Lecture 6

Standard economic models typically assume that interaction between agents is anonymous, and the pattern of social interactions irrelevant. However, in reality individuals often rely on, benefit from and try to shape their ‘network’, i.e. their set of social contacts.

Moreover, the pattern of network interactions has been shown to influence outcomes and decision-making, e.g. the probability to find a job, to adopt a new technology, to get a career promotion, or to engage in criminal activity.

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Review

- Network visualization (Pajek + R + igraph)
    - Principal features of network structure
- Mathematics of model networks
- Poisson Random graphs

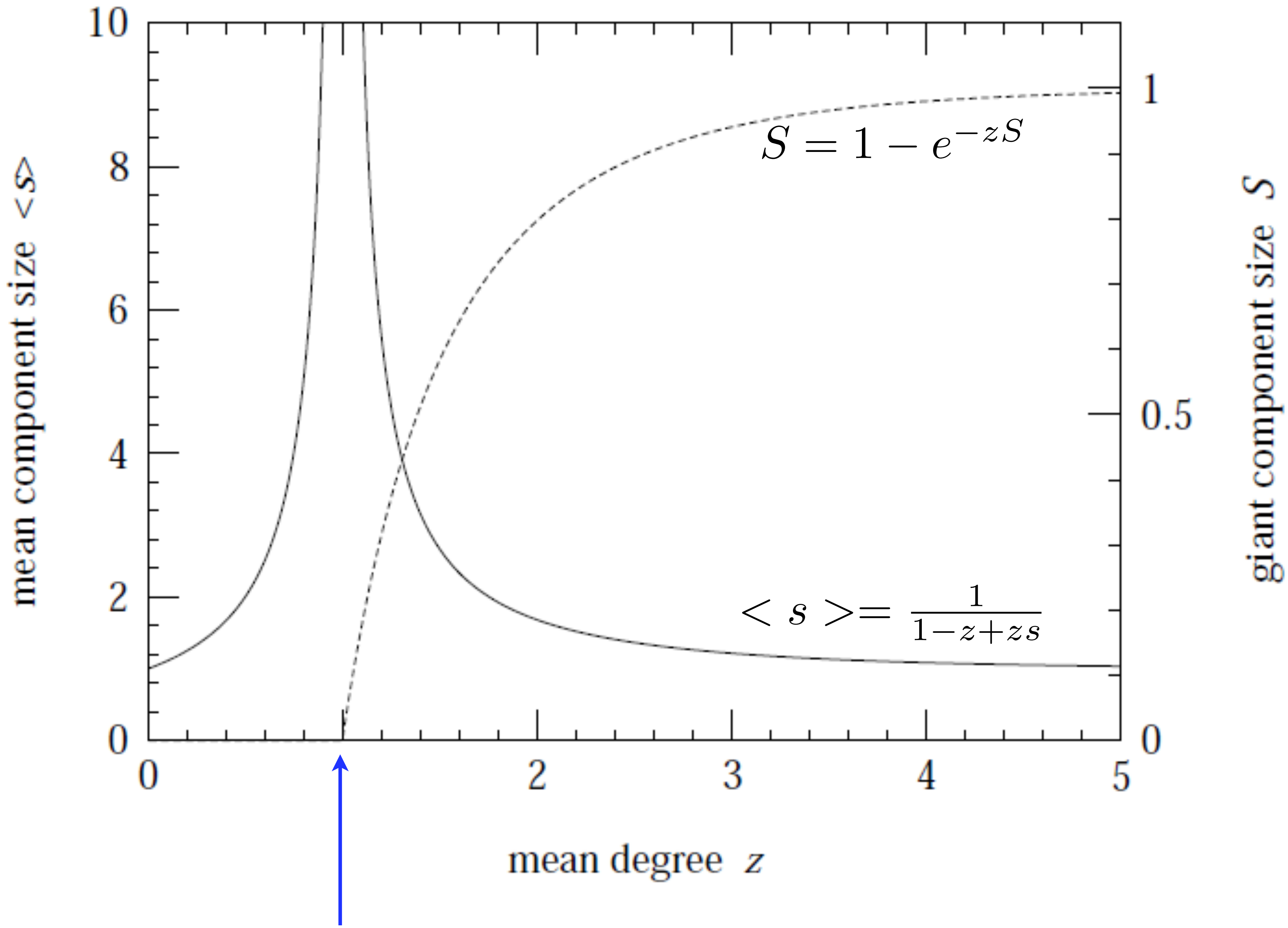
- caveat -
In almost all respects,
the properties of random networks do not match those of real world networks

Other properties...

- Low transitivity, C=p
- Poisson distribution (unlikely)
- Random mixing pattern
- No correlation degree between degrees of adjacent nodes
- No community structure
- fully mixed approximation: contacts are random and uncorrelated

A good straw man but rarely taken seriously in modeling of real systems!

Look at generalized models...



phase transition at which a giant component first forms

Today

- Generalized random graphs
- Bipartite graphs

Next class...

