# Lecture 4

In the short run, there are plenty of good problems about the nonlinear dynamics of systems coupled according to small-world, scale-free or generalized random connectivity. The speculations that these architectures are dynamically advantageous need to be sharpened, then confirmed or refuted mathematically for specific examples. Other ripe topics include the design of self-healing networks, and the relationships among optimization principles, network growth rules and network...

In the longer run, network thinking will become essential to all branches of science as we struggle to interpret the data pouring in from neurobiology, genomics, ecology, finance and the World-Wide Web.

Steven Strogatz
Exploring complex networks
Nature 2001

# Review

- Common Properties:
- small-world effect
- transitivity or clustering
- network resilience
- mixing patterns

Can have a profound effect on the structural properties of a network!

# Today

- Mixing patterns (scalar properties)
- degree correlation
- community structure

# Mixing patterns

| $E$ | | woman | | | |
|---|---|---|---|---|---|
| | | black | hispanic | white | other |
| men | black | 506 | 32 | 69 | 26 |
| | hispanic | 23 | 308 | 114 | 38 |
| | white | 26 | 46 | 599 | 68 |
| | other | 10 | 14 | 47 | 32 |

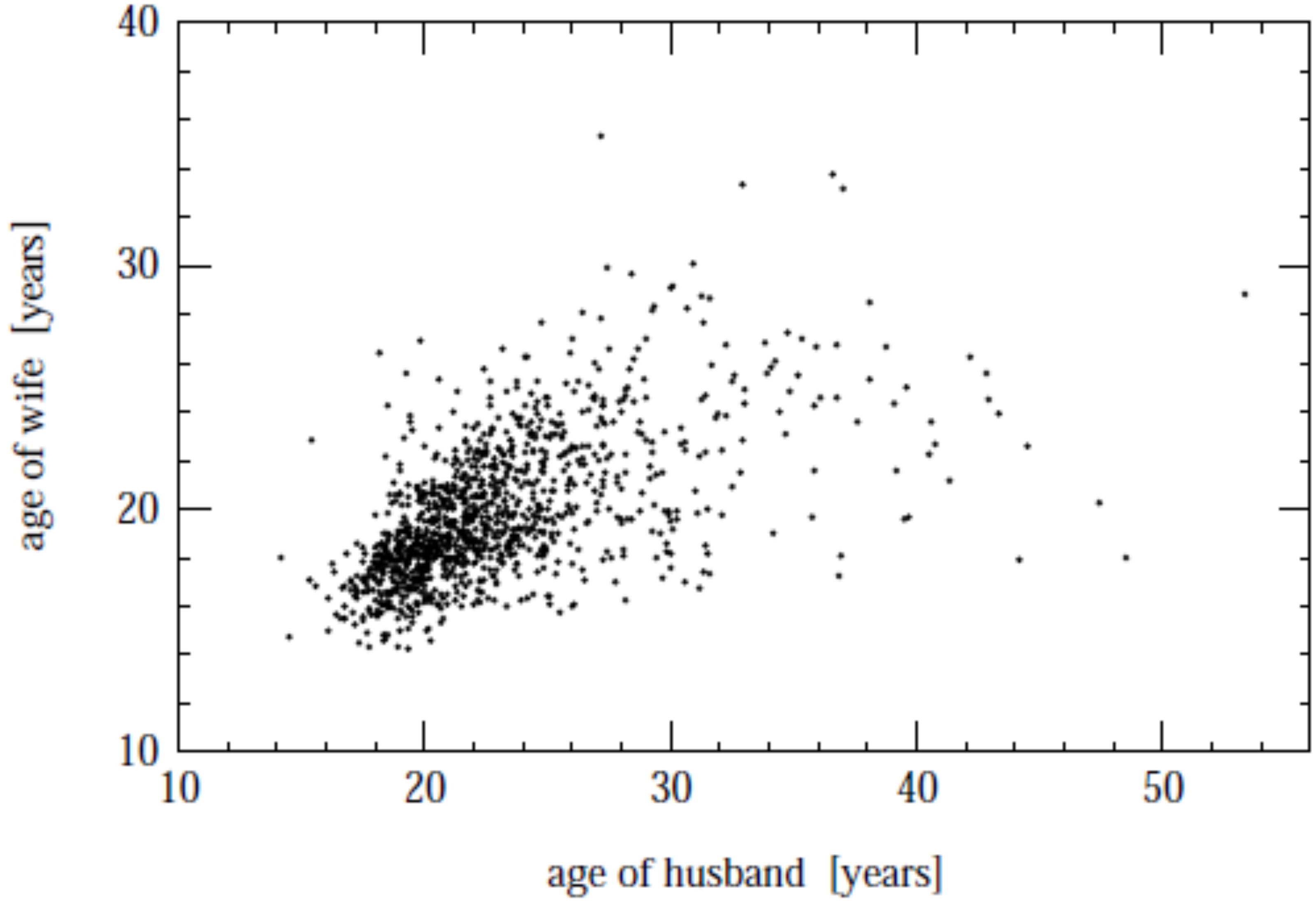
| | | woman | | | |
|---|---|---|---|---|---|
| | | black | hispanic | white | other |
| men | black | 0.258 | 0.016 | 0.035 | 0.013 |
| | hispanic | 0.012 | 0.157 | 0.058 | 0.019 |
| | white | 0.013 | 0.023 | 0.306 | 0.035 |
| | other | 0.005 | 0.007 | 0.024 | 0.016 |

