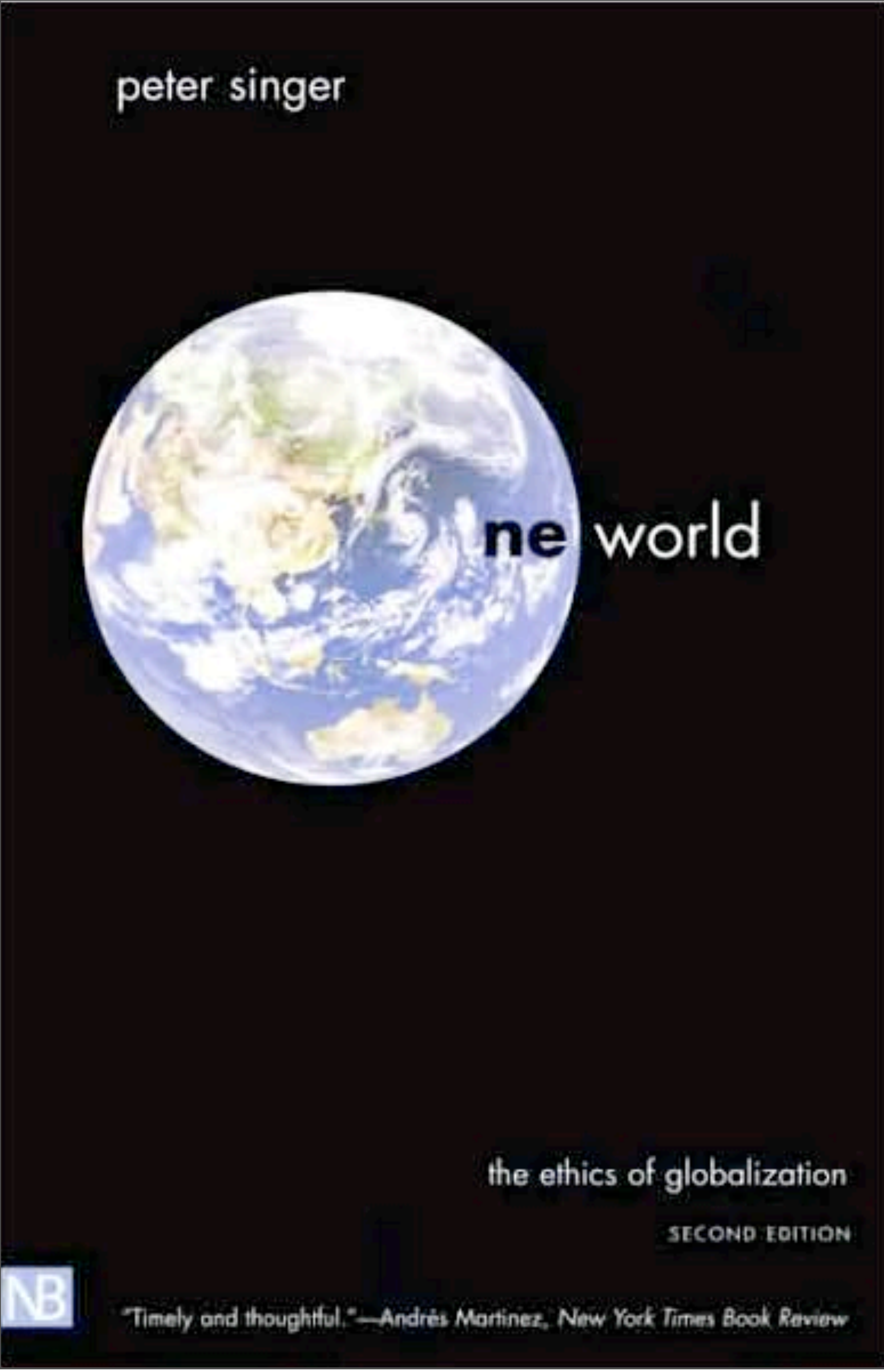
Lecture 13

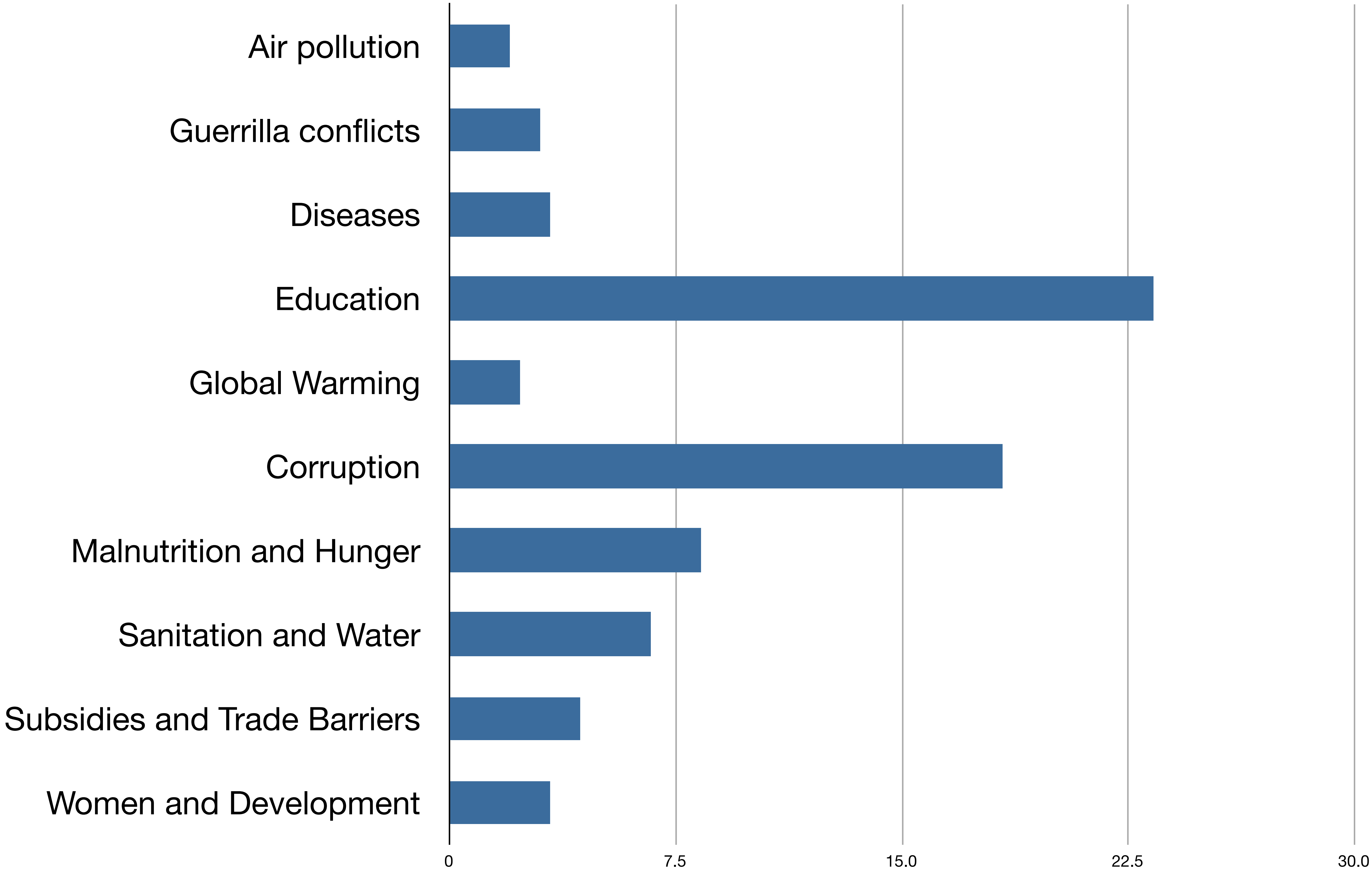
Final Review

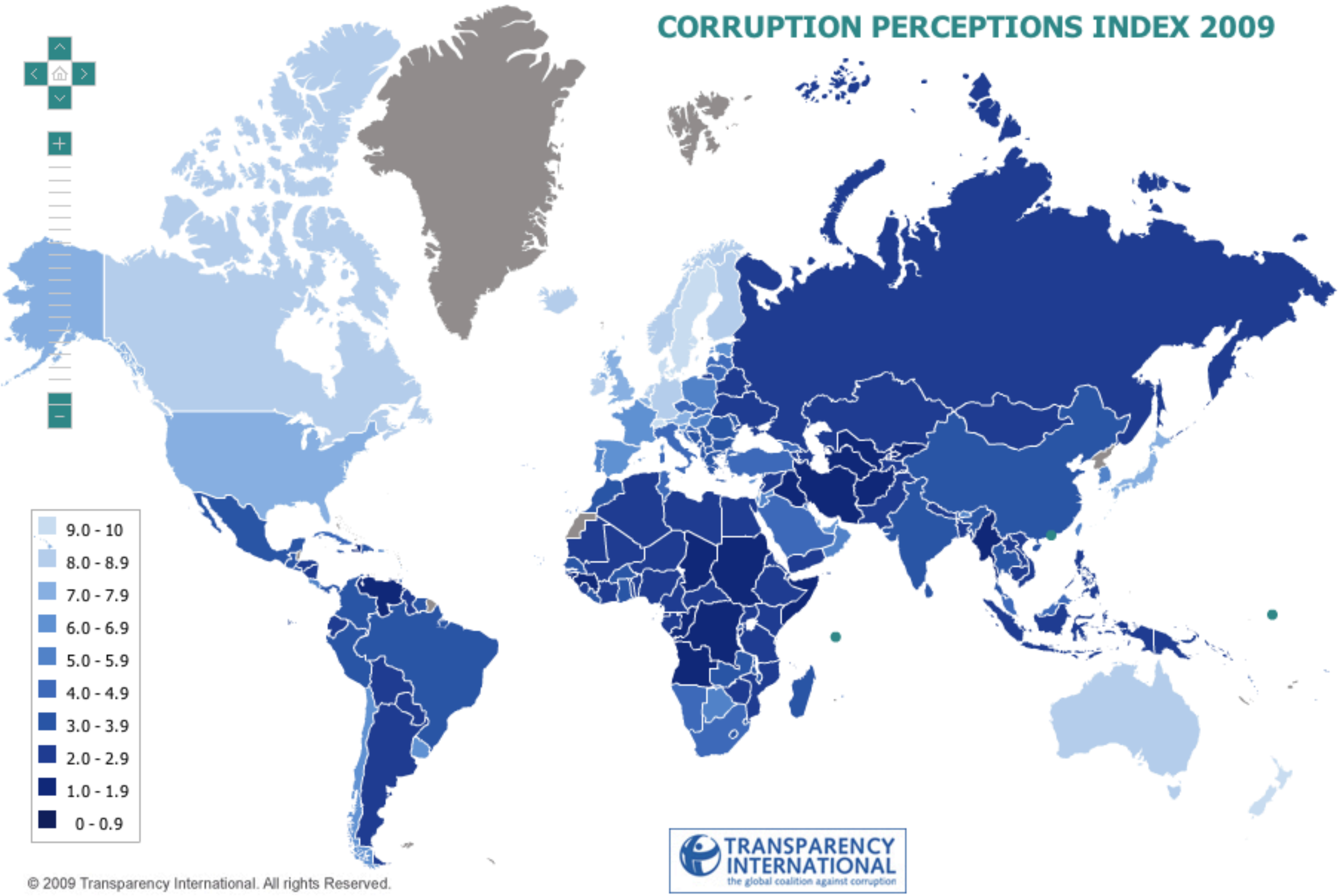
WE HAVE A PROBLEM!



GLOBALIZATION

Survey: Priorities among a series of proposals for confronting ten great challenges (grad students)




CORRUPTION PERCEPTIONS INDEX 2009
9.0 - 10
8.0 - 8.9
7.0 - 7.9
6.0 - 6.9
5.0 - 5.9
4.0 - 4.9
3.0 - 3.9
2.0 - 2.9
1.0 - 1.9
0 - 0.9
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the global coalition against corruption

Local solution?

High-tech solutions?

Technology is anything invented after you were born

Alan Kay

Technology is anything
that doesn't work yet

Danny Hillis

tech•nol•o•gy |tekˈnäləjē|

noun (pl. **-gies**)

the application of scientific knowledge for practical purposes, esp. in industry

- machinery and equipment developed from such scientific knowledge.
- the branch of knowledge dealing with engineering or applied sciences.

strategy

1. Data about incentives

gather data to quantify influence within an institution

frequent questions:

- what are the sources of income?
- how have those sources changed?
- what policies does the institution use to police these sources of income?
- how careful are they enforced?