- Markov + exponential graphs
- Small-world graphs

Generalized random graphs

Random in all respects other than their degree distribution

“degree sequence”

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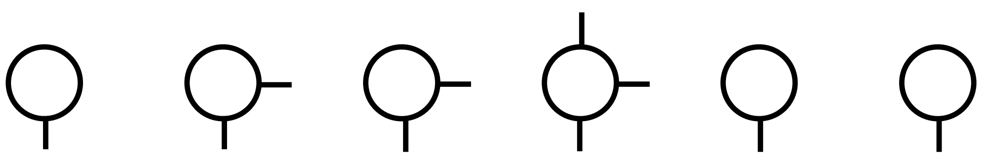
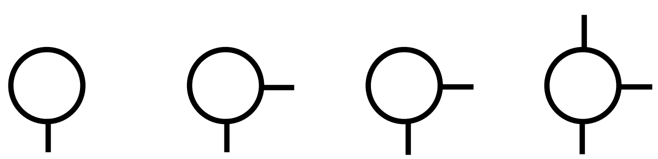


choose pairs of stubs at random

Random in all respects other than their degree distribution

“degree sequence”



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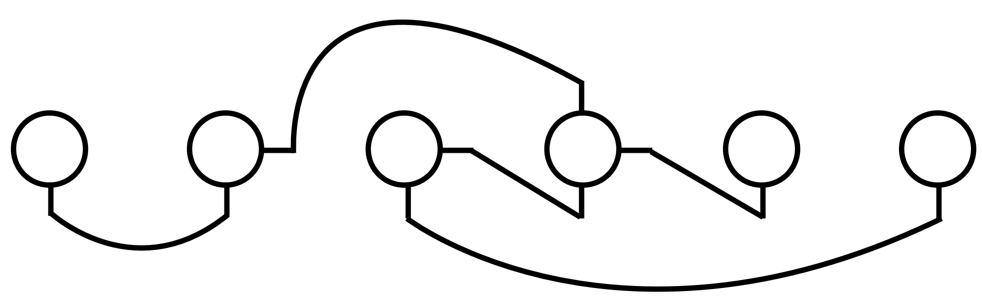
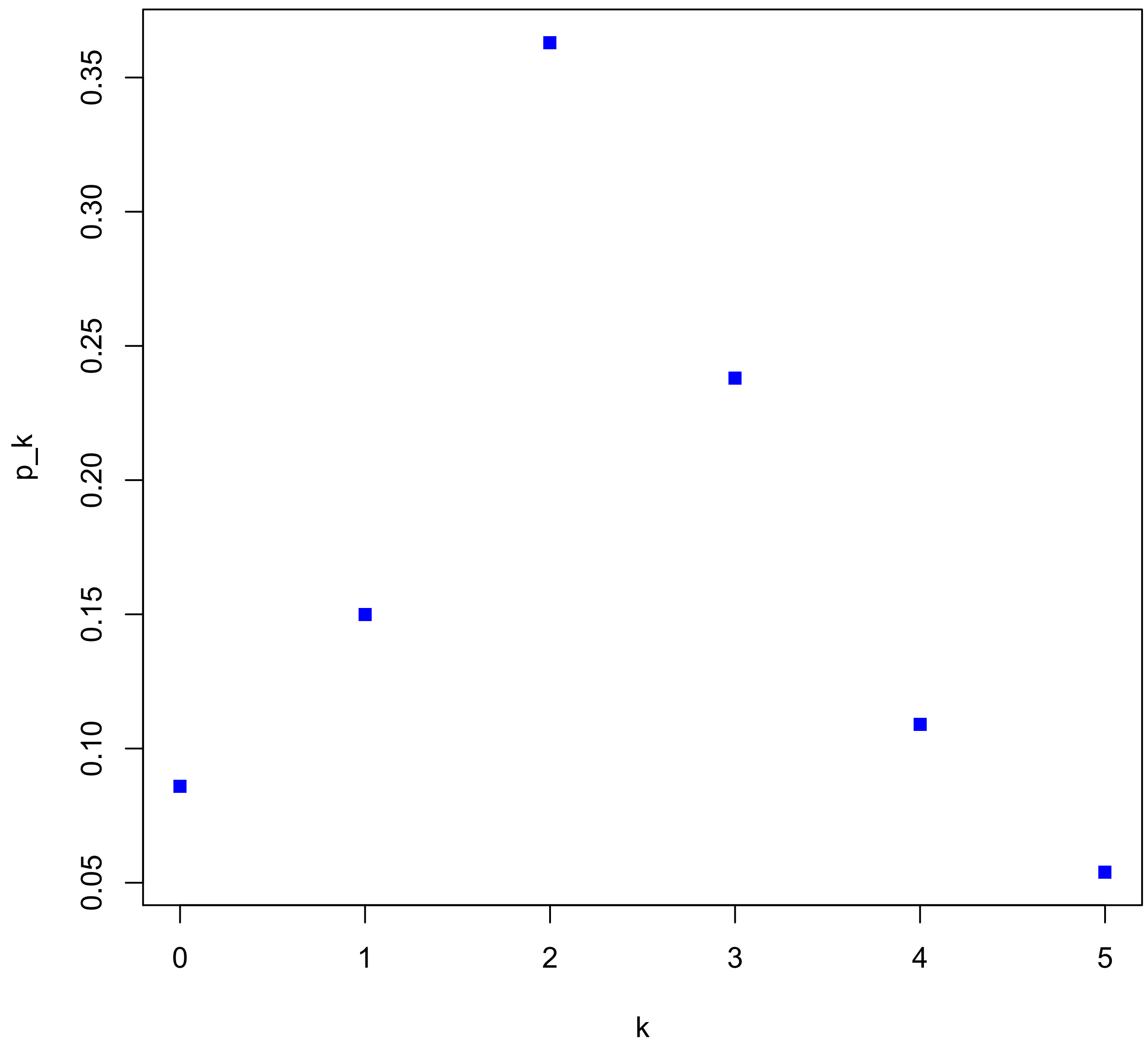


