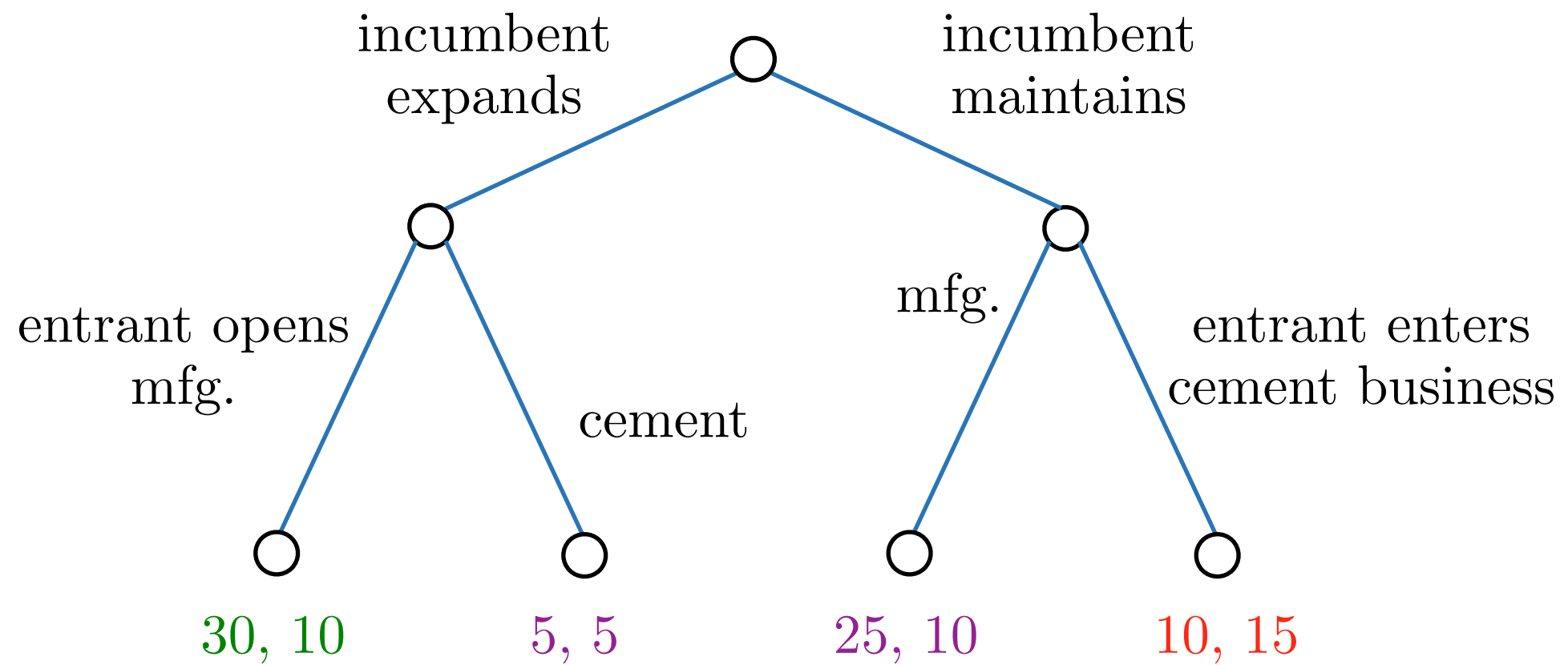


Entrant moves first

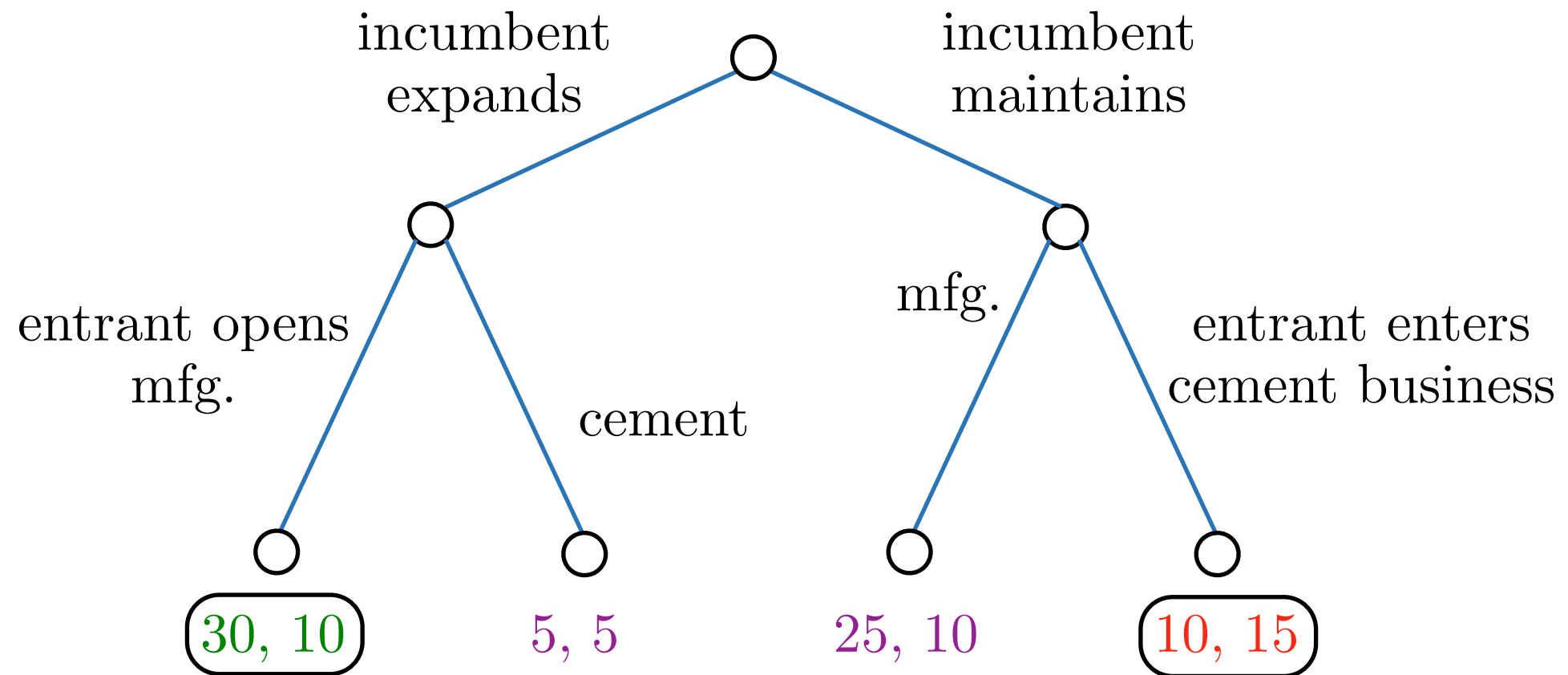


Nash! Does not maximize the joint profits!

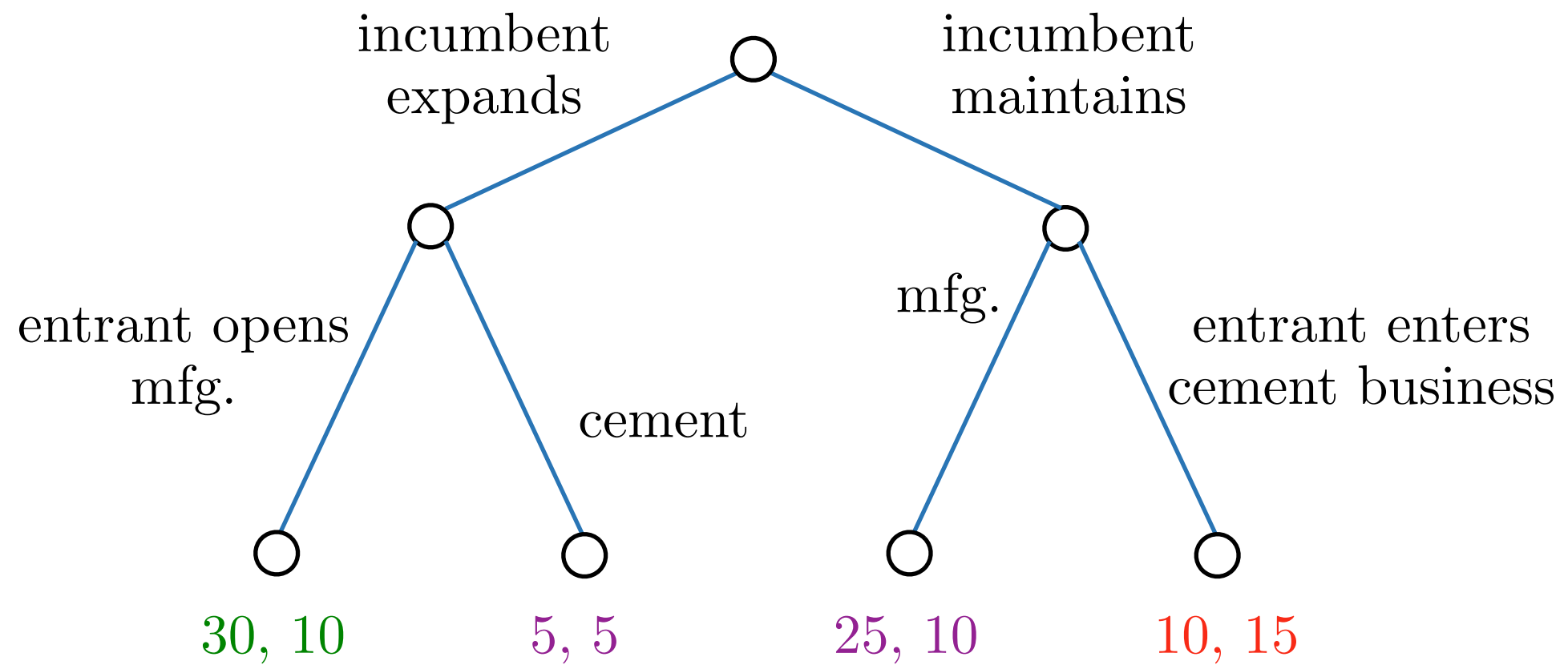
Incumbent moves first



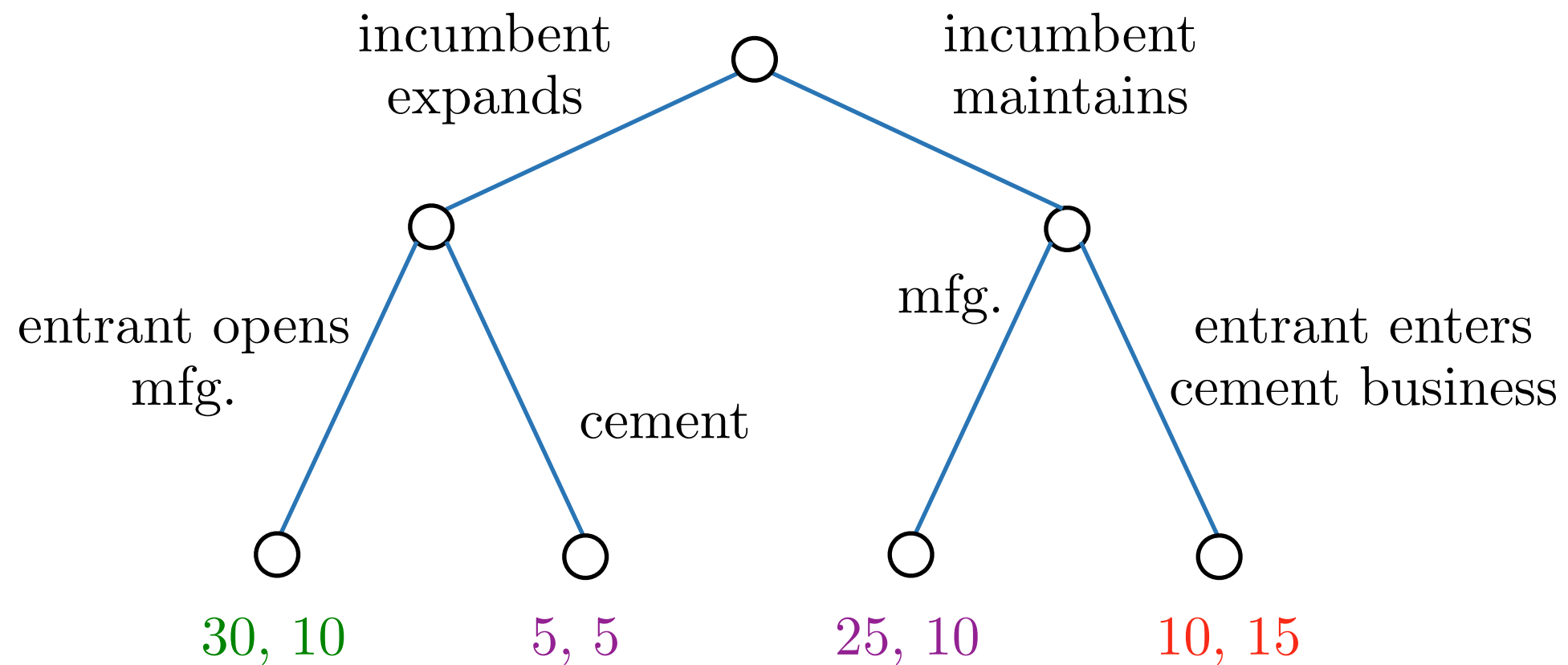
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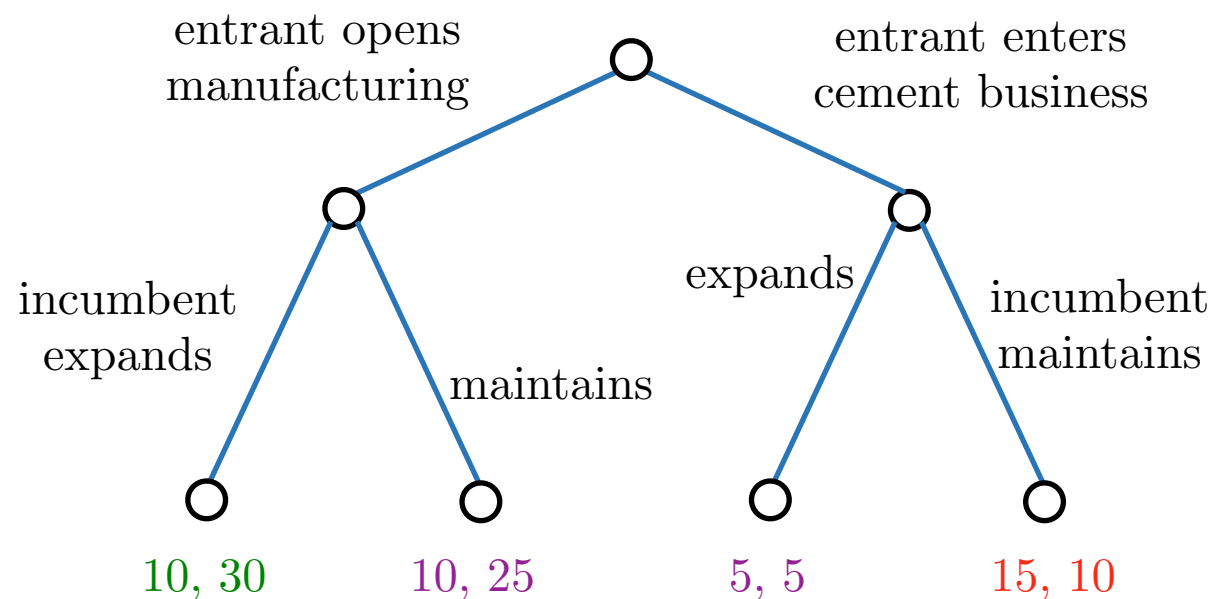


└
backwards
induction

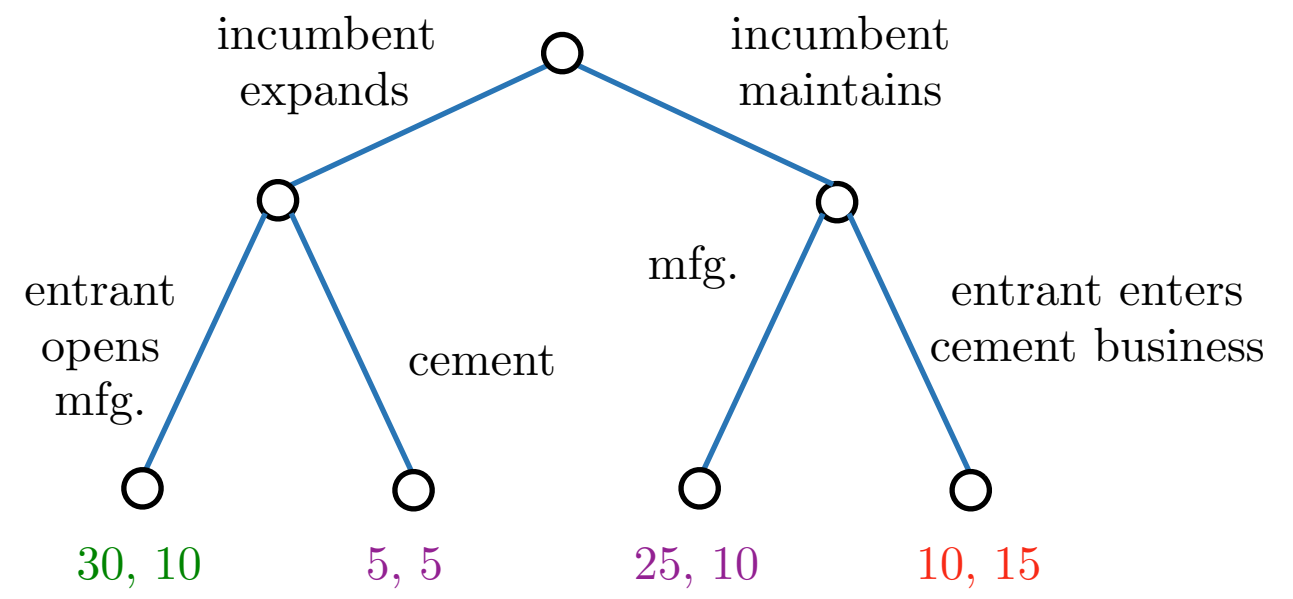
Nash! Maximizes joint outcome!