$$\begin{aligned} P(j|i) &= \frac{e_{ij}}{\sum_j e_{ij}} \\ &= \frac{0.258}{0.258 + 0.016 + 0.035 + 0.013} \\ &= 0.79 \end{aligned}$$

# Mixing by Scalar Properties

e.g.

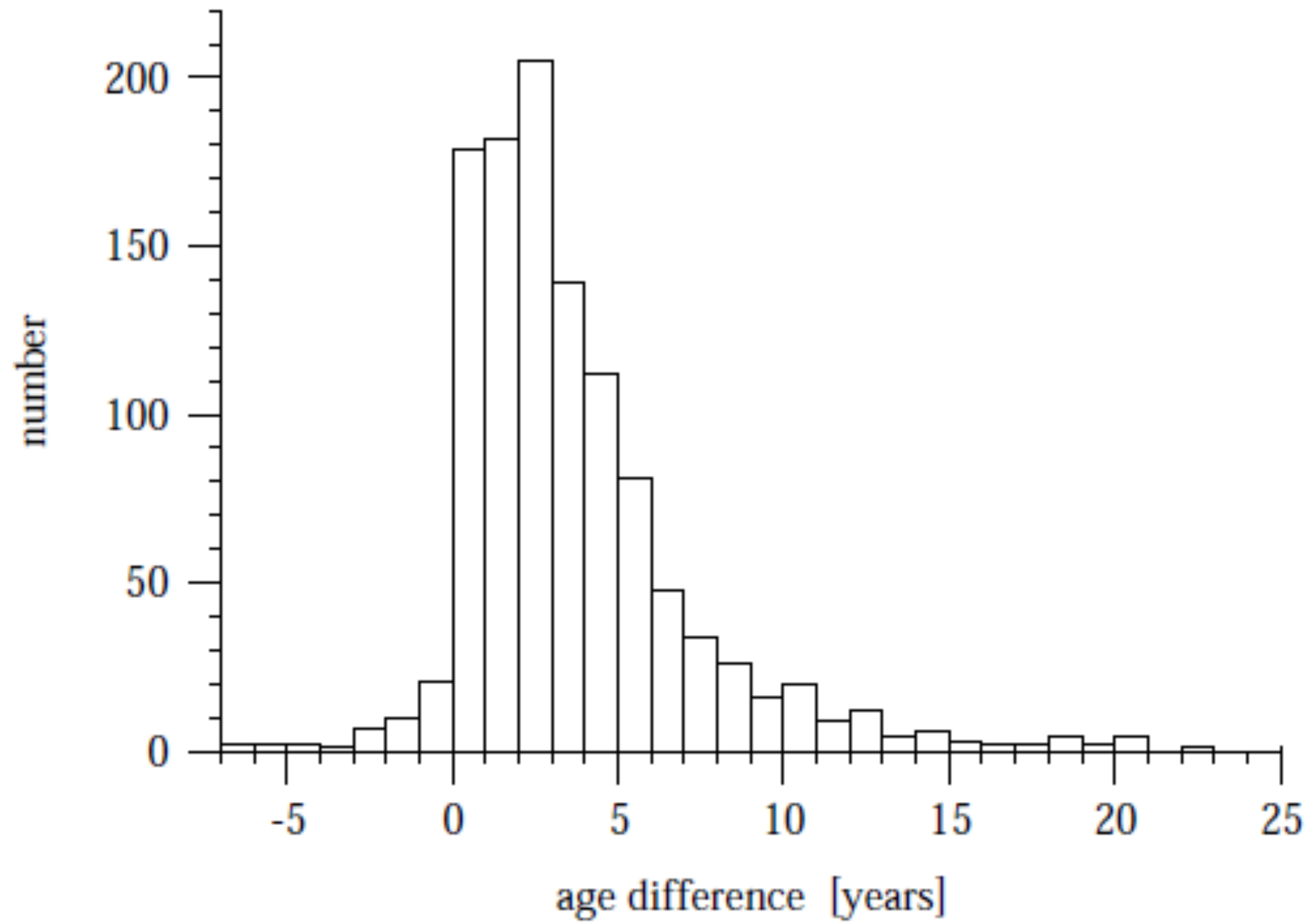
# 1141 married couples at time of marriage