connect them

Random in all respects other than their degree distribution

example

Measured degree distribution




```
p=cbind(0.086 0.15 0.363 0.238 0.109 0.054)
degree=sample(0:5, 1000, replace=T, p)
barplot(table(degree)/length(degree))
generate_graph
source("analyzenetw.R")
network.stats(g, compute.distance=TRUE)
```

```
> network.stats(g, compute.distance=TRUE)
Number of nodes: 1000

Degree
  Average: 2.332
  Std.Dev.: 1.291295

Giant component
  Size: 895
  As perc.: 0.895
Second largest component: 5

Isolated nodes
  Number: 80
  As perc.: 0.08

Clustering coefficient
  Total: 0.001267427
  Average: 0.002759527

Degree correlation: 0.04015807

Distance in giant component
  Average: 8.884626
  Std.Dev.: 2.709888
Diameter: 27
```

```
Number of nodes: 1000

Degree
  Average: 2.328
  Std.Dev.: 1.232857

Giant component
  Size: 913
  As perc.: 0.913
Second largest component: 3

Isolated nodes
  Number: 78
  As perc.: 0.078

Clustering coefficient
  Total: 0.002620087
  Average: 0.003633291

Degree correlation: 0.01986788

Distance in giant component
  Average: 9.152858
  Std.Dev.: 2.612548
Diameter: 24
```

```
Number of nodes: 1000