Rationalizing the game

- The game's outcome: who moves first?!
- Can Incumbent change the outcome of the game?
- Incumbent's threat is not credible - Incumbent makes itself worse off by expanding
- How can Incumbent commit itself to expanding before Entrant decides what to do?



Entrant moves first



Incumbent moves first

Sink investment in expansion

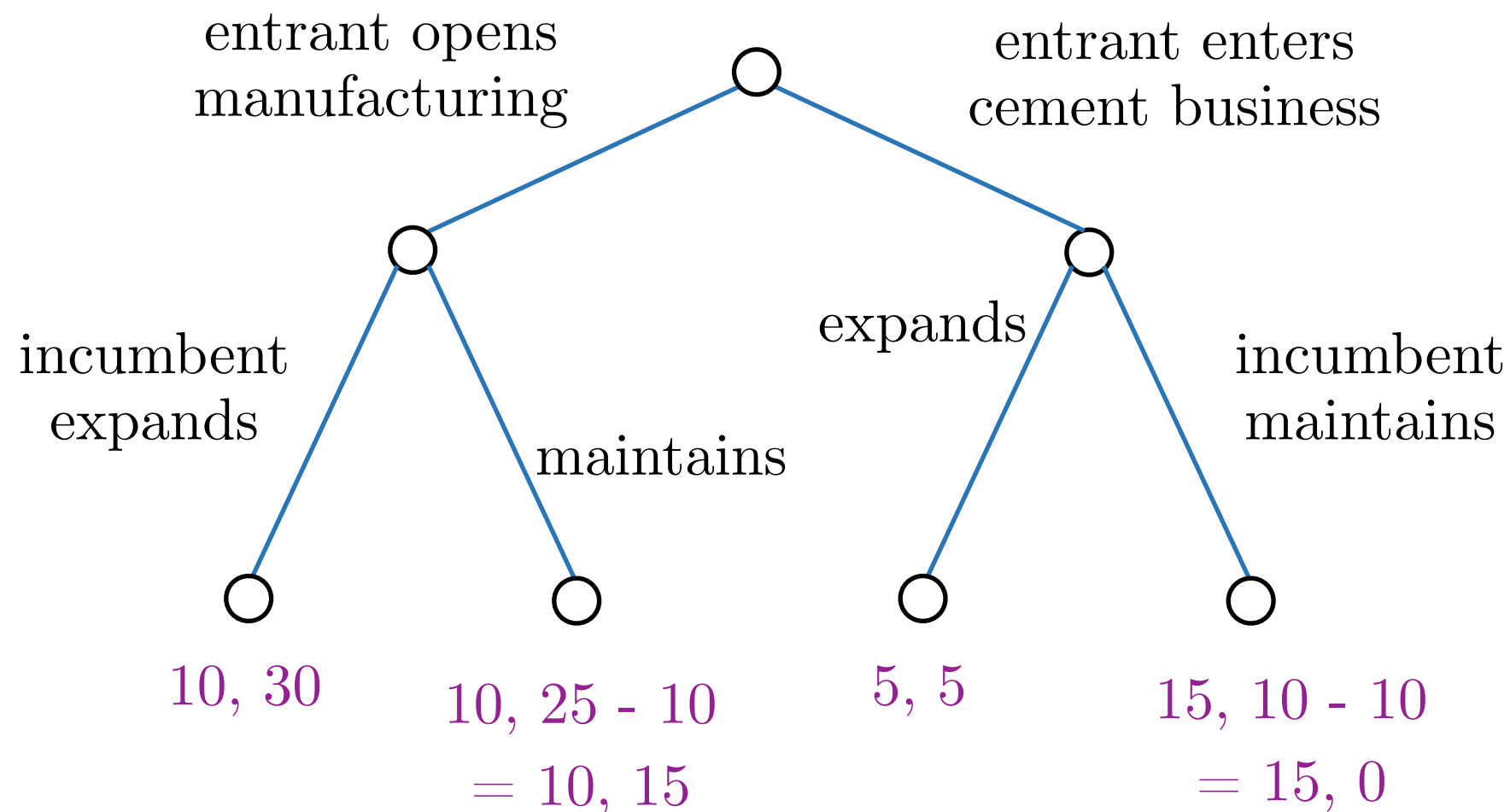
- Spend some money early
- Invest in expansion equipment
- Expansion prior to when the capacity is needed

Sink investment in expansion

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Remarks

- DuPont had reason to commit itself to increase production
 - No sense for DuPont to expand after competitors enter the market
 - Company's decision did not affect the strategies available to other players

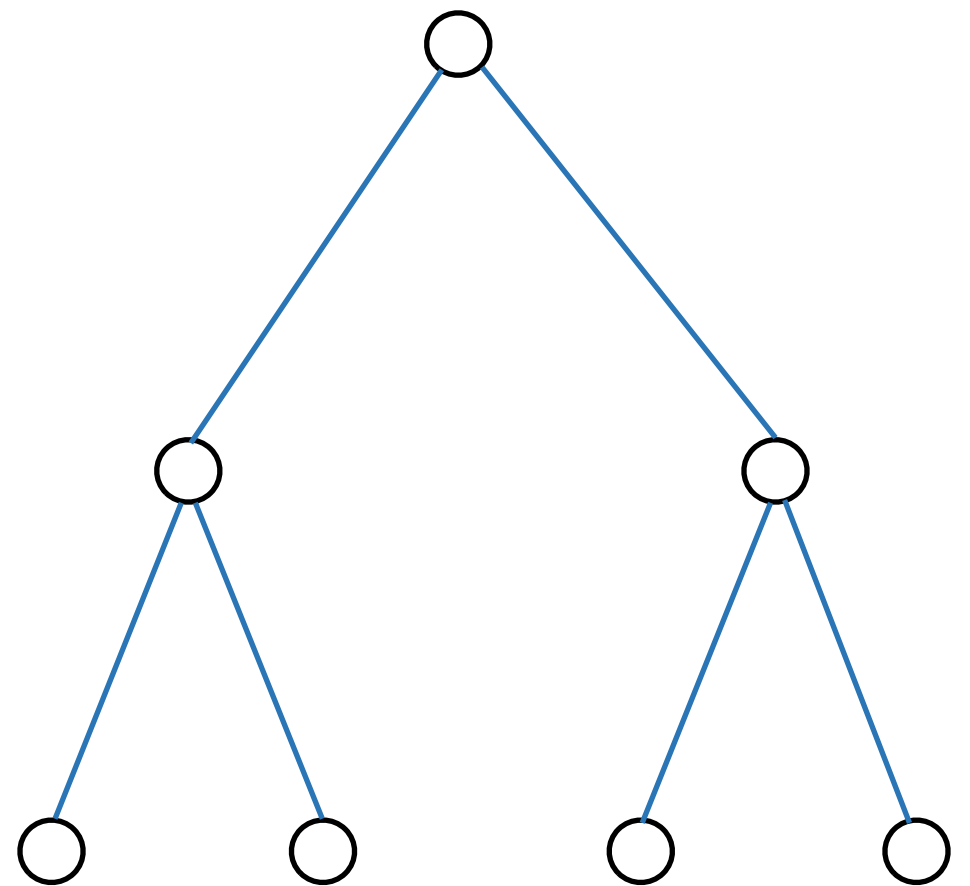
Remarks

- DuPont had reason to commit itself to increase production
 - No sense for DuPont to expand after competitors enter the market
 - Company's decision did not affect the strategies available to other players
- By making its investments early, DuPont changed the equilibrium strategies of the other players
 - No sanctions
 - Risk shielding less efficient producers

Nash Refinement

- Isolate strategies parties are **more likely to adopt**
- Any solution found through backwards induction is a Nash
- Backwards induction not applicable when a player **does not know previous move**

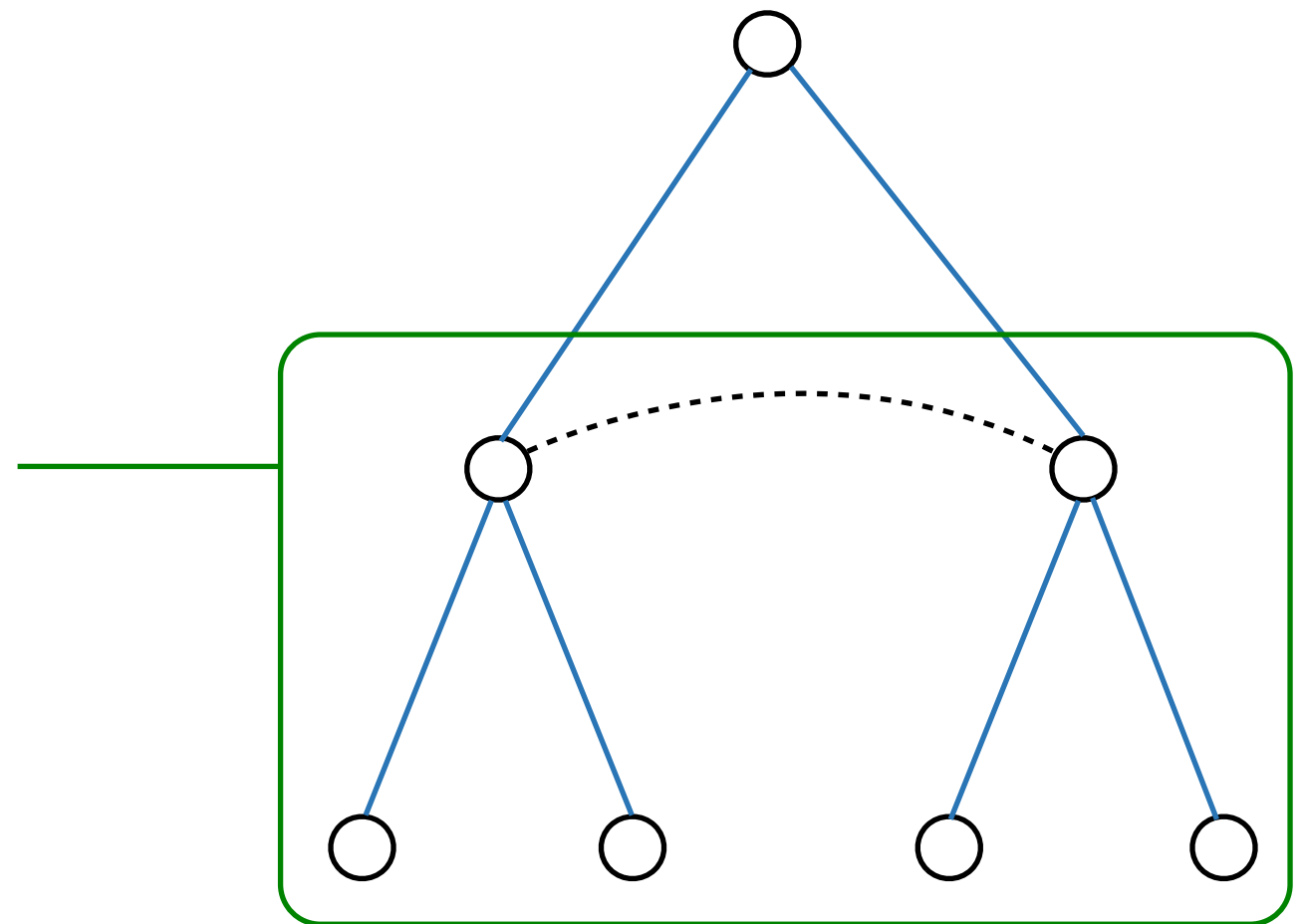
each information set
is a single node



Nash Refinement

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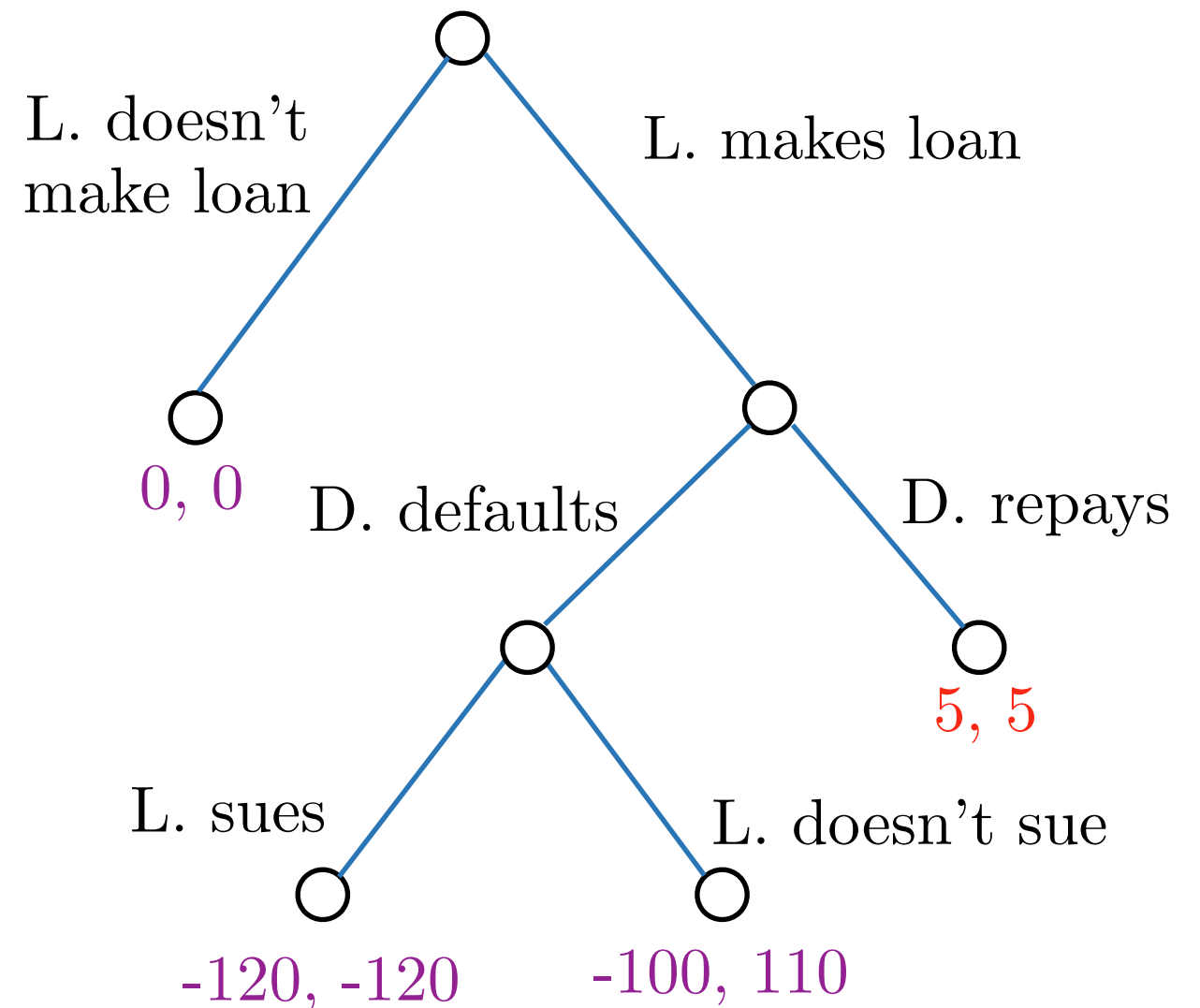
no proper subgame
because they share
same information set