strong positive correlation between the ages

M. E. J. Newman, Mixing patterns in networks, Phys. Rev. E, 67, 016128, 2003

# 1141 married couples at time of marriage

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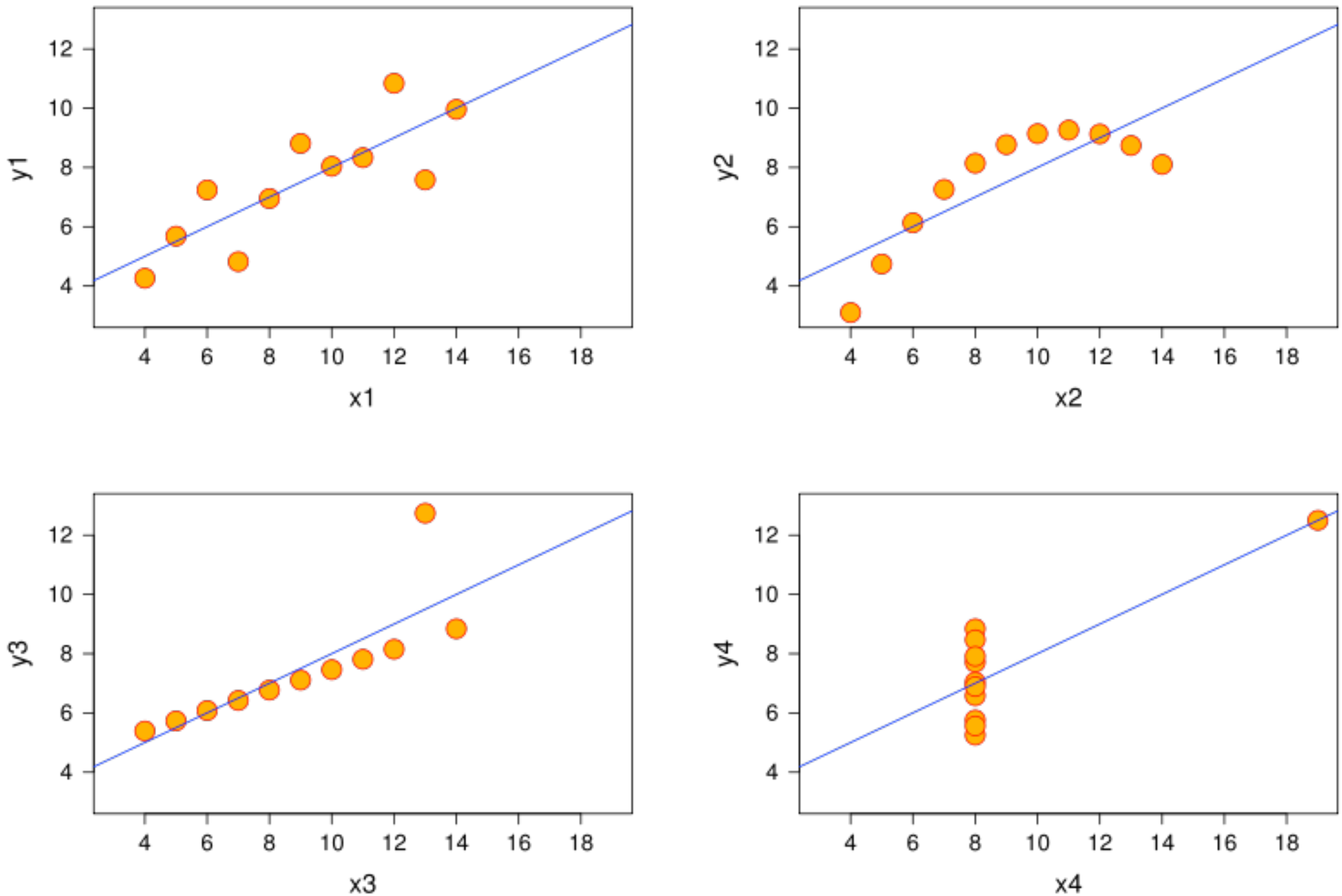



The four $y$ variables have the same mean (7.5), standard deviation (4.12), correlation (0.816) and regression line ($y$ = 3 + 0.5$x$)