Degree
  Average: 2.32
  Std.Dev.: 1.305873

Giant component
  Size: 871
  As perc.: 0.871
Second largest component: 8

Isolated nodes
  Number: 104
  As perc.: 0.104

Clustering coefficient
  Total: 0.002549936
  Average: 0.001763668

Degree correlation: 0.03241888

Distance in giant component
  Average: 8.479615
  Std.Dev.: 2.290376
Diameter: 19
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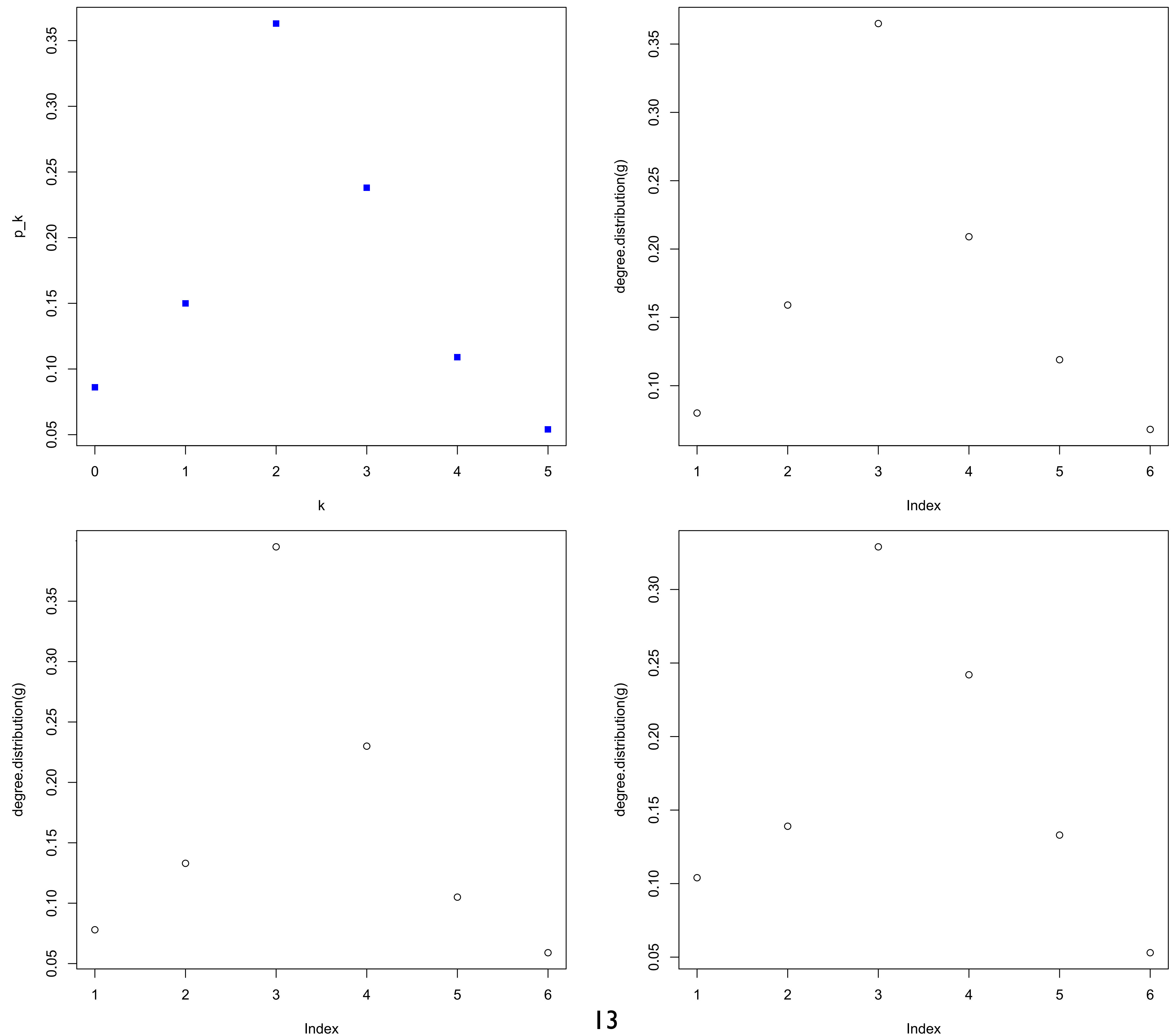
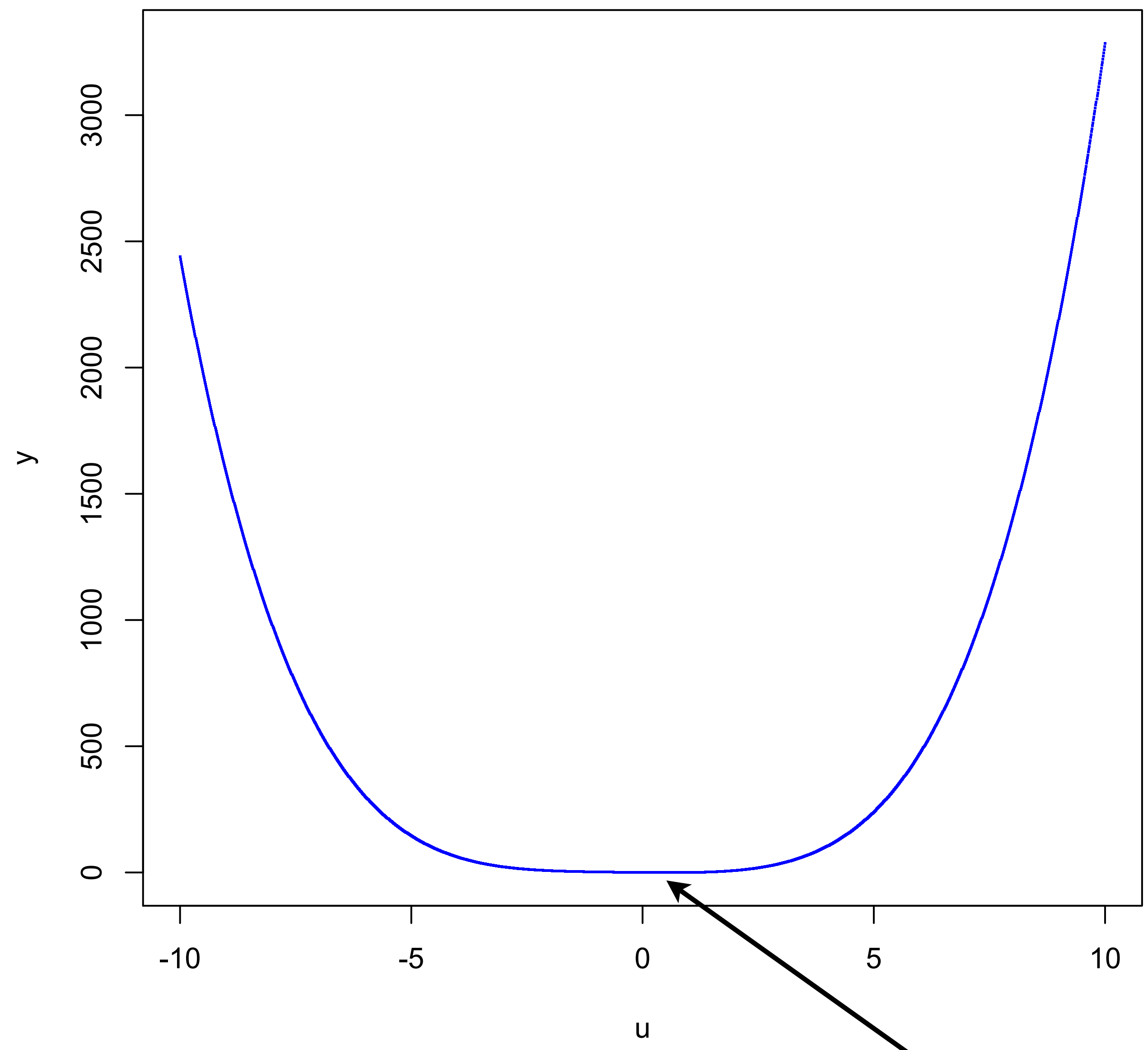


p_k
k
degree.distribution(g)
Index
degree.distribution(g)
Index
degree.distribution(g)