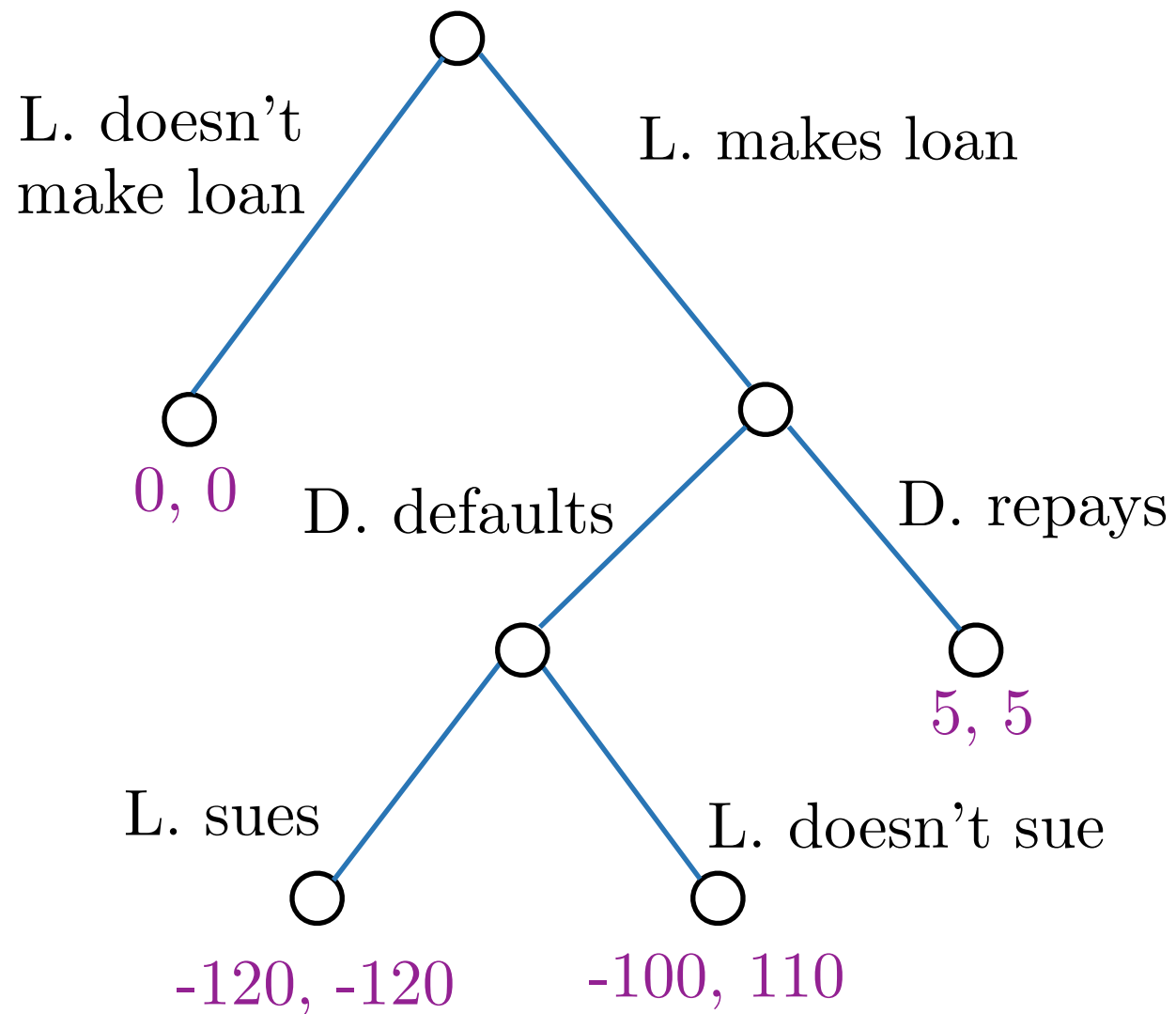
Example

Lender/debtor problem

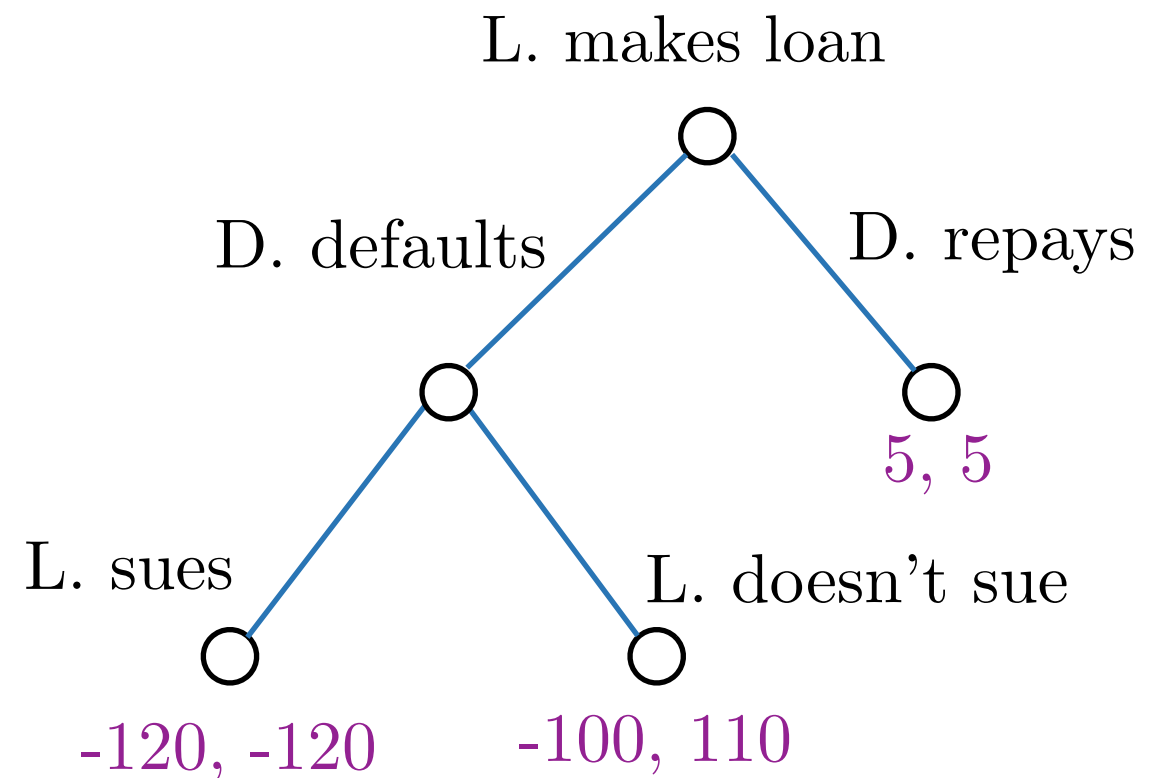
- Contract **with costly** enforcement (125)
- L. makes a loan and sues if debtor does not repay, D. repays is a Nash, **but not plausible**
- Debtor knows lender will not sue
- **Subgame perfection:** candidate be a Nash in every subgame



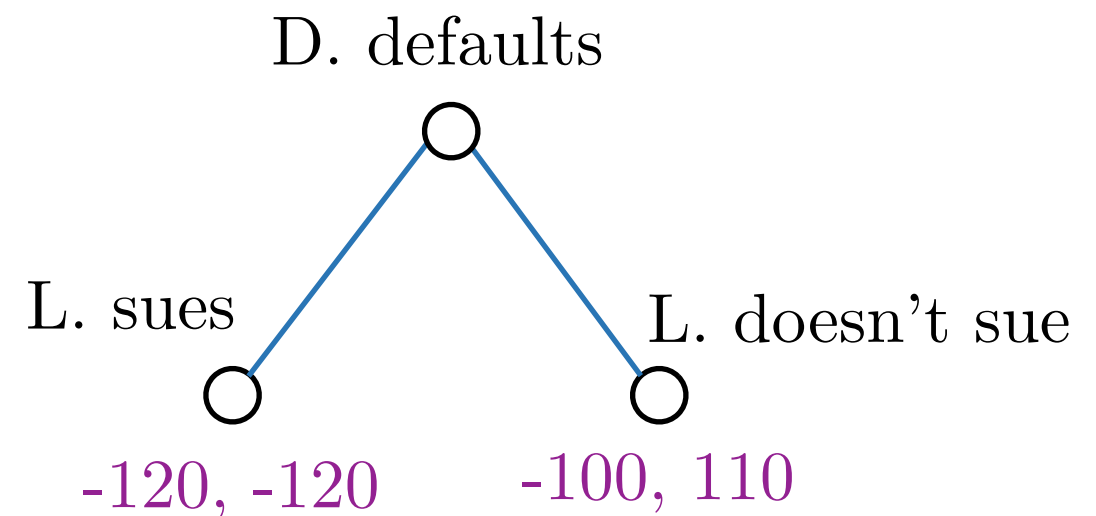
Game three



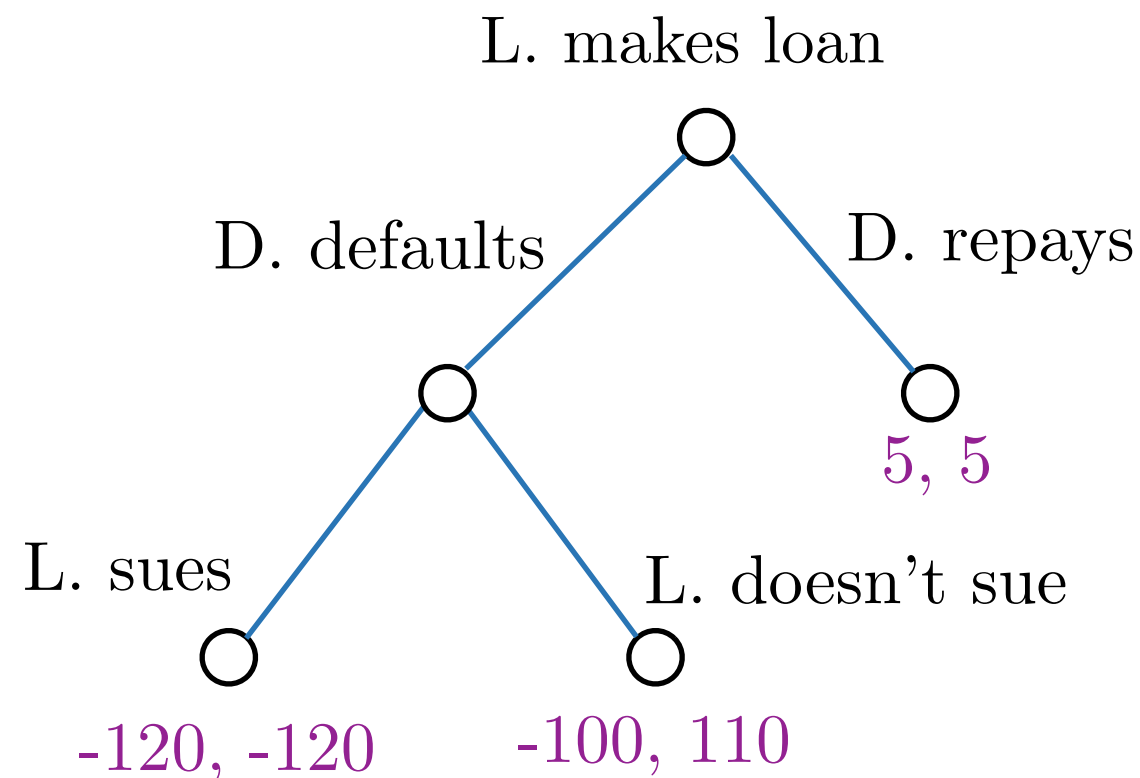
Subgame 1



Subgame 2

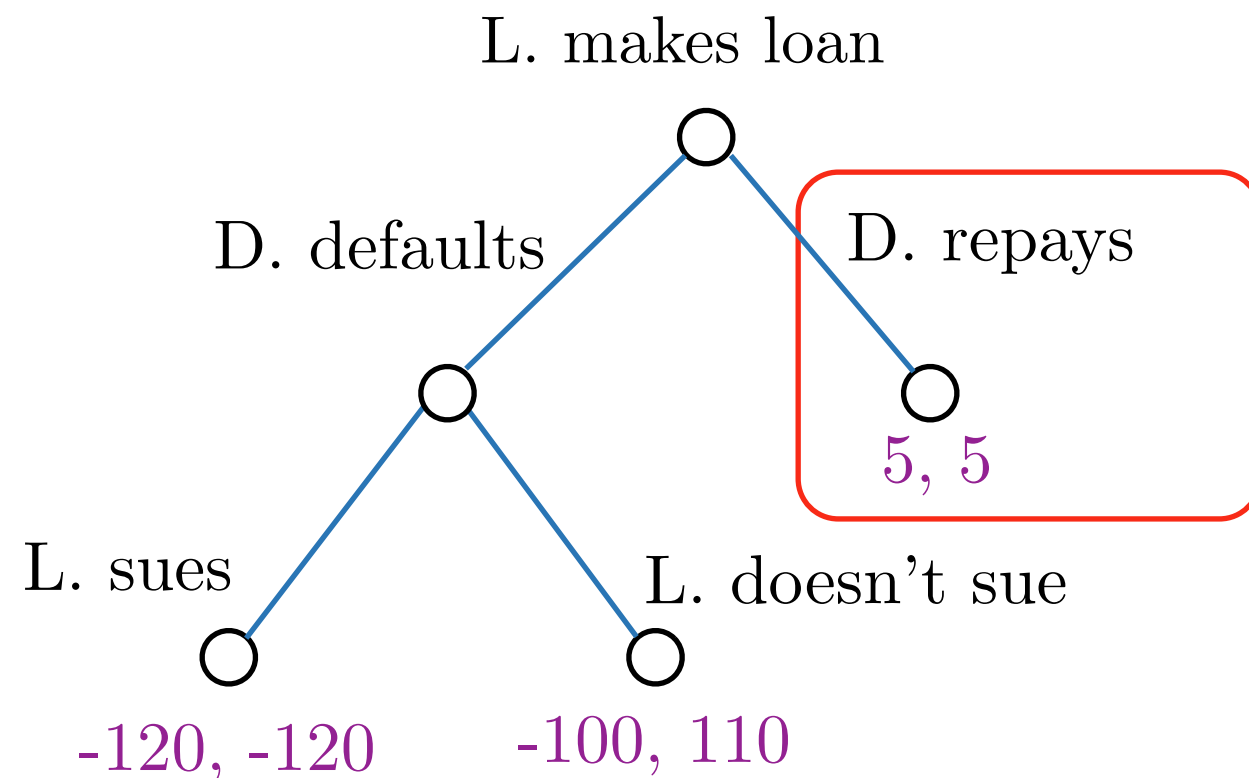


Subgame 1



- To be subgame perfect, a solution to the game cannot adopt a strategy in which Lender sues were Debtor to default
- The only subgame perfect solution is the one reach through backwards induction
- Eliminates Nash equilibria that are not plausible!

Subgame 1



Not a Nash in subgame 1!!

- To be subgame perfect, a solution to the game cannot adopt a strategy in which Lender sues were Debtor to default
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A mere verbal declaration of firmness
may not be credible

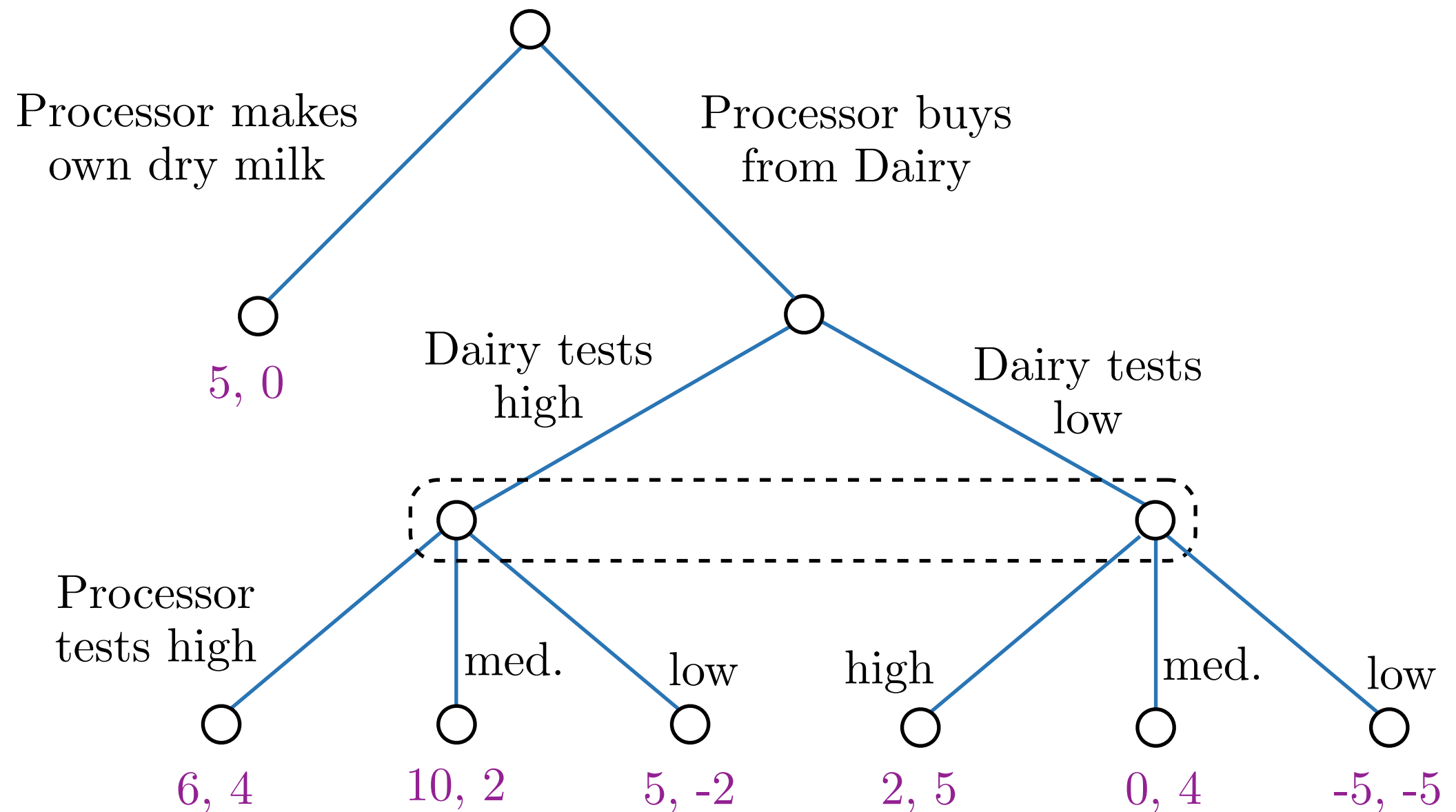
Making one's commitments credible is one of the finer arts in the realm of strategic games

How to make threats/promises credible?

- No value if the promise is broken
- Making a commitment:
 - sinking investment in expansion
 - may not waste resources
 - labor union may send their leaders into wage bargaining with firm instructions that tie their hands
 - examples?

Example 2

General Foods vs. Valley Lea Dairy



Backwards induction?