# Degree Correlation

| network | type | $n$ | $m$ | $z$ | $\ell$ | $\alpha$ | $C^{(1)}$ | $C^{(2)}$ | $r$ |
|---|---|---:|---:|---:|---:|---:|---|---|---:|
| film actors | undirected | 449 913 | 25 516 482 | 113.43 | 3.48 | 2.3 | 0.20 | 0.78 | 0.208 |
| company directors | undirected | 7 673 | 55 392 | 14.44 | 4.60 | – | 0.59 | 0.88 | 0.276 |
| math coauthorship | undirected | 253 339 | 496 489 | 3.92 | 7.57 | – | 0.15 | 0.34 | 0.120 |
| physics coauthorship | undirected | 52 909 | 245 300 | 9.27 | 6.19 | – | 0.45 | 0.56 | 0.363 |
| biology coauthorship | undirected | 1 520 251 | 11 803 064 | 15.53 | 4.92 | – | 0.088 | 0.60 | 0.127 |
| telephone call graph | undirected | 47 000 000 | 80 000 000 | 3.16 | | 2.1 | | | |
| email messages | directed | 59 912 | 86 300 | 1.44 | 4.95 | 1.5/2.0 | | 0.16 | |
| email address books | directed | 16 881 | 57 029 | 3.38 | 5.22 | – | 0.17 | 0.13 | 0.092 |
| student relationships | undirected | 573 | 477 | 1.66 | 16.01 | – | 0.005 | 0.001 | −0.029 |
| sexual contacts | undirected | 2 810 | | | | 3.2 | | | |
| WWW `nd.edu` | directed | 269 504 | 1 497 135 | 5.55 | 11.27 | 2.1/2.4 | 0.11 | 0.29 | −0.067 |
| WWW Altavista | directed | 203 549 046 | 2 130 000 000 | 10.46 | 16.18 | 2.1/2.7 | | | |
| citation network | directed | 783 339 | 6 716 198 | 8.57 | | 3.0/– | | | |
| Roget's Thesaurus | directed | 1 022 | 5 103 | 4.99 | 4.87 | – | 0.13 | 0.15 | 0.157 |
| word co-occurrence | undirected | 460 902 | 17 000 000 | 70.13 | | 2.7 | | 0.44 | |