Index

Largest component



$$S = 1 - G_0(u)$$

$$u = \frac{G_0'(u)}{z}$$

$$S = 0.914$$

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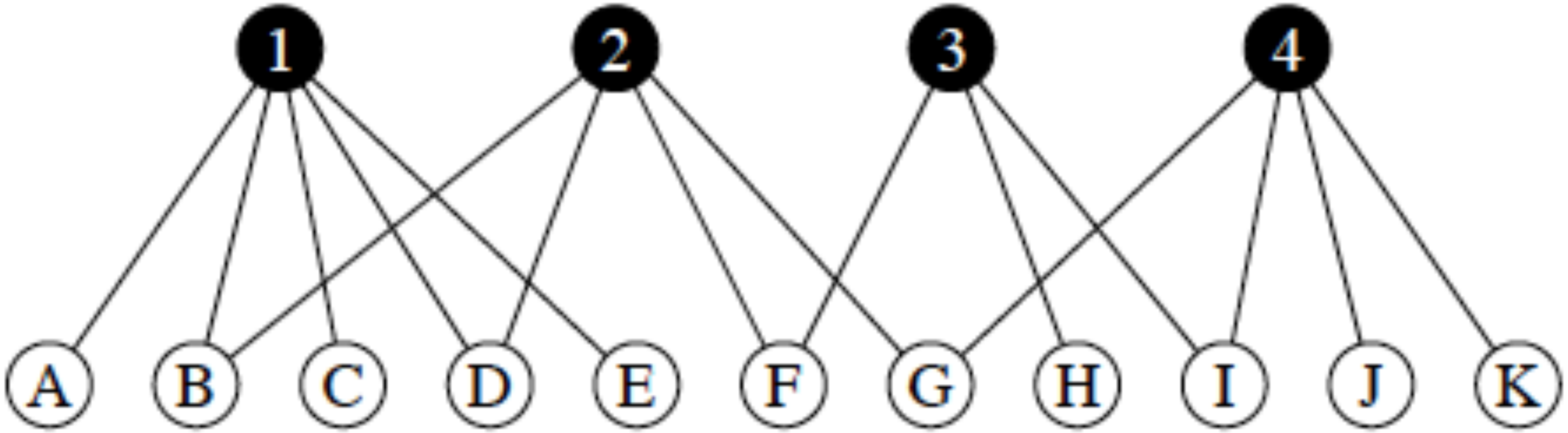
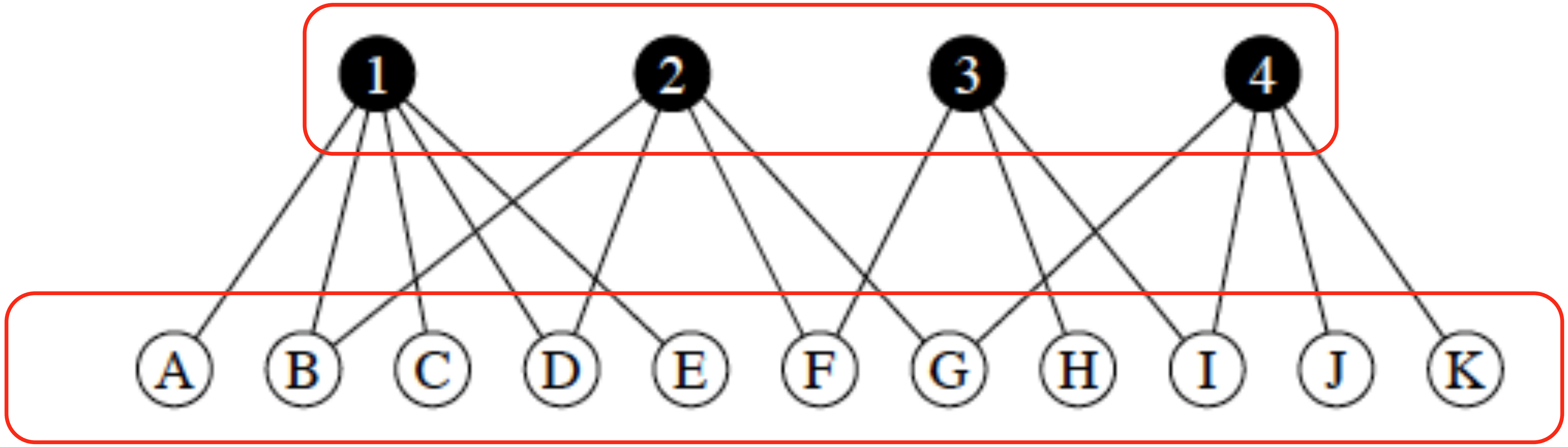
bipartite graphs
- with arbitrary degree -



from Random graphs with arbitrary degree distributions and their applications by Newman, Strogatz and Watts



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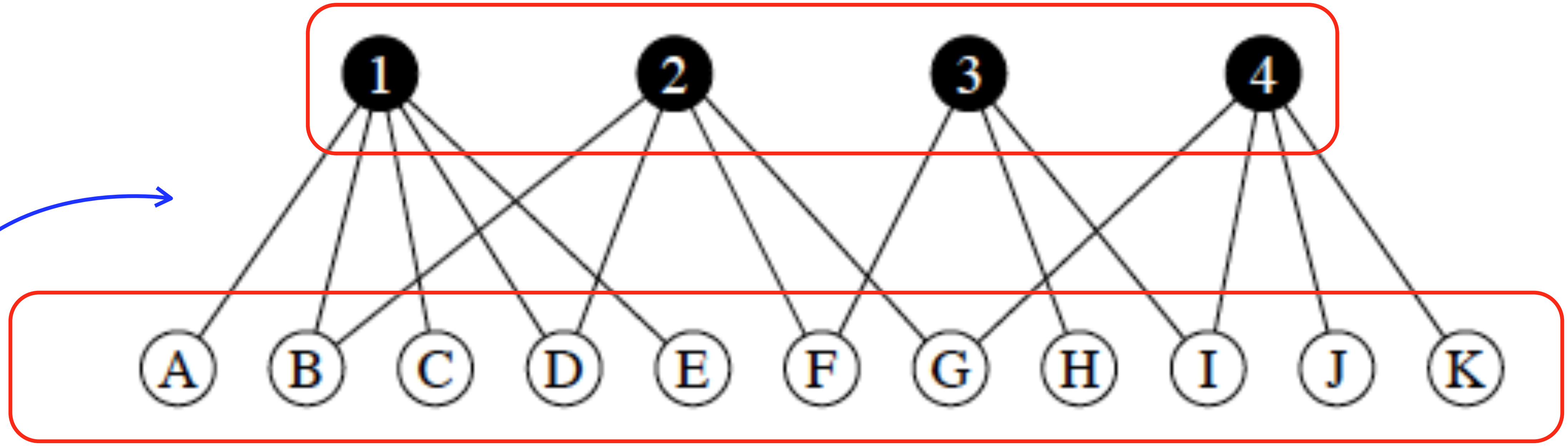
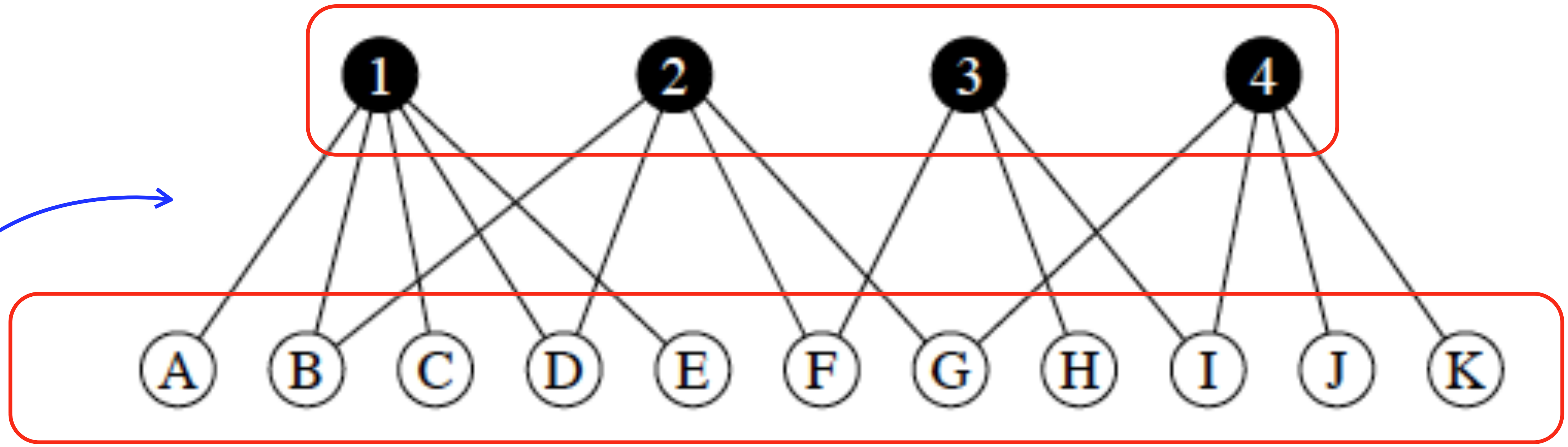


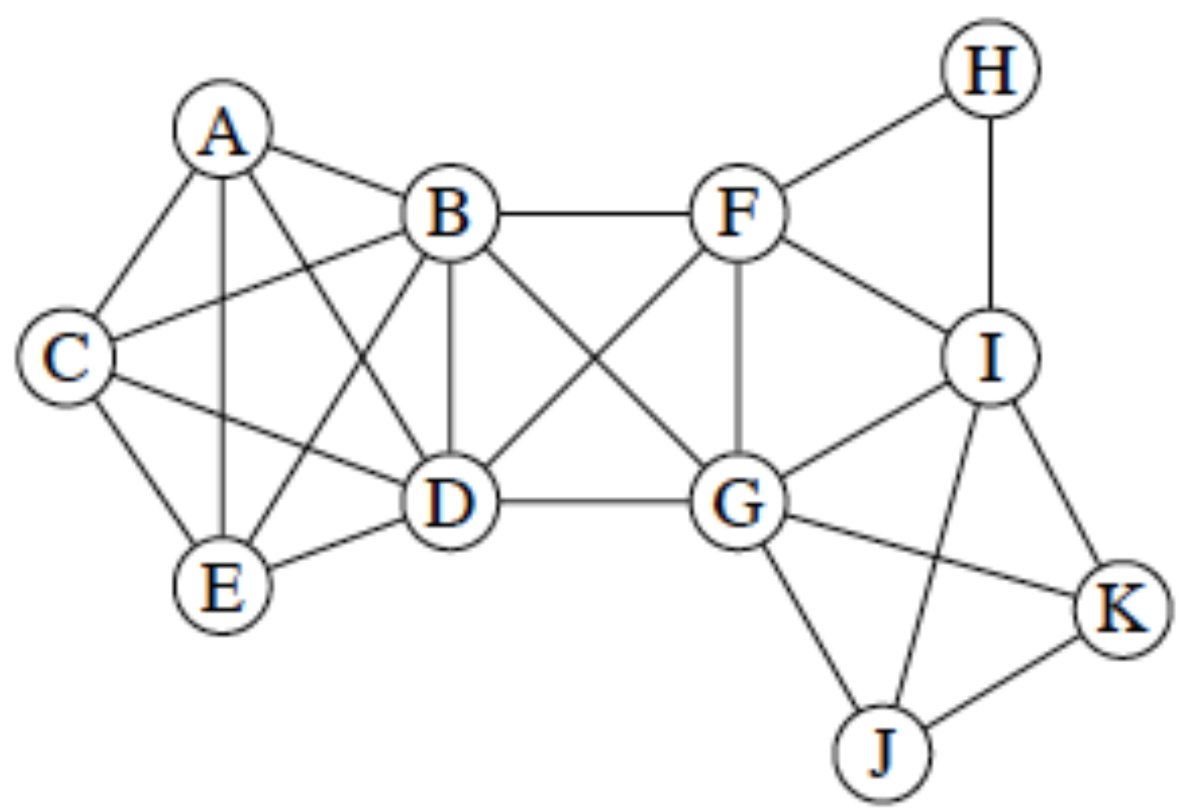