General Foods vs. Valley Lea Dairy

- Cannot reason backwards as in lender/debtor problem

General Foods vs. Valley Lea Dairy

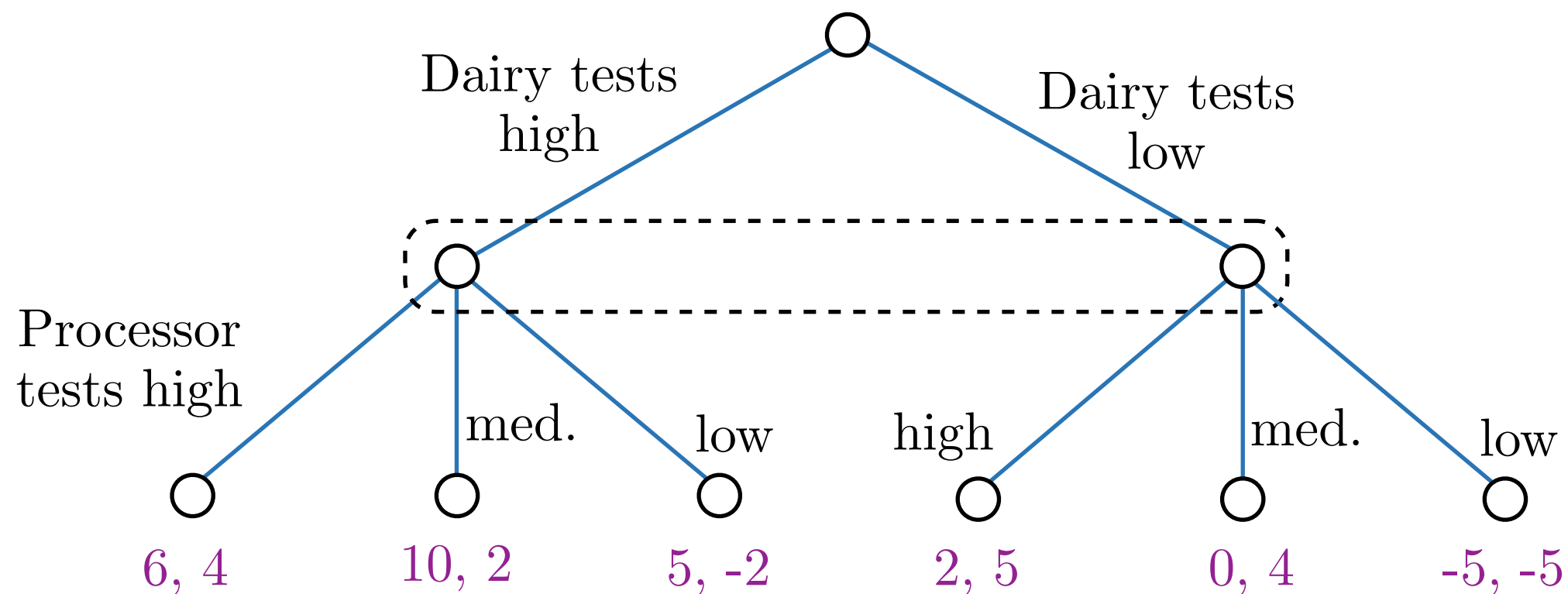
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- Solve using subgames
- **Normal form:** players don't know what the others have done

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Normal form

Dairy's level of care

High

Low

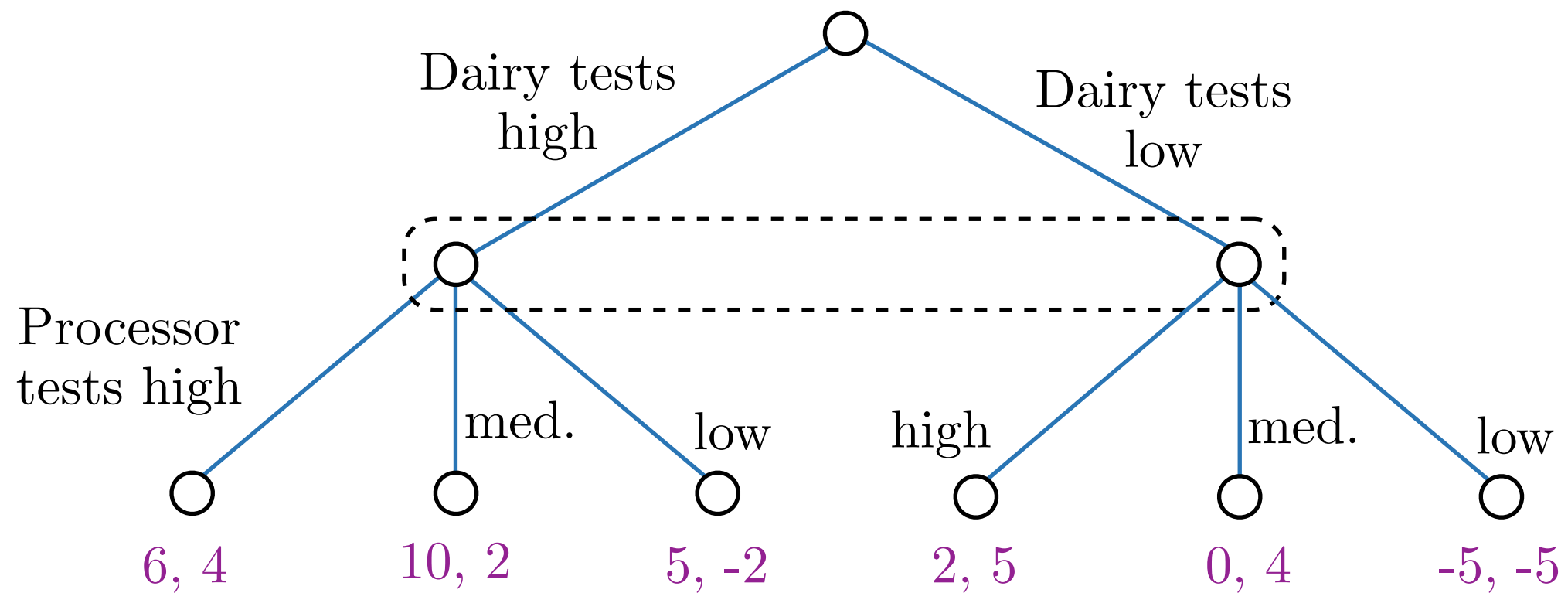
Processor's
level of care

High

Medium

Low

6, 4	2, 5
10, 2	0, 4
5, -2	-5, -5



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- Subgame has only one Nash
- Solution to the entire game becomes clear!
- Processor better off making its own dry milk
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- Perturb the payoffs by introducing a schedule of
 - damages Dairy owes
 - damages Processor recovers
- Both parties act with the interest of the other in mind!

Sale with warranty

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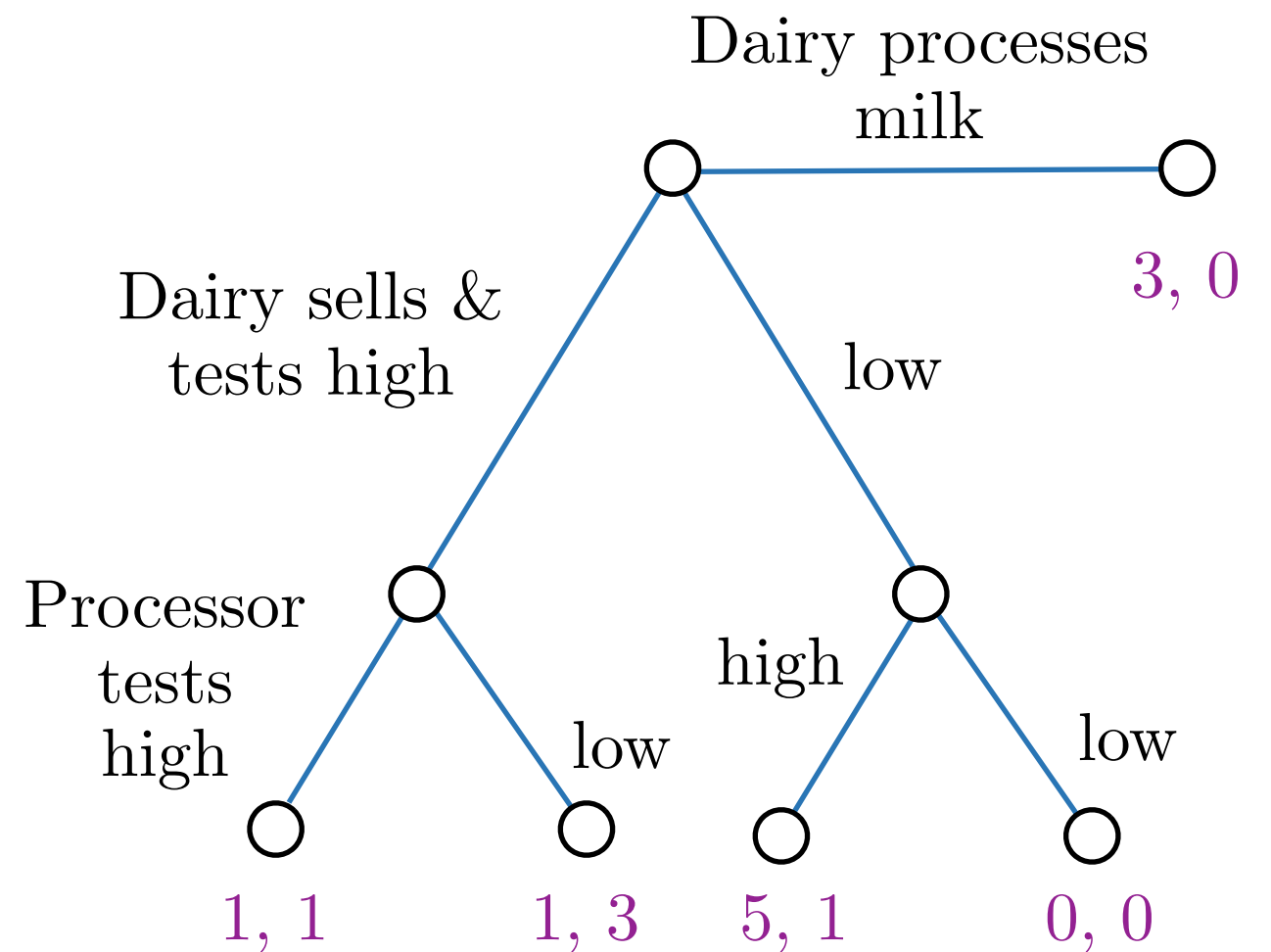
Remarks

- Uniform Commercial Code
- Provide incentives to exercise care
 - fewest rationality assumptions?
 - rule informationally parsimonious?

No dominated strategies?

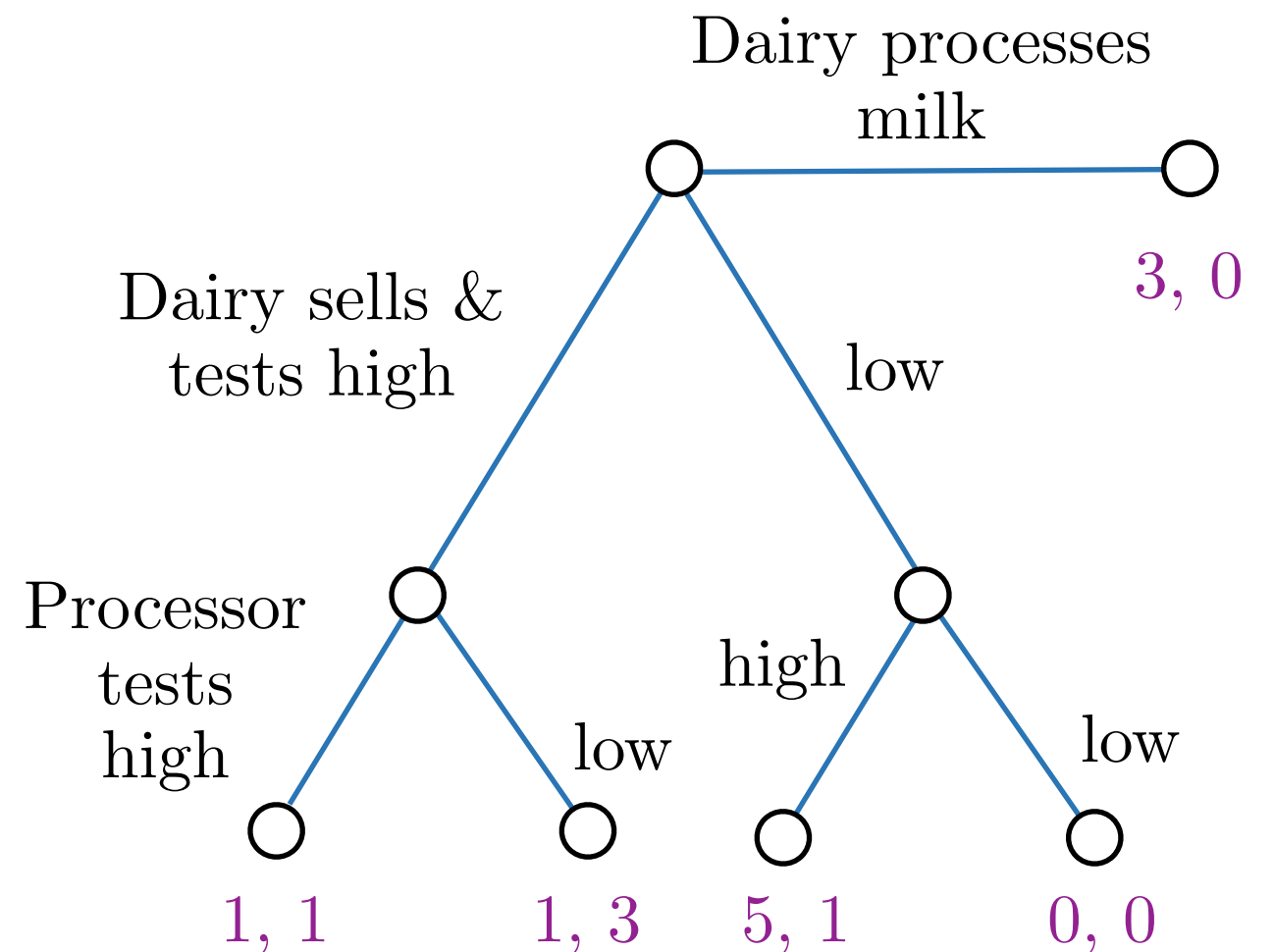
Example

- Use backwards induction



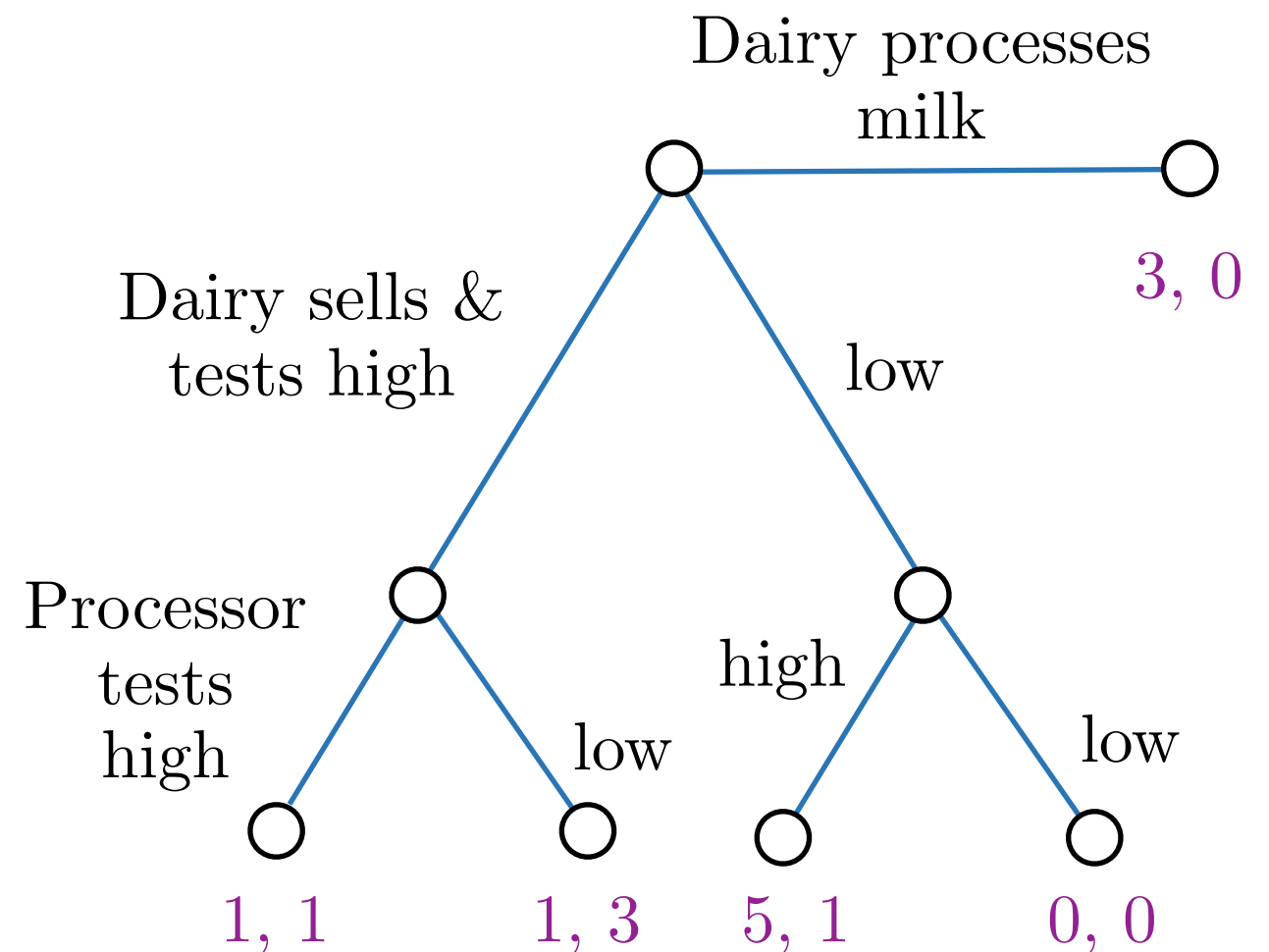
Example

- Use backwards induction
- **Focus:** who moves last?