| Internet | undirected | 10 697 | 31 992 | 5.98 | 3.31 | 2.5 | 0.035 | 0.39 | −0.189 |
| power grid | undirected | 4 941 | 6 594 | 2.67 | 18.99 | – | 0.10 | 0.080 | −0.003 |
| train routes | undirected | 587 | 19 603 | 66.79 | 2.16 | – | | 0.69 | −0.033 |
| software packages | directed | 1 439 | 1 723 | 1.20 | 2.42 | 1.6/1.4 | 0.070 | 0.082 | −0.016 |
| software classes | directed | 1 377 | 2 213 | 1.61 | 1.51 | – | 0.033 | 0.012 | −0.119 |
| electronic circuits | undirected | 24 097 | 53 248 | 4.34 | 11.05 | 3.0 | 0.010 | 0.030 | −0.154 |
| peer-to-peer network | undirected | 880 | 1 296 | 1.47 | 4.28 | 2.1 | 0.012 | 0.011 | −0.366 |
| metabolic network | undirected | 765 | 3 686 | 9.64 | 2.56 | 2.2 | 0.090 | 0.67 | −0.240 |
| protein interactions | undirected | 2 115 | 2 240 | 2.12 | 6.80 | 2.4 | 0.072 | 0.071 | −0.156 |
| marine food web | directed | 135 | 598 | 4.43 | 2.05 | – | 0.16 | 0.23 | −0.263 |
| freshwater food web | directed | 92 | 997 | 10.84 | 1.90 | – | 0.20 | 0.087 | −0.326 |
| neural network | directed | 307 | 2 359 | 7.68 | 3.97 | – | 0.18 | 0.28 | −0.226 |

# Possible explanations

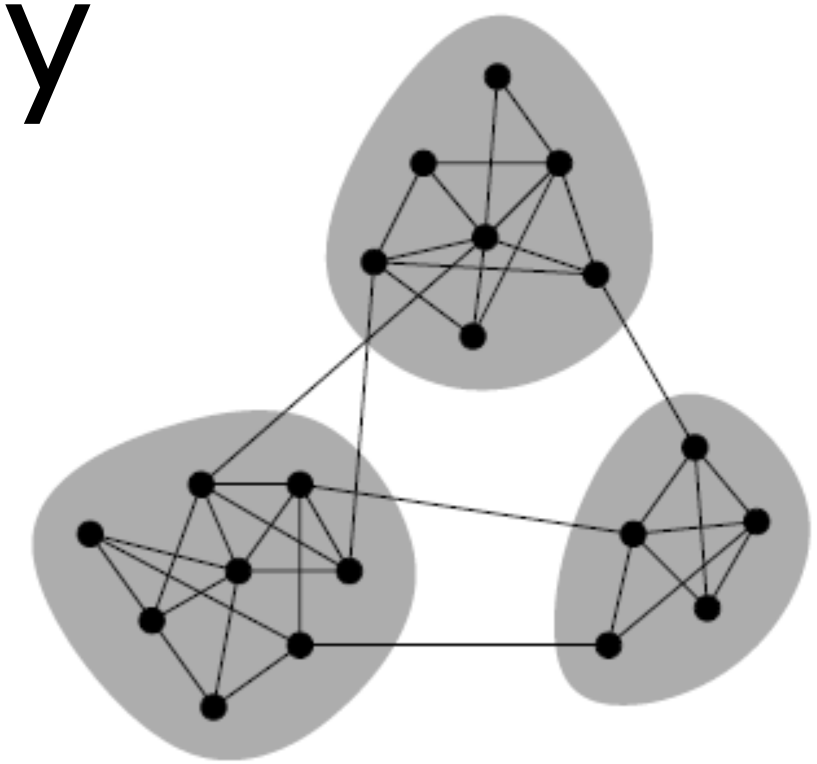
"In the social networks it is entirely possible, and is often assumed in the sociological literature, that similar people do attract one another, and therefore that there could be a real preference among gregarious people for association with other gregarious people, and similarly for hermits."