links running only between
vertices of unlike kind

from Random graphs with arbitrary degree distributions and their applications by Newman, Strogatz and Watts



links running only between vertices of unlike kind

discarding some information



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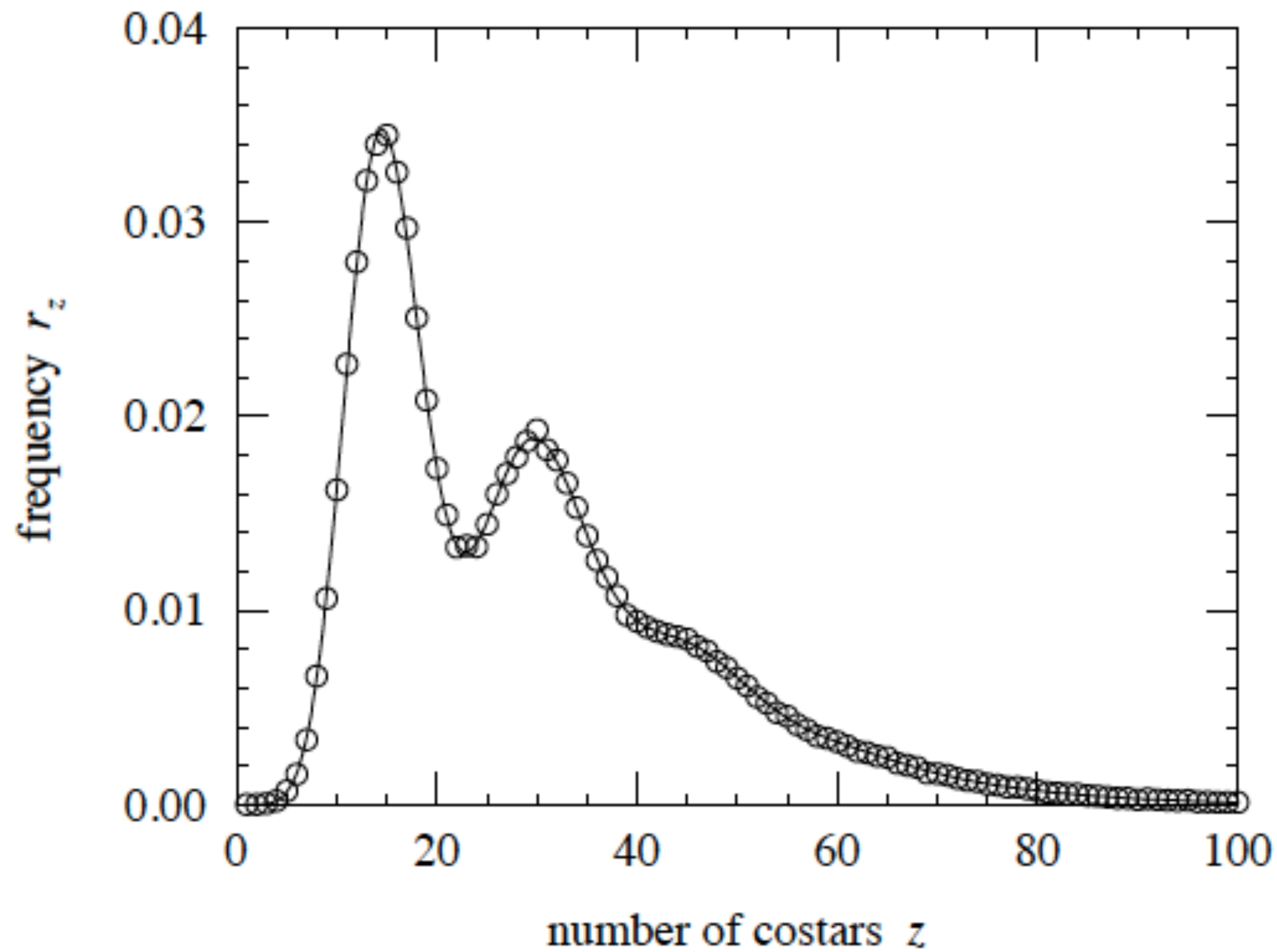
example



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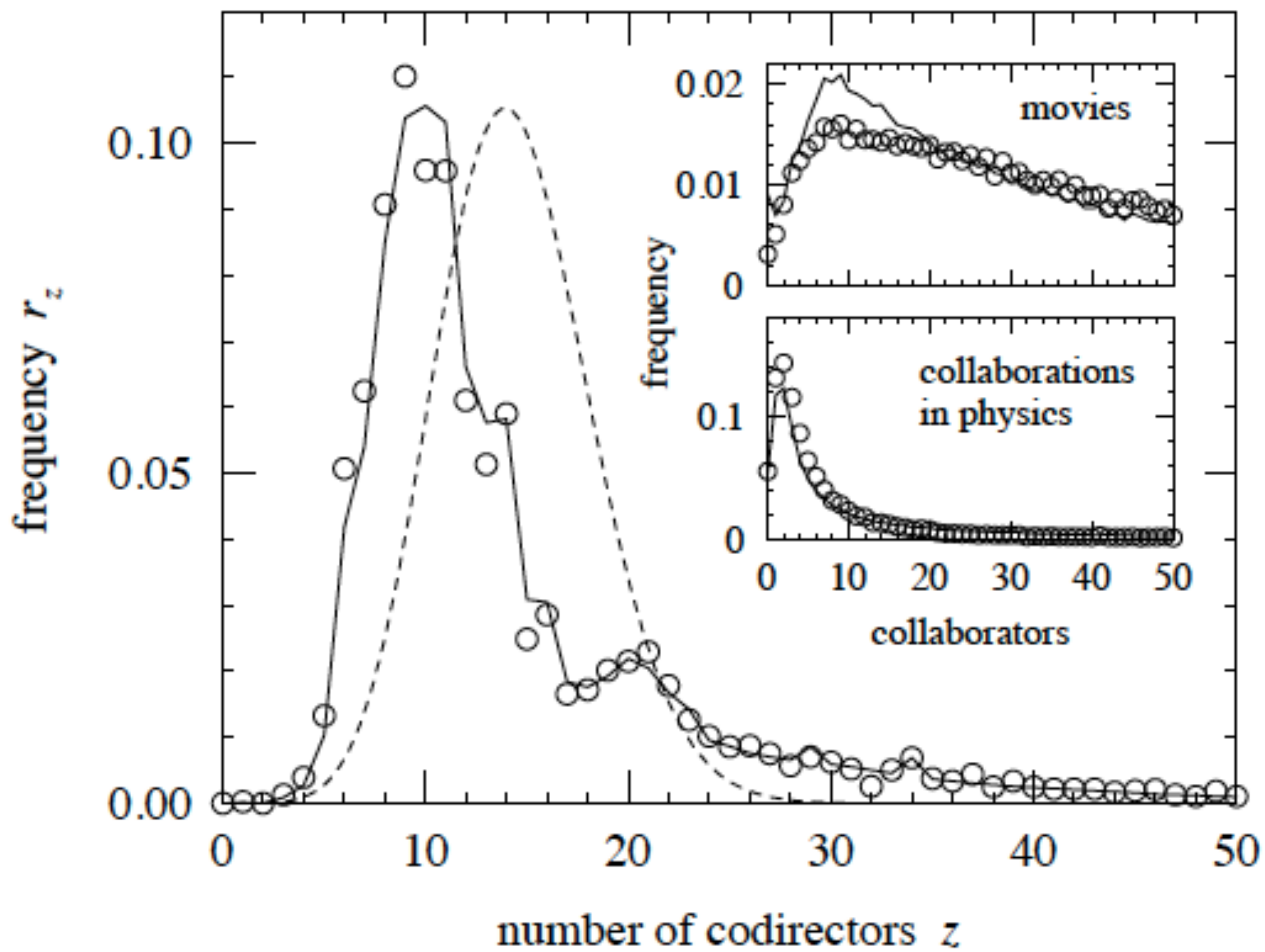
Real-world networks



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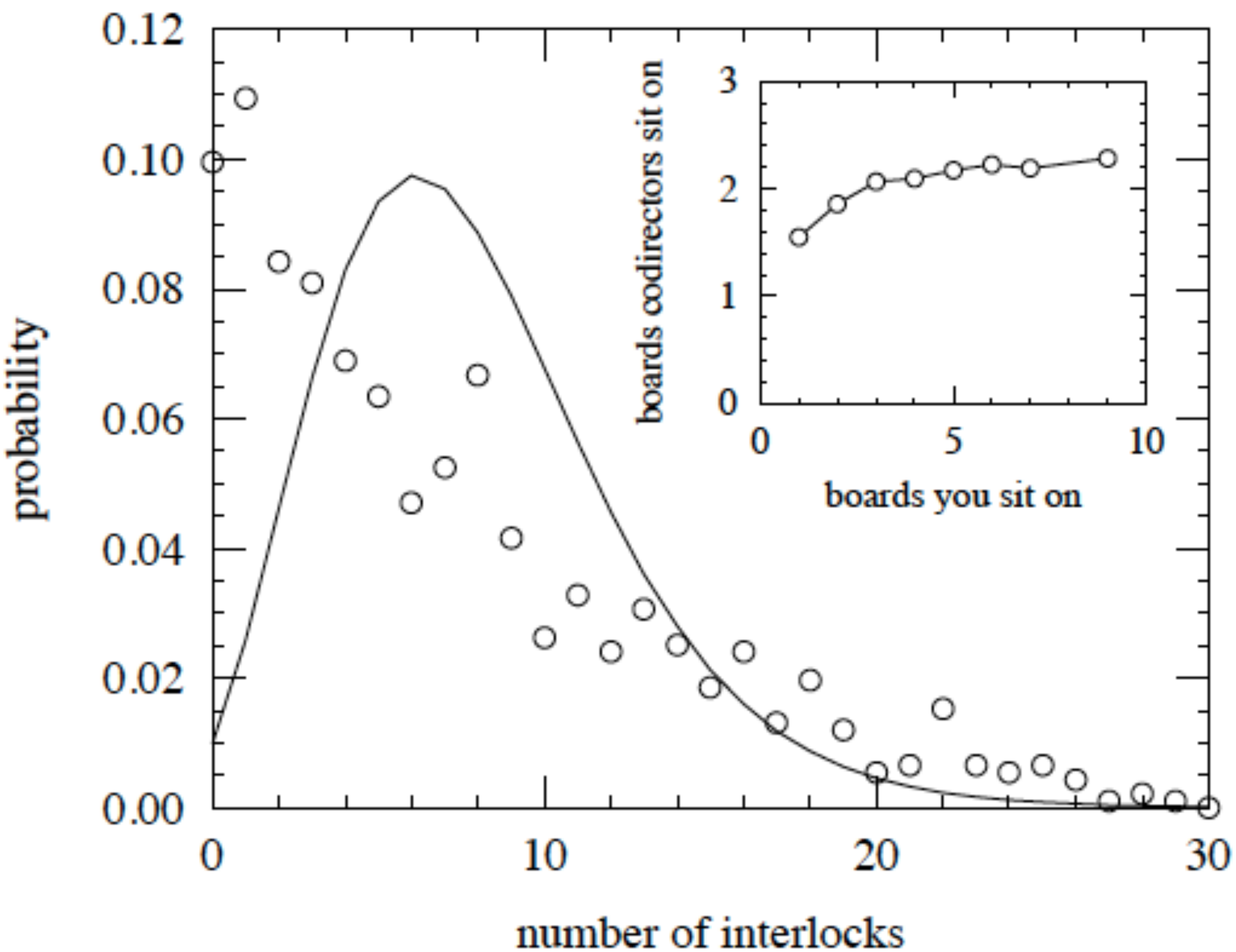
In <u>some</u> cases random graphs with appropriate distributions of vertex degree predict with surprising accuracy the behavior of the real worlds



from Random graphs with arbitrary degree distributions and their applications by Newman, Strogatz and Watts

Often, it is deviations from the random graph behavior, not agreement with it, that allows us to draw conclusions about the real world network understand discrepancy between theory and reality

$\longrightarrow$

indicates the presence of additional structure not captured by a generalized random graph

other frameworks needed!