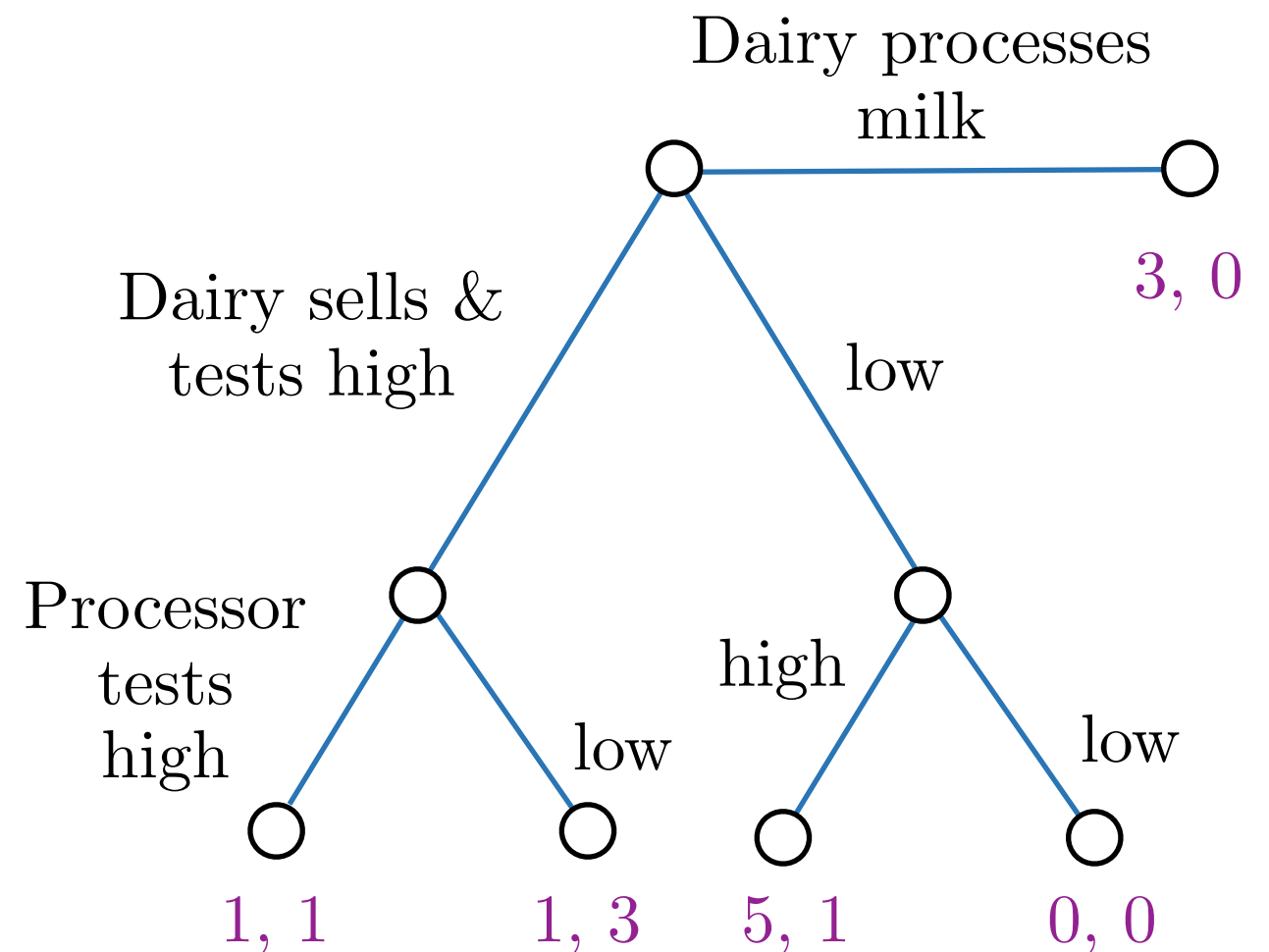
Example

- Use backwards induction
- **Focus:** who moves last?
- Dairy predicts how Processor will move...
- Dairy sells and test low
- Processor test high



Example

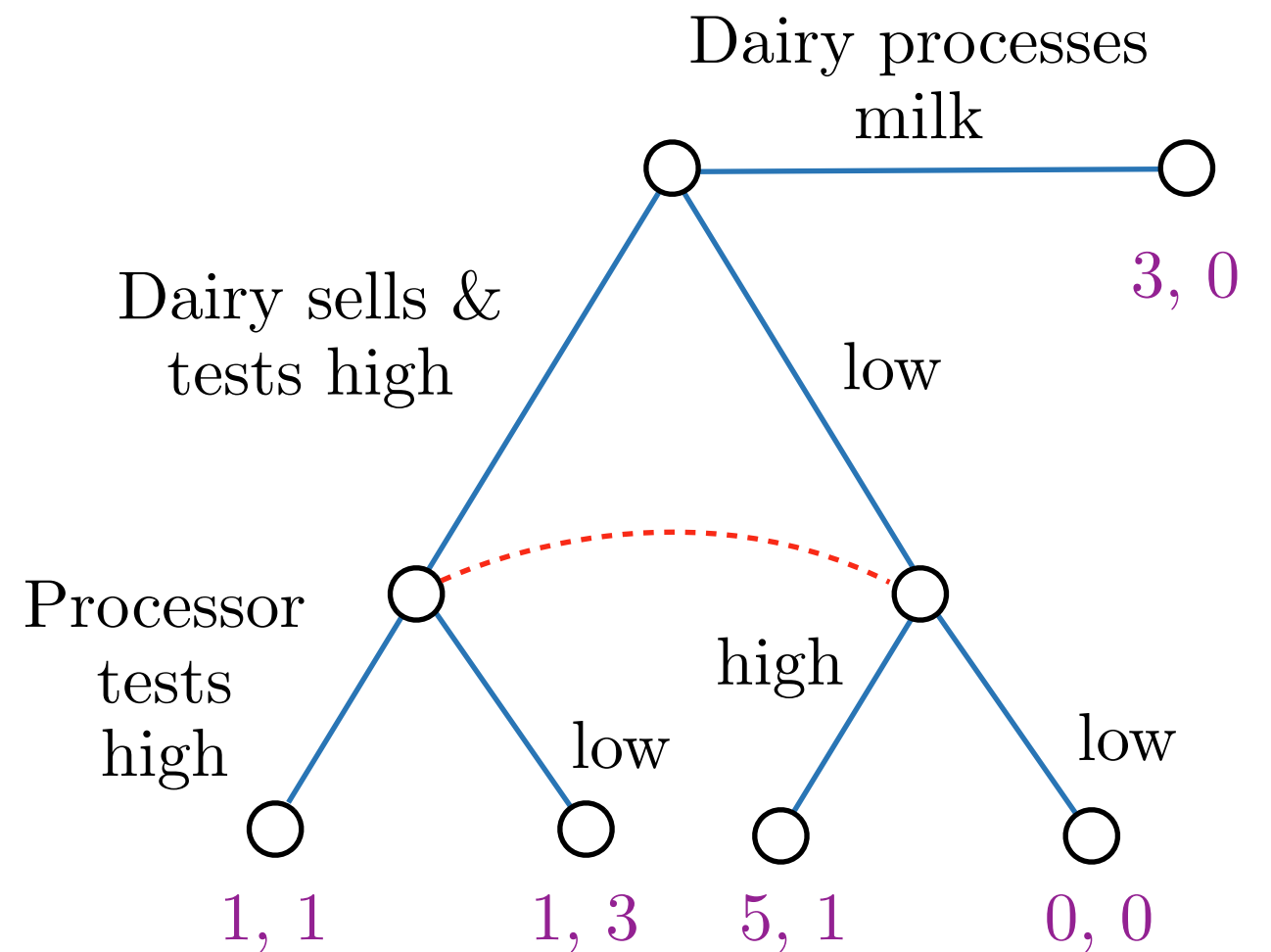
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What happens when P. does not know the test D. ran?

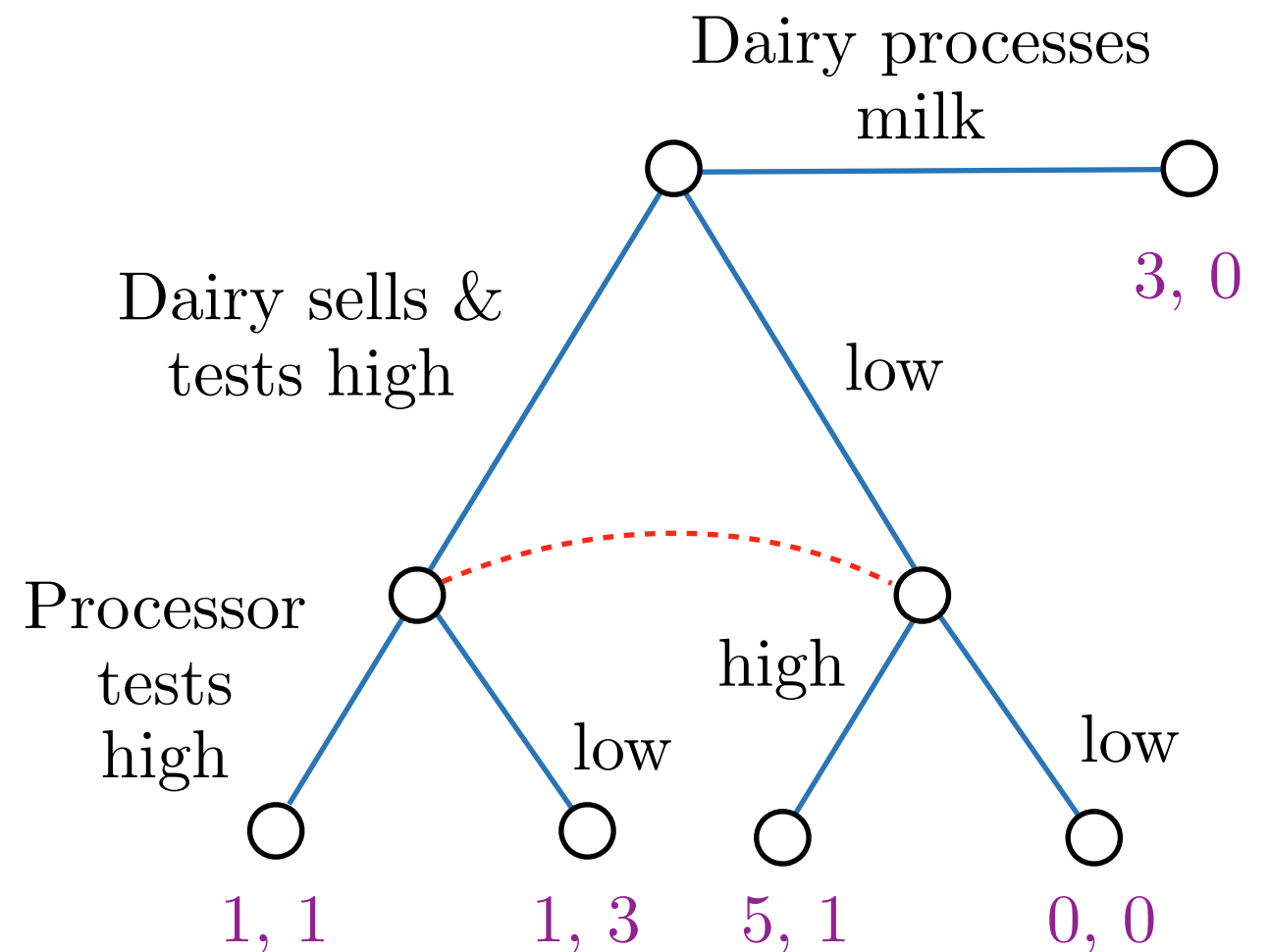
Example

- Knowledge of processors position is not built in!



Example

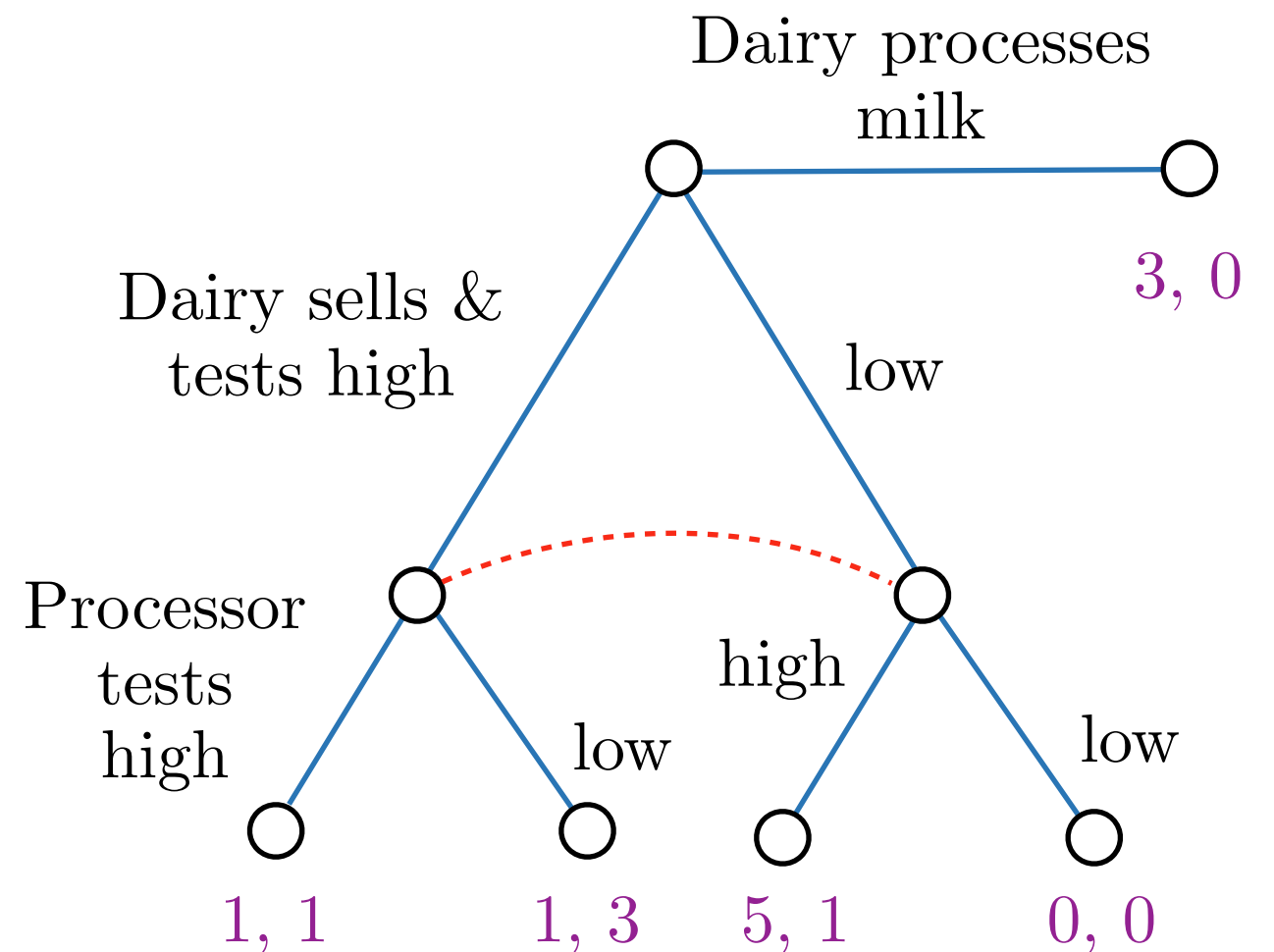
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Example