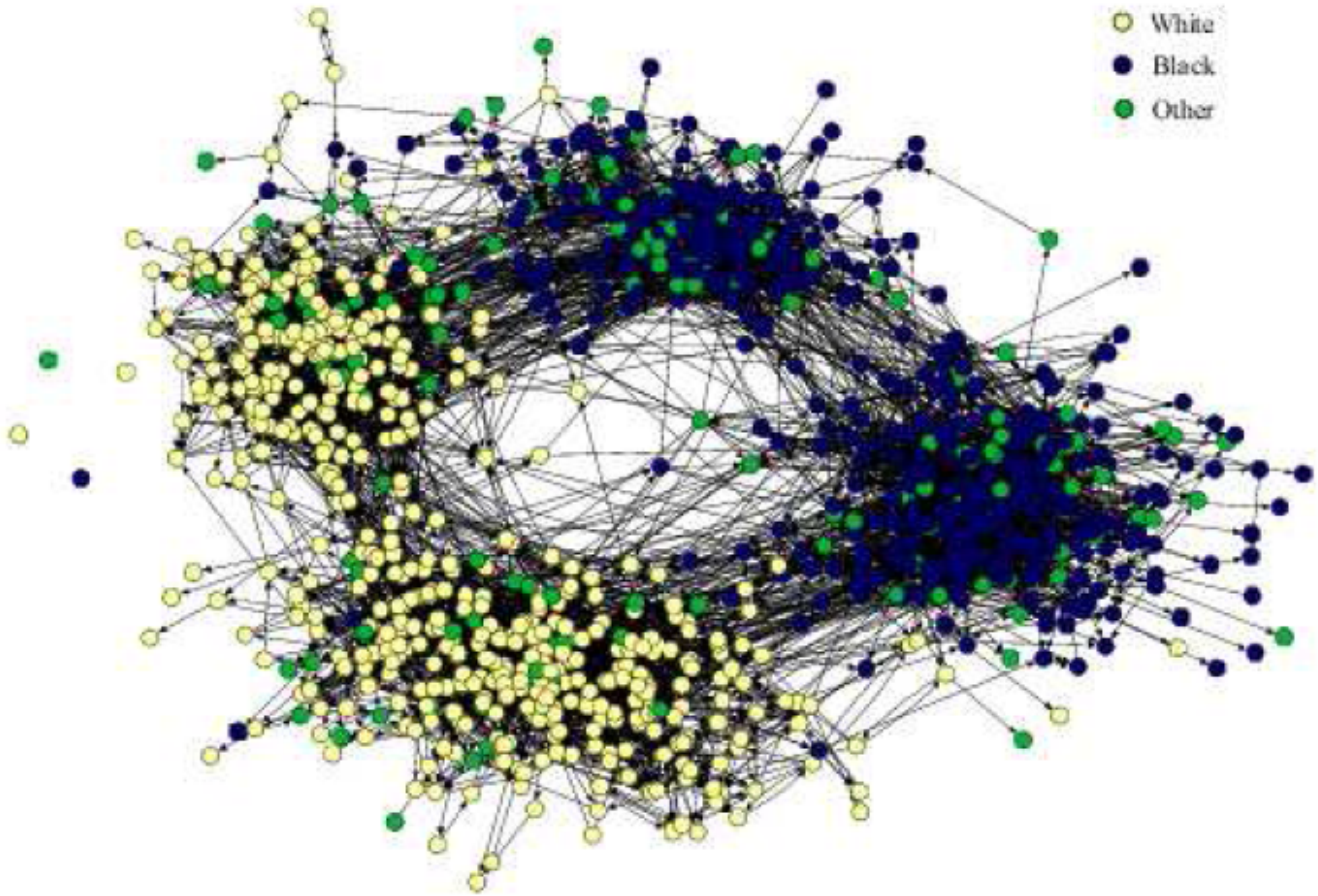
"In the Internet and the World-Wide Web there may be organizational reasons for degree anti-correlation between vertices. The high-degree vertices in these networks are often connectivity providers (Internet) or directories (Web), which by definition tend to be connected to the "little people"—the individual service subscribers in the case of the Internet or the individual web-pages on the Web."