- Knowledge of processors position is not built in!
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		D.	
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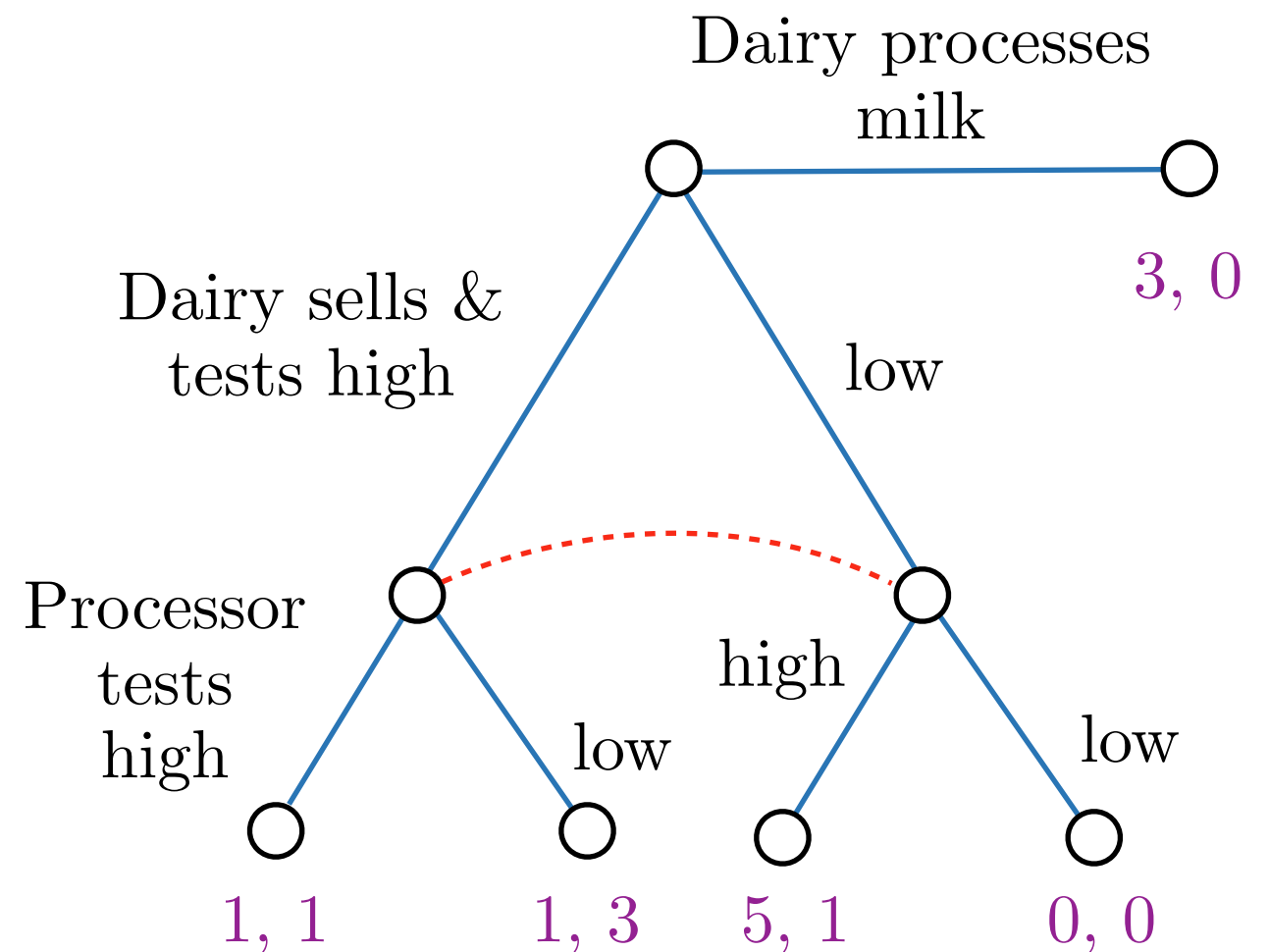


Example

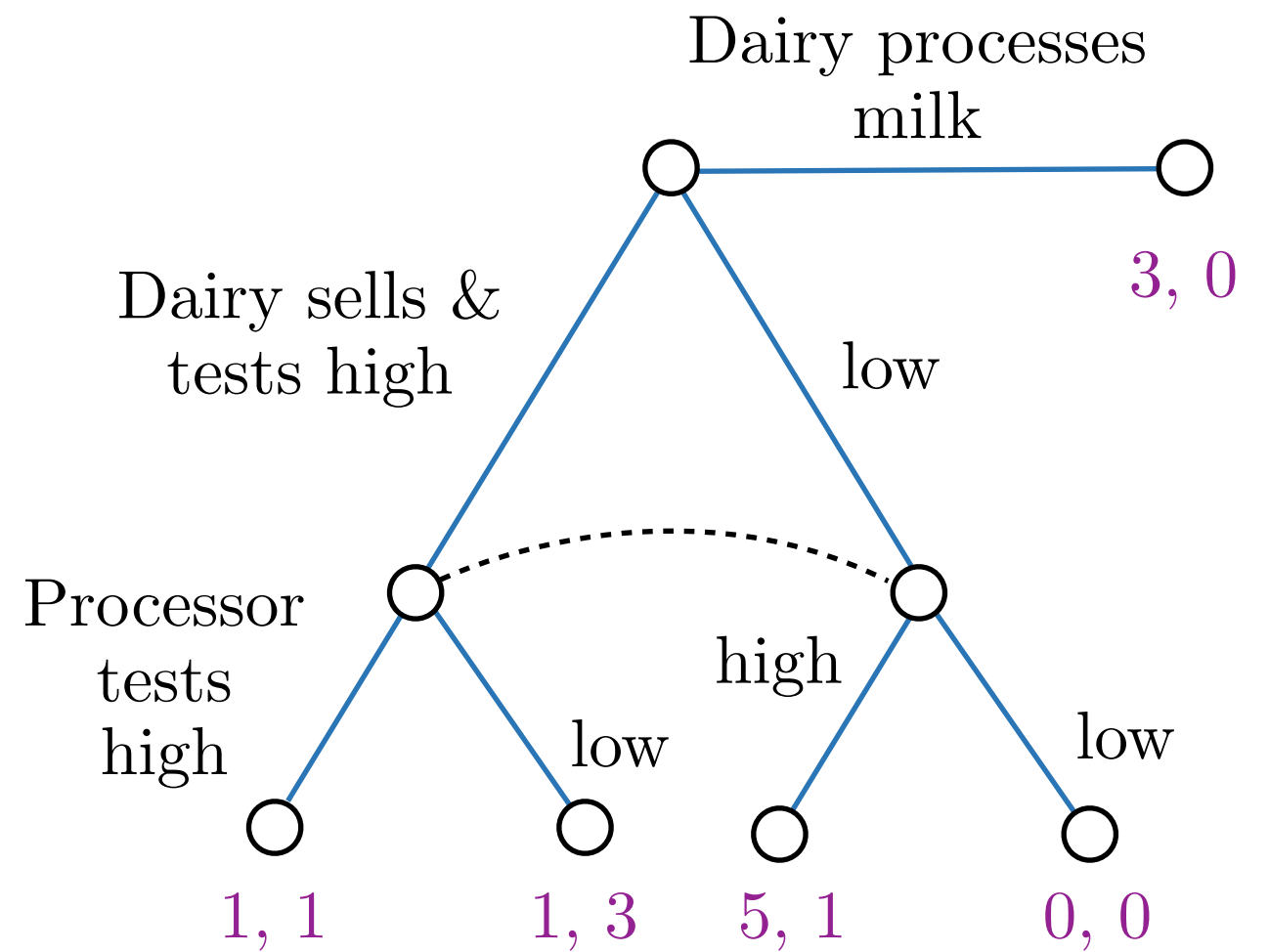
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- No dominated strategies
- Incorporating beliefs?

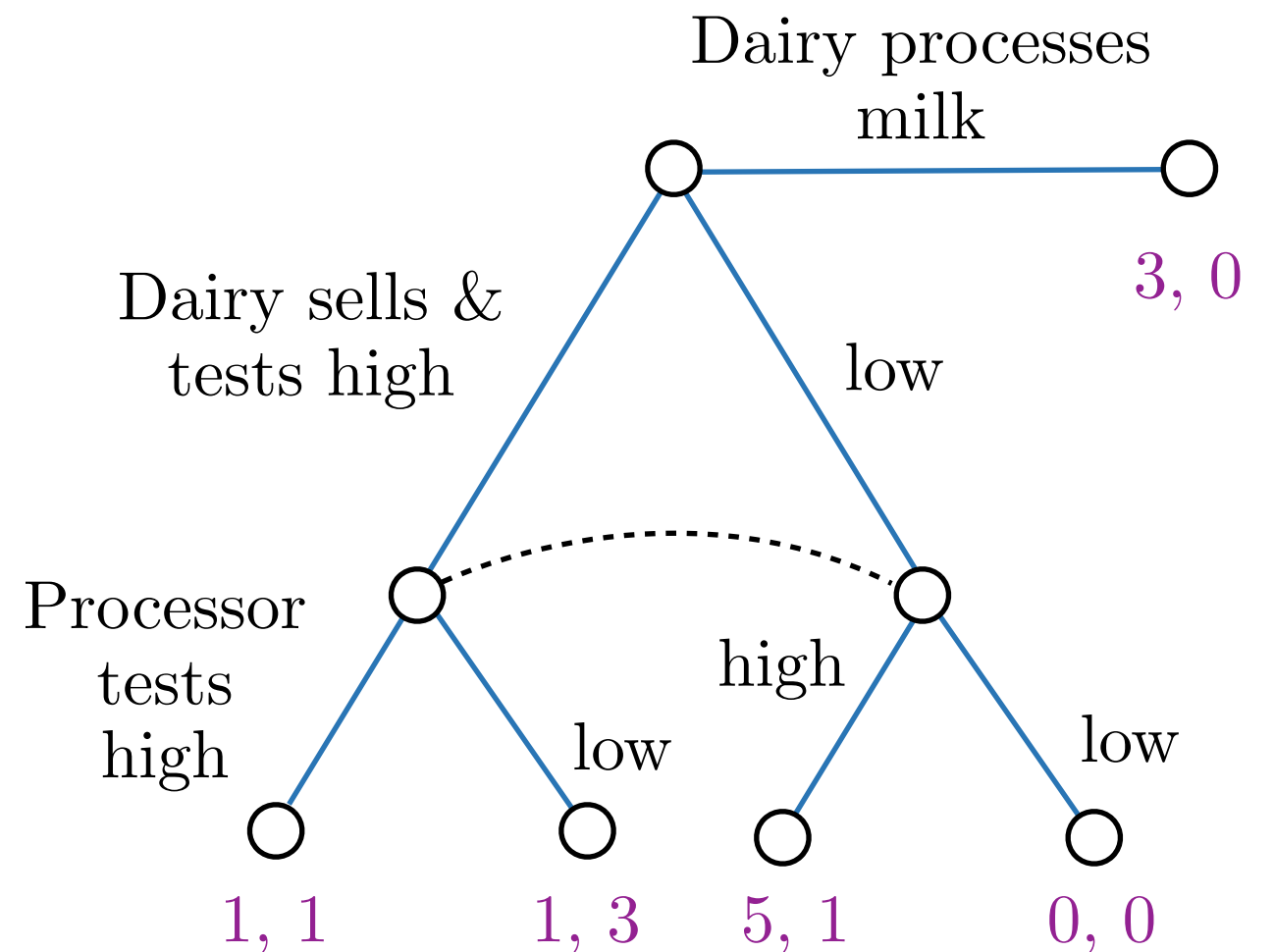


Incorporating beliefs...



Incorporating beliefs...

- What does D. believe about P.'s position?



Incorporating beliefs...

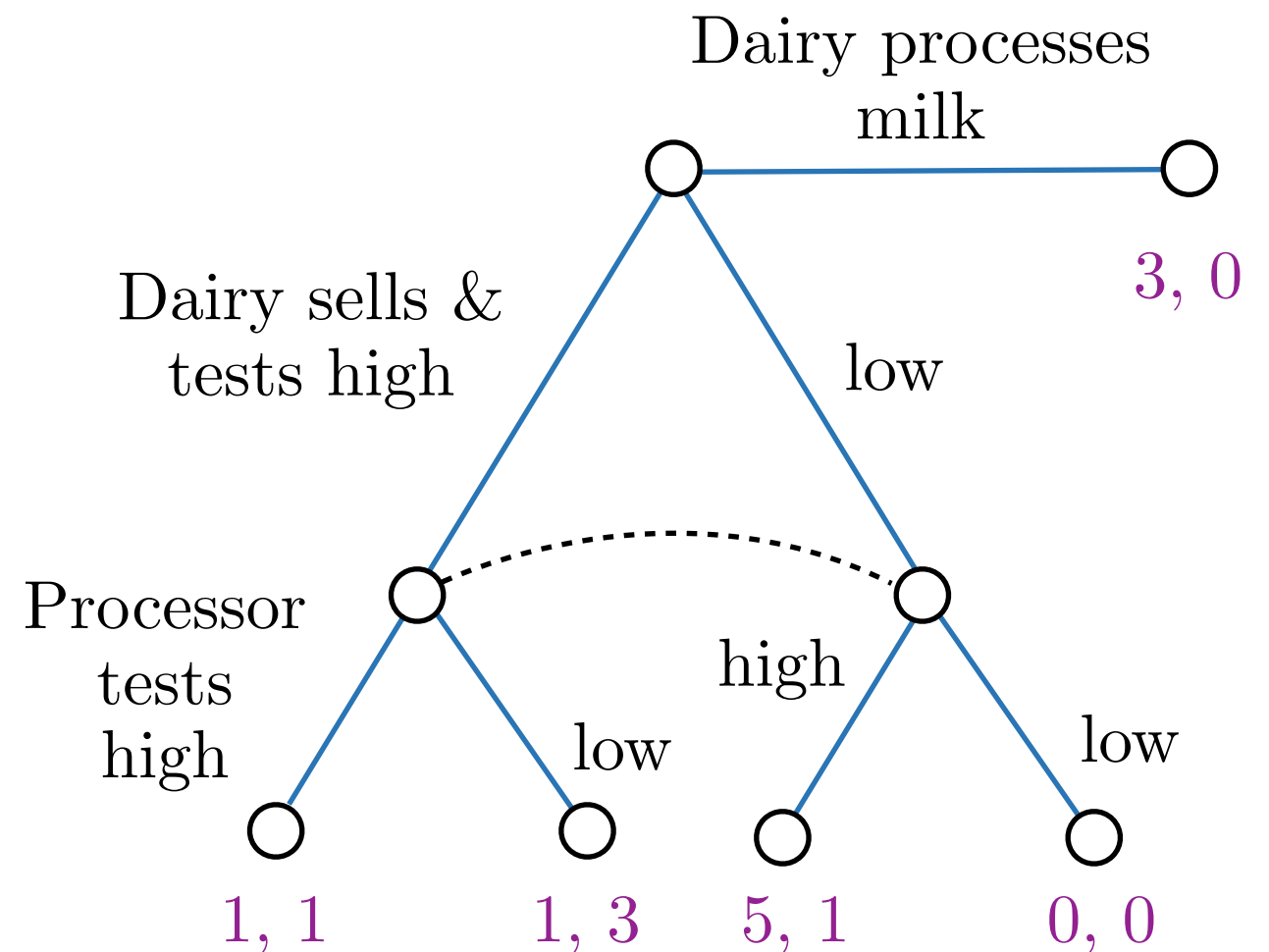
- What does D. believe about P.'s position?
- E.g., 50-50 chance that D. tests high
- If D. sells...

- P. tests high:

$$u_2 = .5(1) + .5(1) = 1$$

- P. tests low:

$$u_1 = .5(3) + .5(0) = 1.5$$



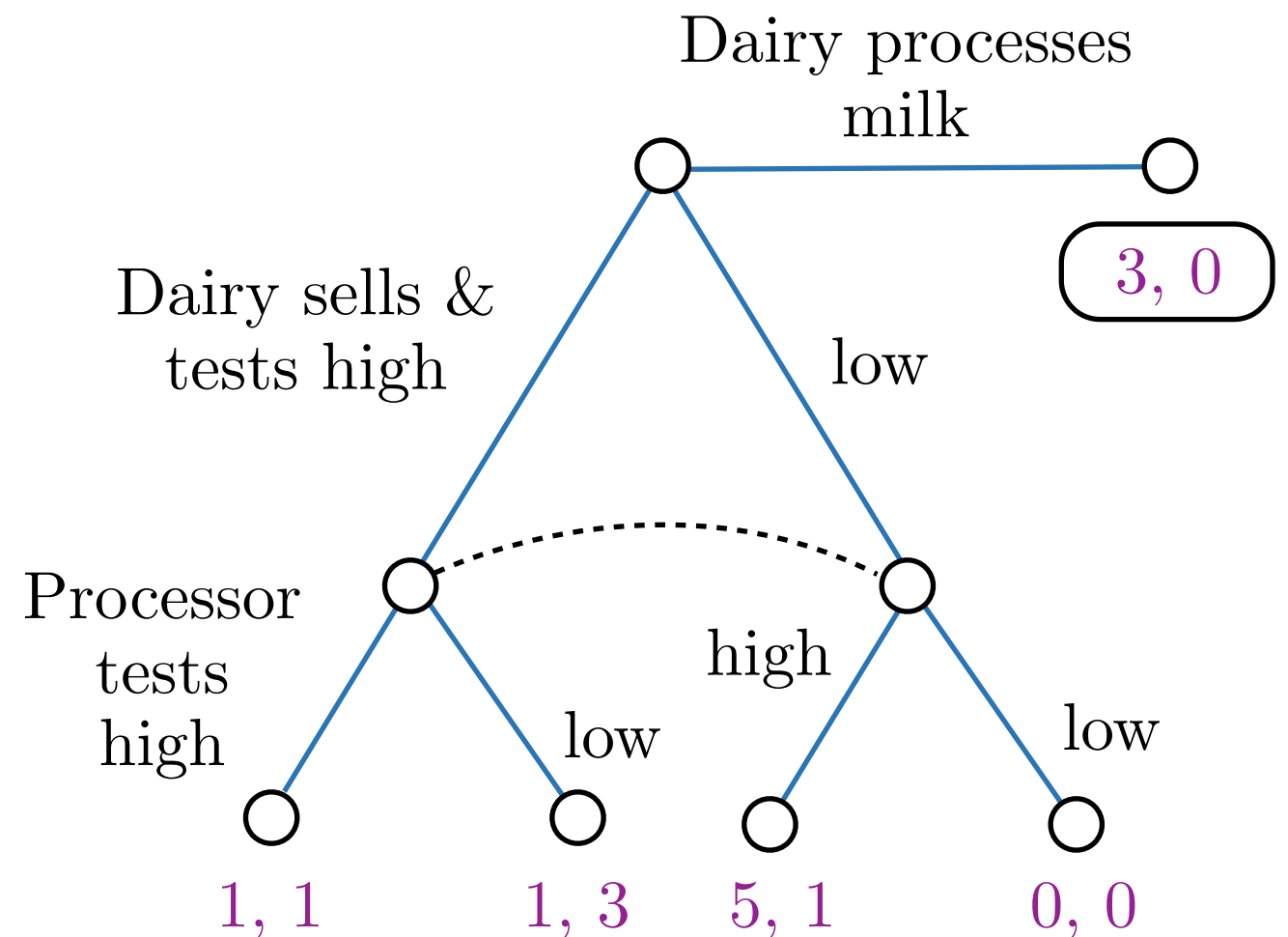
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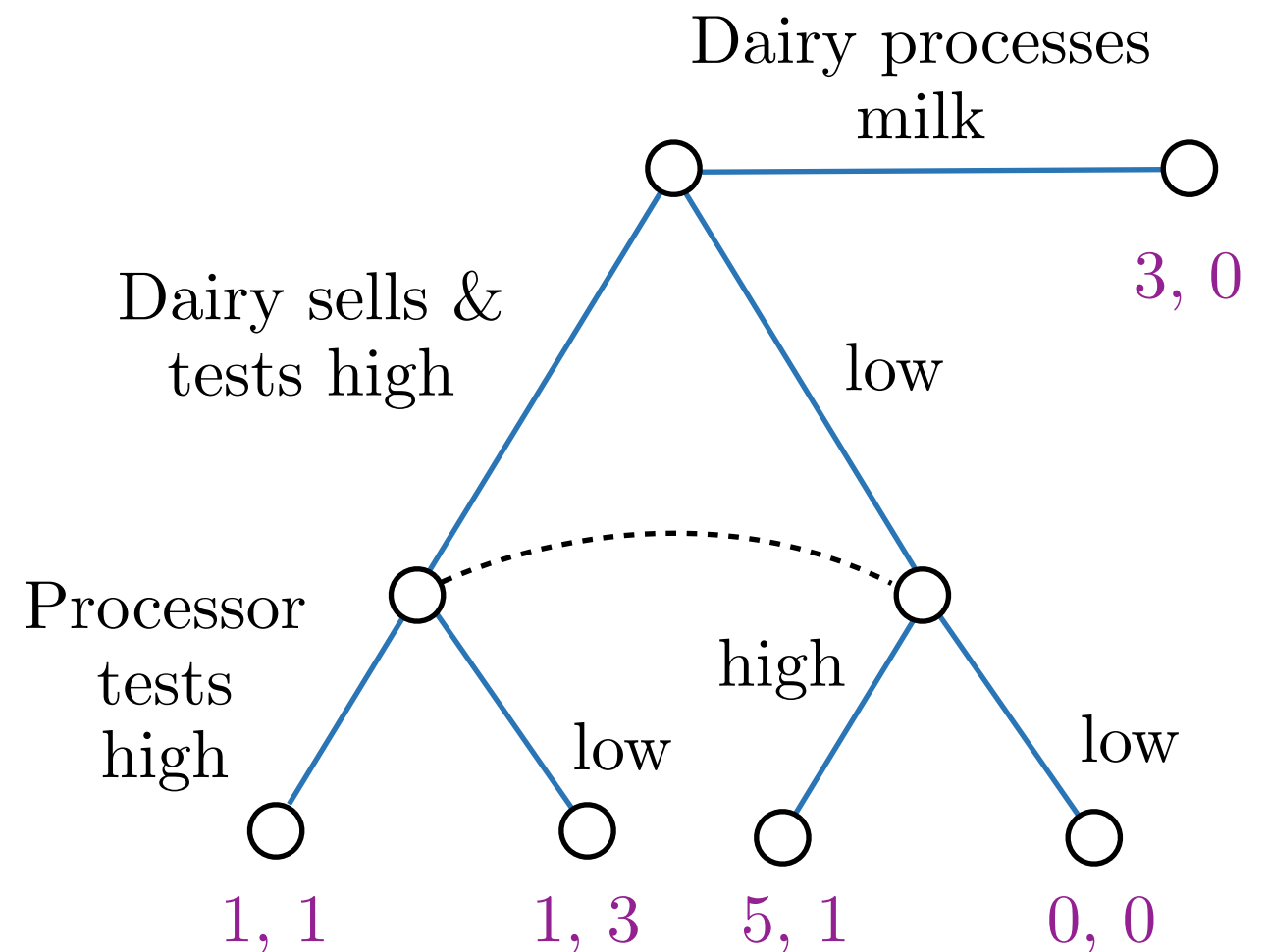
$$u_1 = .5(3) + .5(0) = 1.5$$



Any other course is not bayesian rational
(given its belief!)

Assignment

- Propose a perfect Bayesian equilibrium
- D. sells the milk



Solution 1

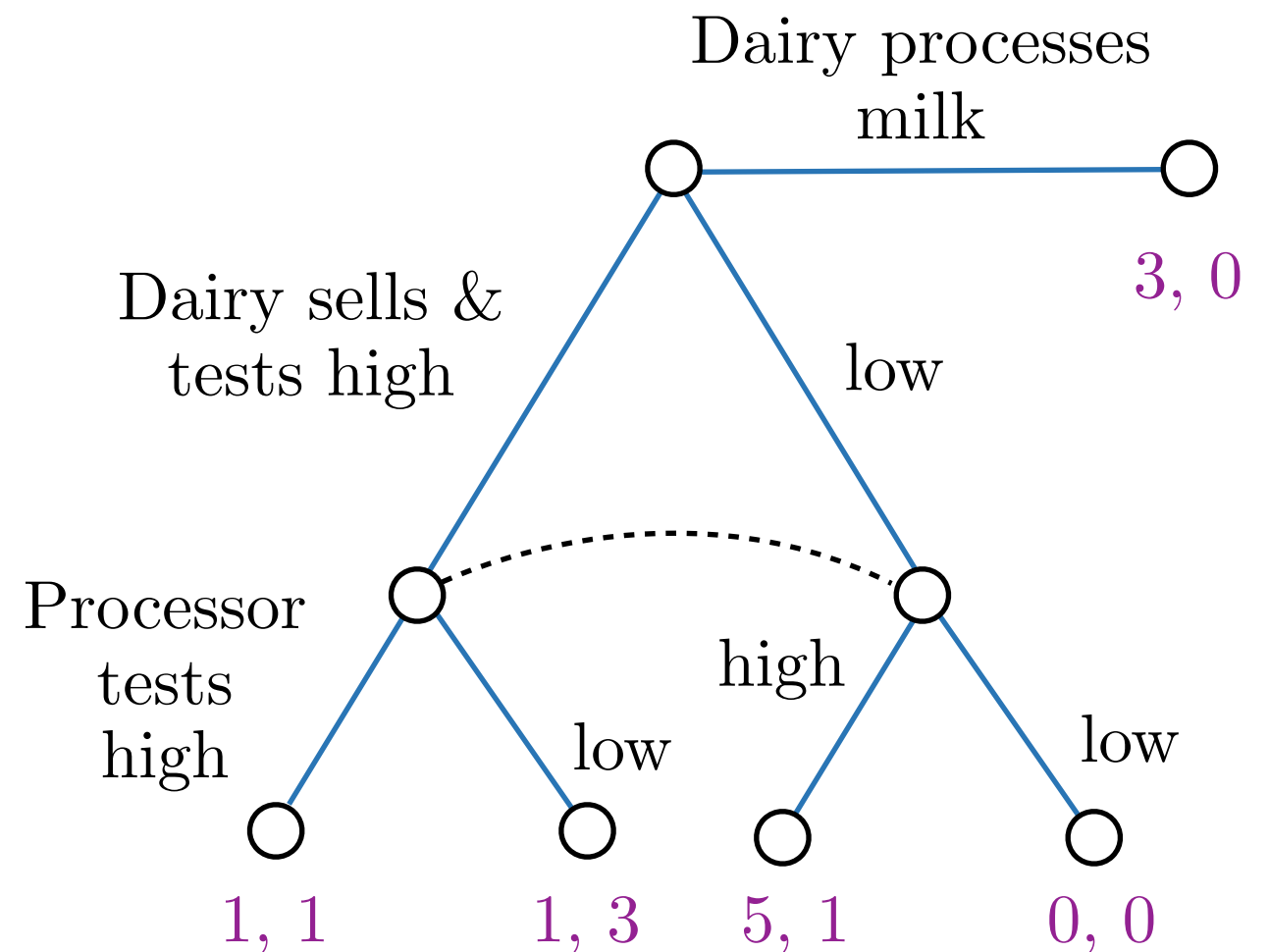
- P. believes in a 30-70 chance that D. tests high
- If D. sells...

- P. tests high:

$$u_2 = .3(1) + .7(1) = 1$$

- P. tests low:

$$u_1 = .3(3) + .7(0) = .9$$



Solution 1

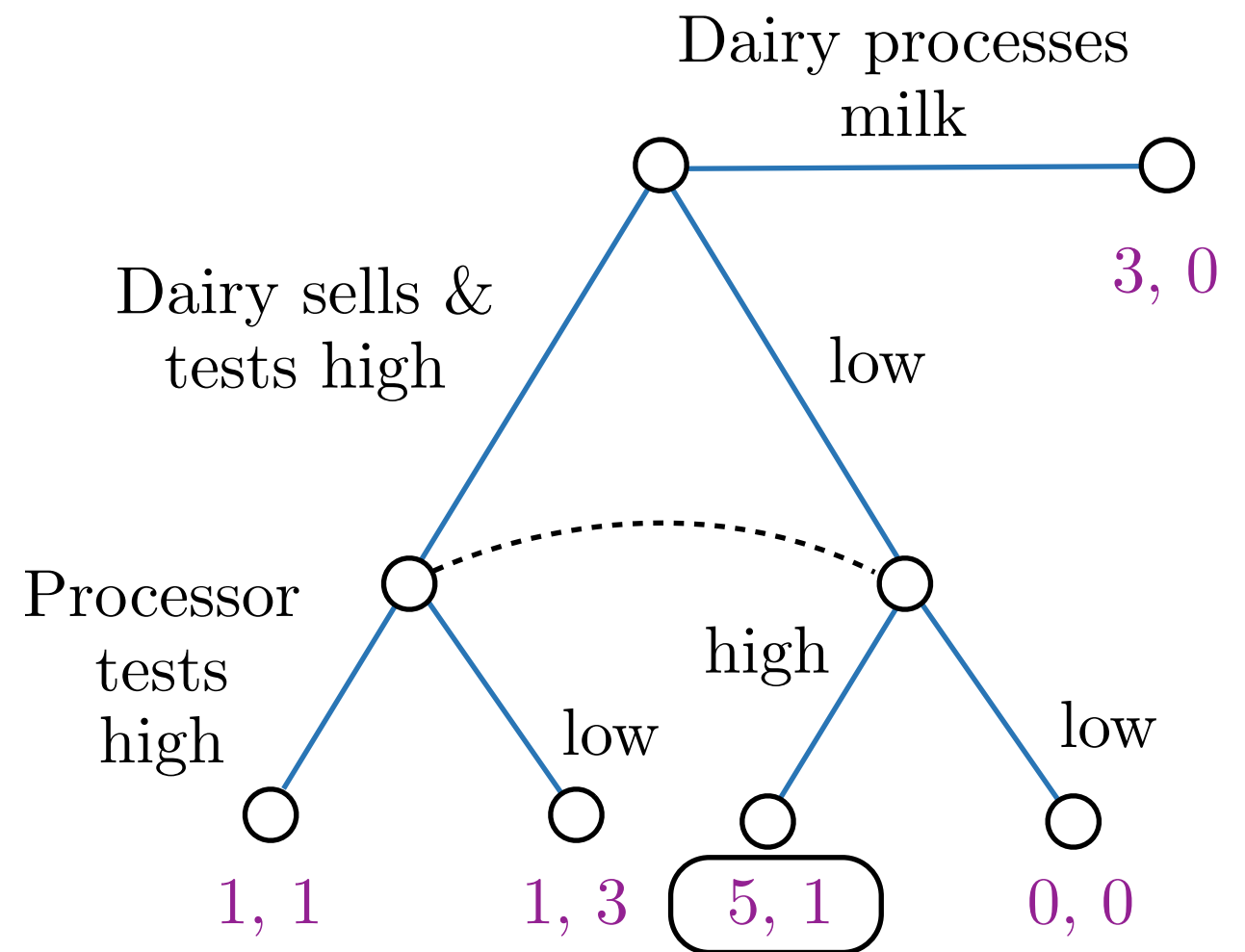
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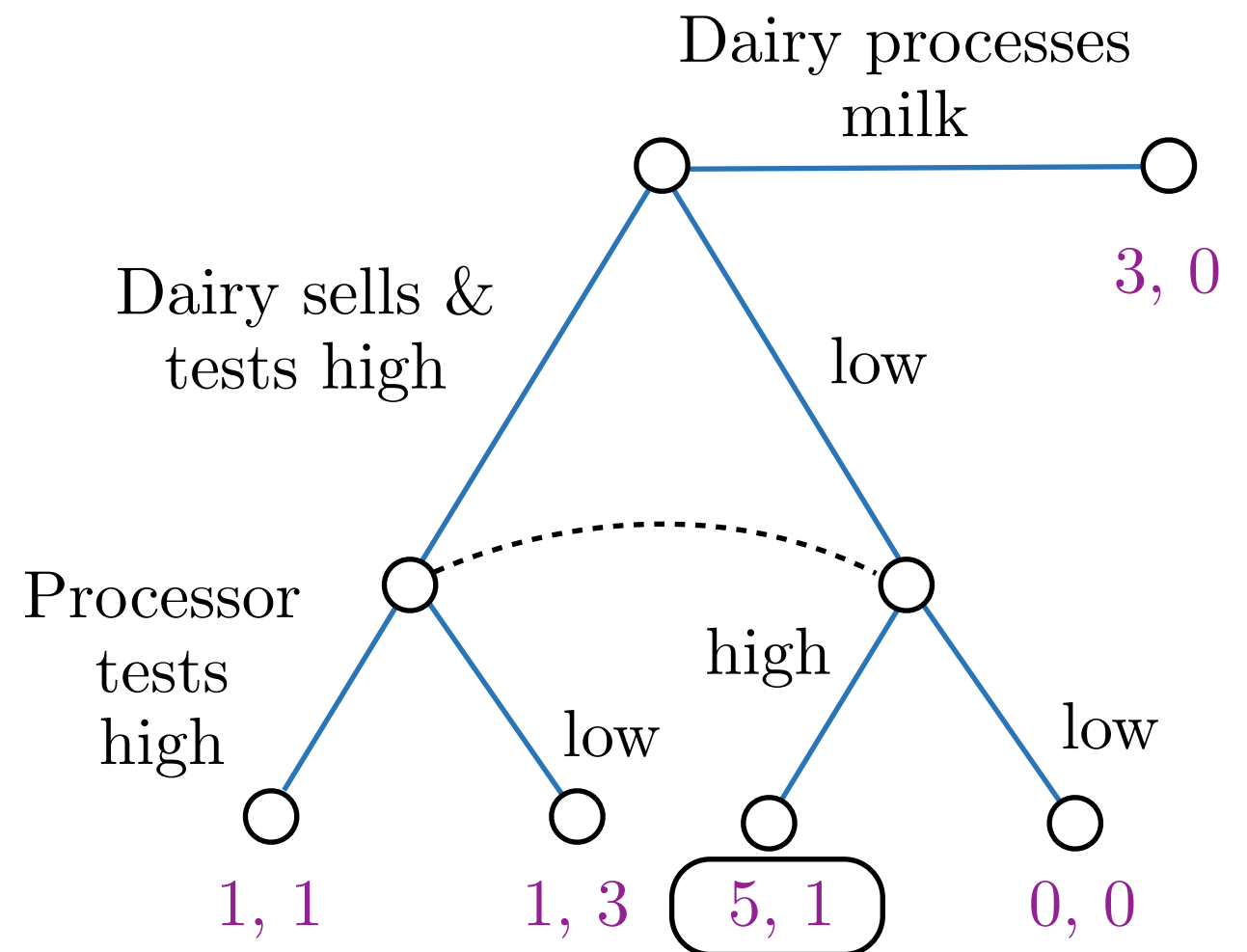
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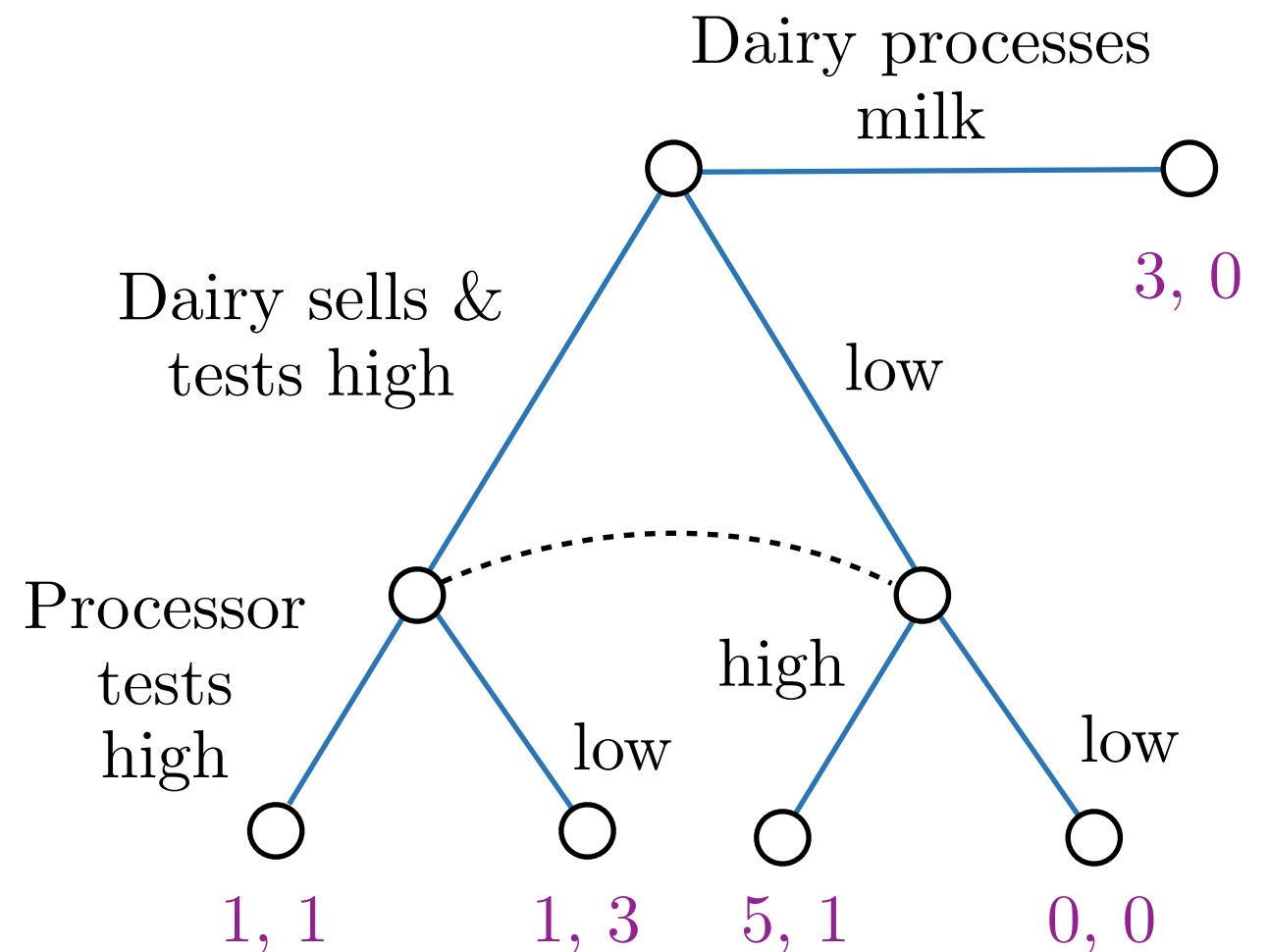
$$u_1 = .3(3) + .7(0) = .9$$