M. E. J. Newman, Mixing patterns in networks, Phys. Rev. E, 67, 016128, 2003

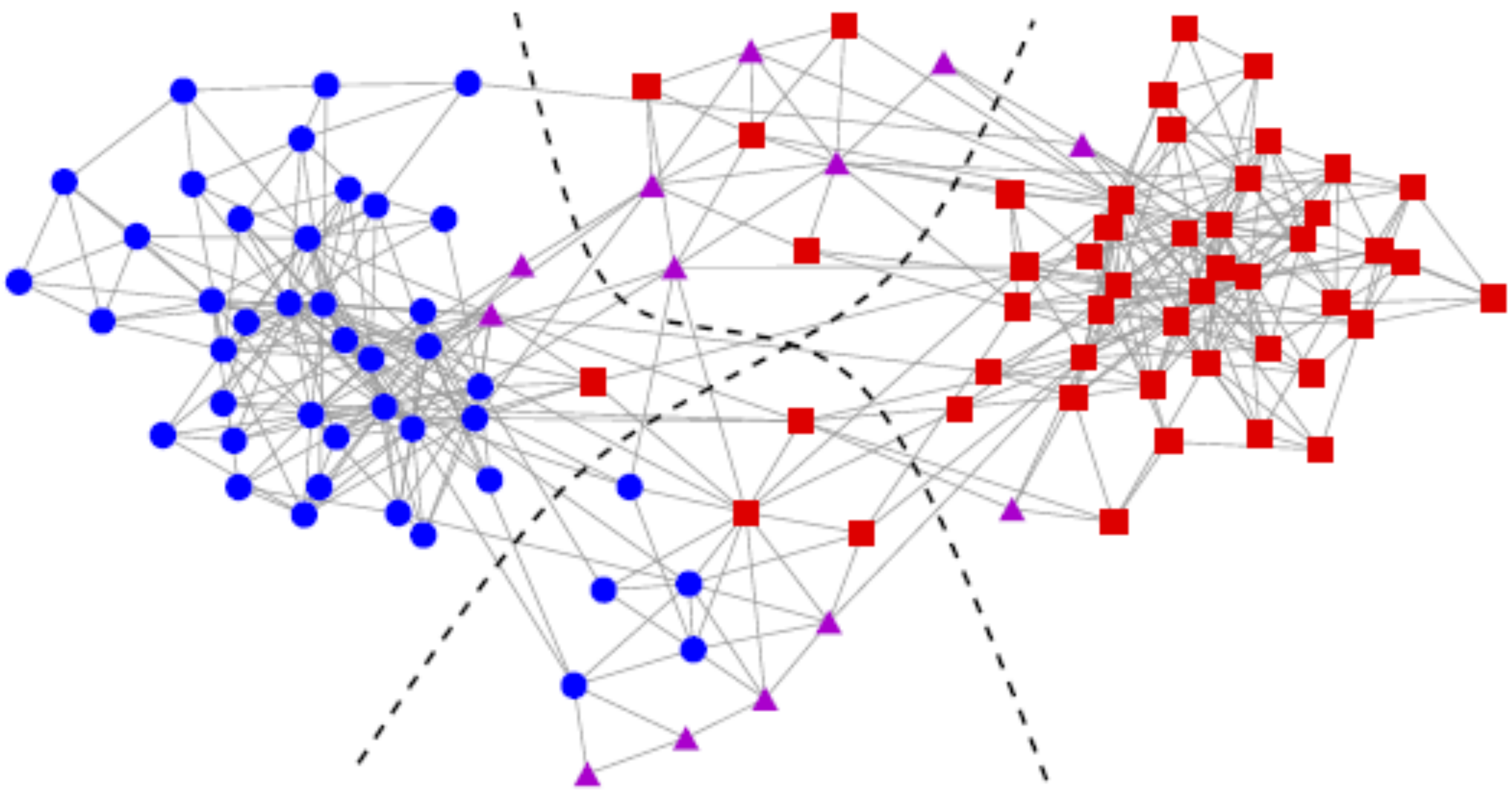
some of the degree correlations we see in our networks could have real social or organizational origins, while others may be artifacts of the types of networks we are looking at and the constraints that are placed on their structure.