D. receives highest possible payoff

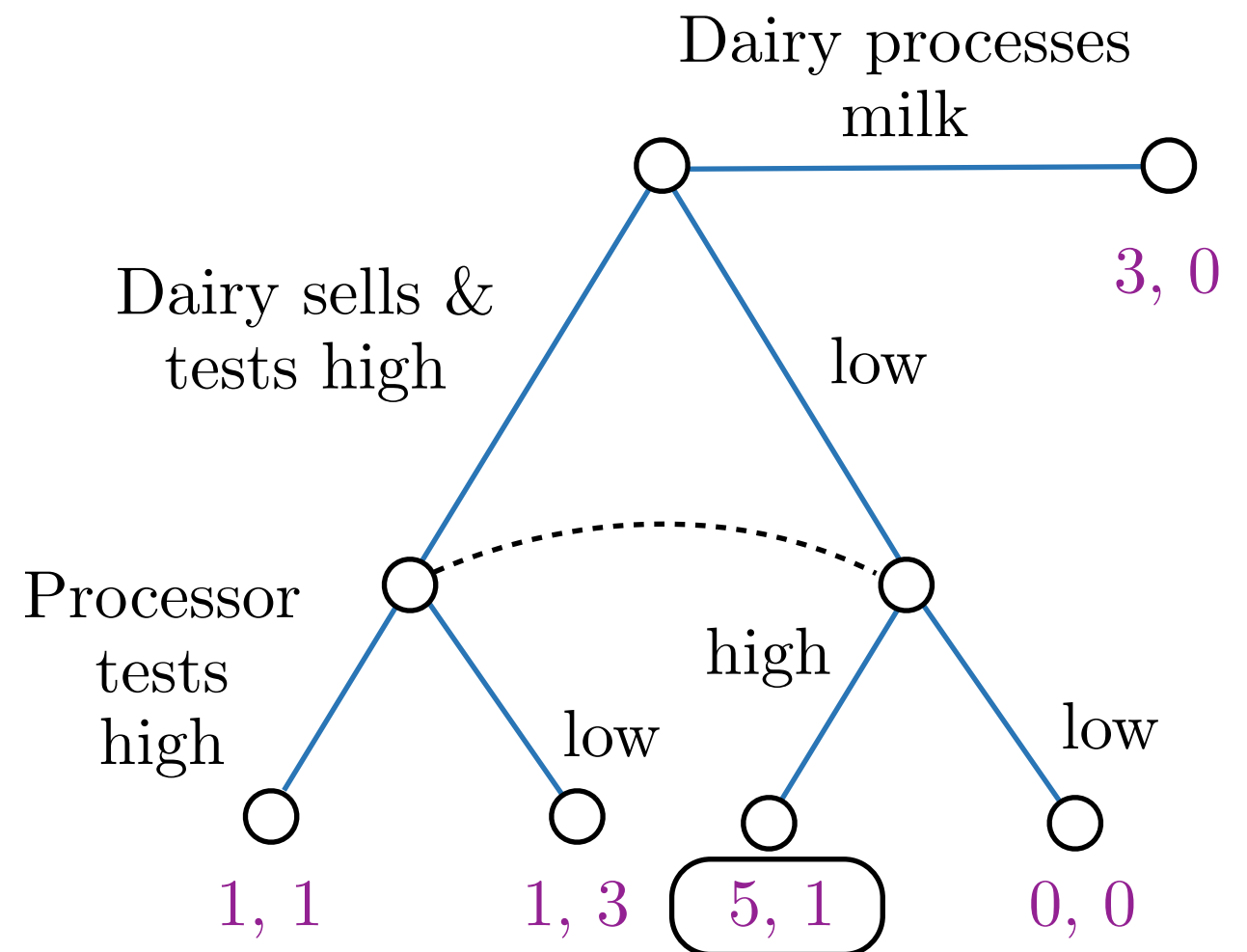
Solution 2

- P. believes that **in any case** in which D. sells, it will test low
- P. tests high



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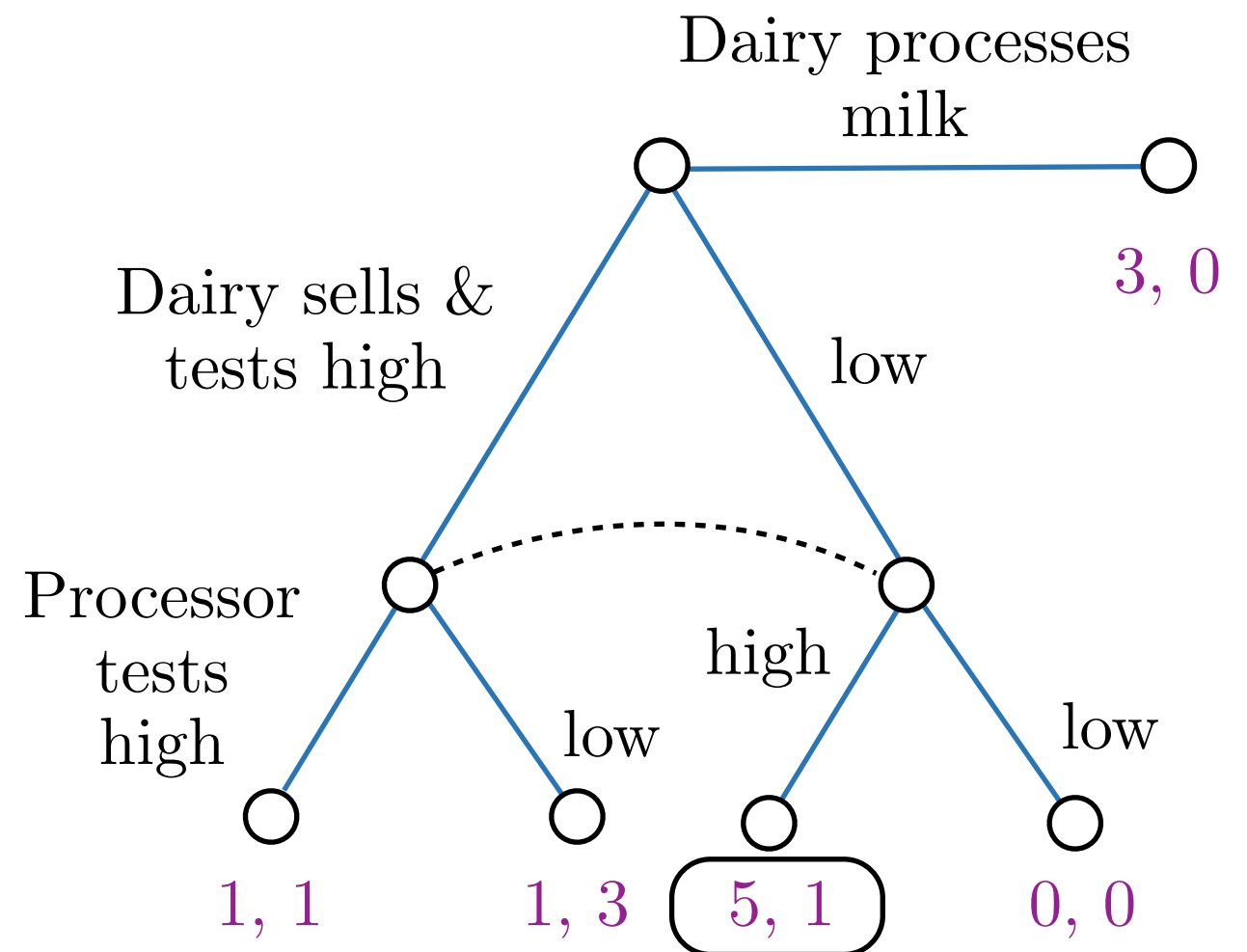


Solution 2

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D. always does better processing its own milk than it does by selling and testing high

P. **should infer** that whenever D. sells the milk it will test low

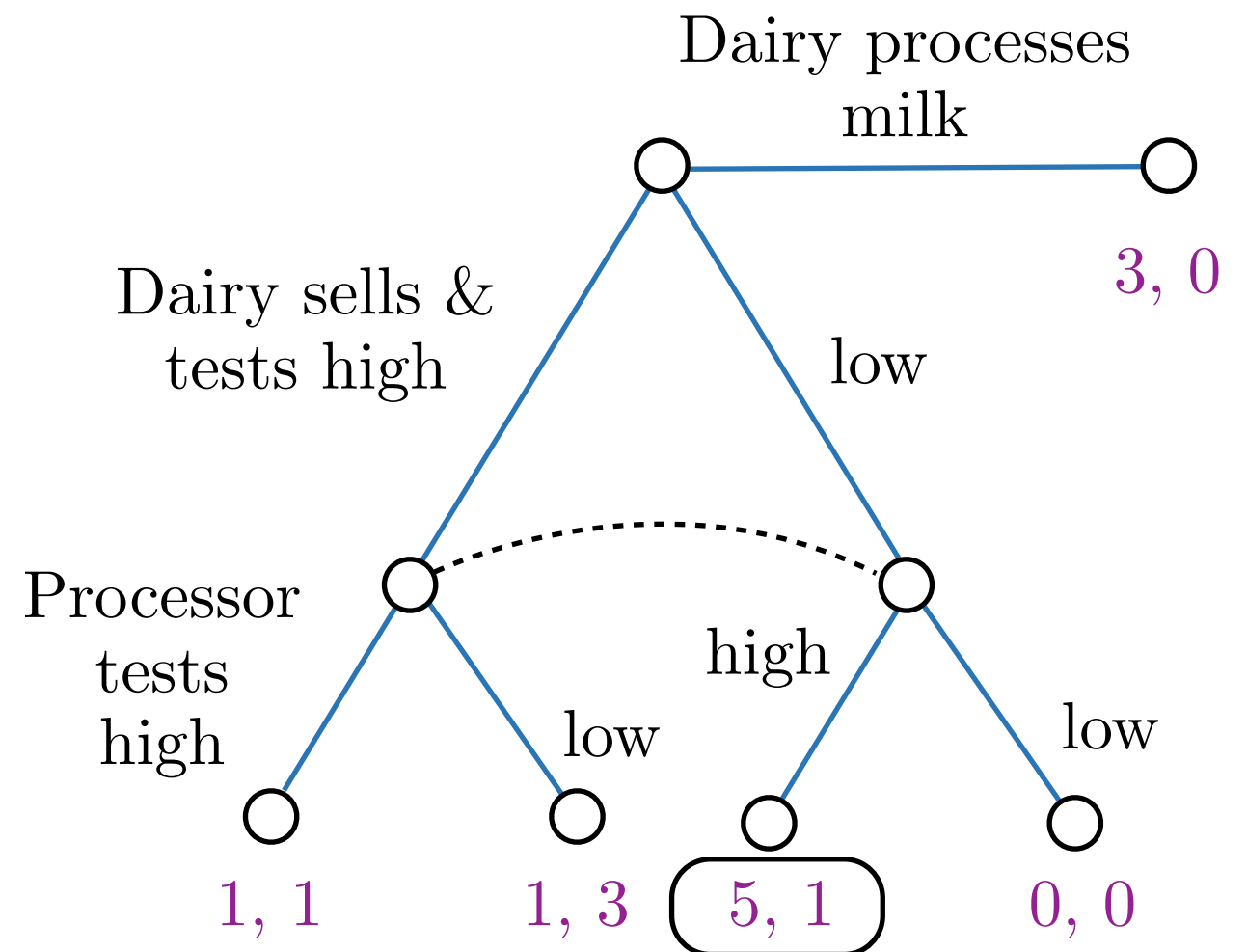


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Only the proposed solution survives the refinement

Combination of strategies and beliefs forms
the perfect Bayesian Nash equilibrium

Disclosing information

- **Before the sale:** buyer doesn't know the number of apples in a box
- Seller knows the number of apples; buyer knows he/she knows this
- **Buyer can retaliate:** can sue at no cost if seller lied about the number of apples
- Seller with 100 apples in a box. Disclose information?

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verifiable
information
+
legal remedy

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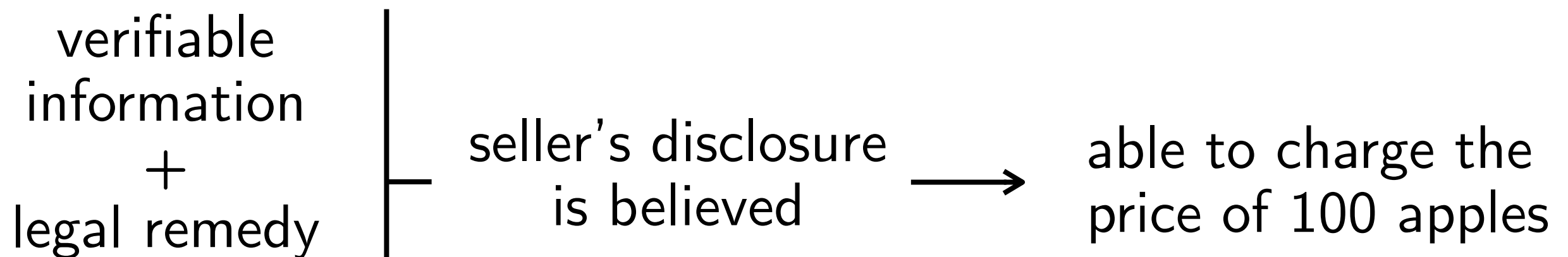
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- Seller with ~~100~~ apples in a box. Disclose information?
99



Disclosing information

- If seller remain silent, buyer will infer seller has less apples
- What about seller with 98 apples? 97 apples? ...
- All disclose **verifiable information**

The unraveling result

- Person who has favorable verifiable information has an incentive to reveal it voluntarily
- high-value sellers will want to distinguish themselves from low-value sellers though voluntary disclosure
- Silence cannot be sustained
- Important implications on approaches to reveal information
- Inquiry limits + disclosure duties

Inquiry limits

- The Americans with Disabilities Act forbids an employer from asking whether an applicant has a disability + pre-offer medical tests
- Educational Amendments of 1972 forbid inquiry into the marital status on an applicant for employment or admission to a school receiving federal funding
- Many states forbid preemployment inquiries into religious or political affiliations
- Limits on the kinds of inquiries that can be made of an applicant to rent a house

Prevent decision-makers from obtaining information thought to be an inappropriate basis for making the decision.

How to make inquiry limits more effective?

- Job applicants may volunteers information (e.g., they never had a drinking problem) even if the employer is not allowed to ask
- How to prevent applicants from volunteering information? Use the unraveling result as guidance...
- Provide some means of keeping the employer from responding adversely upon discovering the applicant had lied
- Assertions that applicants never drank are no longer credible; applicants no longer make them
- Rules determine how parties share information and change the way individuals behave

Remarks

- Tool to analyze how parties act based on information/ beliefs they might infer from
 - initial assessments
 - actions of the other players (repeated games)
- Laws govern the disclosure of information + take into account what type of information
- **Verifiable information:** once revealed can be verified by other players + court
- Unraveling result

The perfect Bayesian equilibrium

- Nash equilibrium
 - Rational players (completely informed)
- Subgame perfection
 - Actions optimal in every subgame
 - Discard solutions that are not plausible, e.g., unconvincing threads
- Perfect Bayesian equilibrium
 - Rational players - Act consistent with their beliefs (incomplete information)
 - Actions optimal given the beliefs of the players

Next class

- Rationality and extensive form CKR