# Community






Friend ship network in a US school (Spring embedding algorithm)



circles (blue) are liberal, squares (red) are conservative, triangles (purple) are centrist or unaligned.

M. E. J. Newman, Modularity and community structure in networks, Proc. Natl. Acad. Sci. USA 103, 8577-8582 (2006)

# Exploring complex networks

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**The study of networks pervades all of science, from neurobiology to statistical physics. The most basic issues are structural: how does one characterize the wiring diagram of a food web or the Internet or the metabolic network of the bacterium *Escherichia coli*? Are there any unifying principles underlying their topology? From the perspective of nonlinear dynamics, we would also like to understand how an enormous network of interacting dynamical systems — be they neurons, power stations or lasers — will behave collectively, given their individual dynamics and coupling architecture. Researchers are only now beginning to unravel the structure and dynamics of complex networks.**

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## Pajek - Program for Large Network Analysis

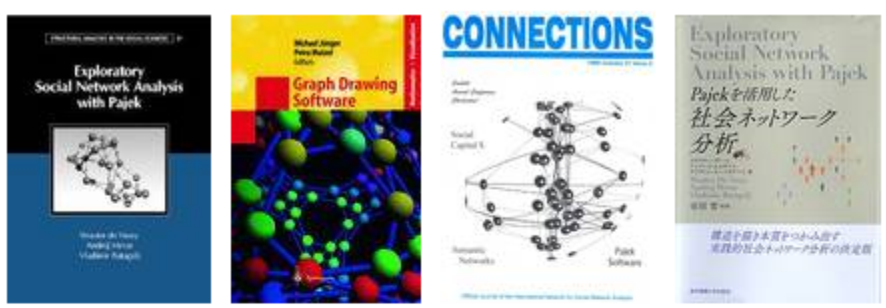
Pajek (Slovene word for Spider) is a program, for Windows, for analysis and visualization of large networks. It is freely available, for noncommercial use, at its download page. See also a reference manual for Pajek (in PDF). The development of Pajek is traced in its History.

Pajek is developed by Vladimir Batagelj and Andrej Mrvar. Some procedures were contributed also by Matjaž Zaveršnik.

A detailed introduction to Pajek is given in the book W. de Nooy, A. Mrvar, V. Batagelj: *Exploratory Social Network Analysis with Pajek*, CUP, 2005. Appendix 4. ESNA in Japanese, TDU, 2010.

For a short description see Batagelj V., Mrvar A.: Pajek - Analysis and Visualization of Large Networks. in Jünger, M., Mutzel, P., (Eds.) Graph Drawing Software. Springer, Berlin 2003. p. 77-103.

An improved version of the paper Batagelj V., Mrvar A.: *Pajek - Program for Large Network Analysis*, presented at Sunbelt'97, was published in Connections 21(1998)2, 47-57 - (preprint Prison Kinemage).

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# Next class

Mathematical models of networks

- Random graph model (generalized)
- Markov graphs

Then...

- Small-worlds models
- Models of growing networks