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frequent questions:

- what are the sources of income?
- how have those sources changed?
- what policies does the institution use to police these sources of income?
- how careful are they enforced?

map economy of influence

use data to identify historical techniques
e.g., to develop and sustain independence

2. Data about perceptions

- track public perception → trust of the institution
- role in institution's success
    - private institutions
    - public institutions

frequent questions:

- how does public trust of the FDA evolve? improved/weaken?

2. Data about perceptions

- track public perception → trust of the institution
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frequent questions:

- how does public trust of the FDA evolve? improved/weaken?

map the relevant public

use data to develop historical understanding
of different perceptions
e.g., independence in attitudes

3. Models of causation

measure causation

→ explore and capture economies of influence - hard!

frequent questions:

- how do influences affect the work product?
- how do influences affect public trust?

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→ explore and capture economies of influence - hard!

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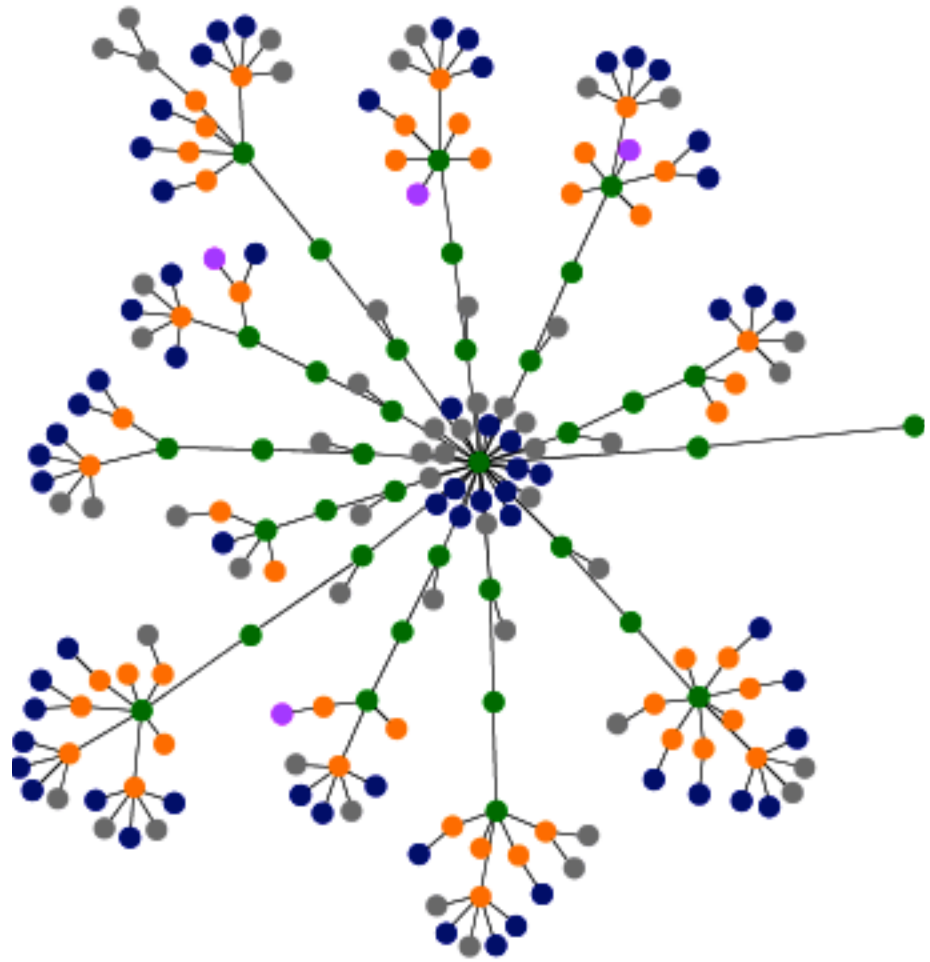
- how do influences affect public trust?

develop quantitative analysis of the
independence of actors within an institution

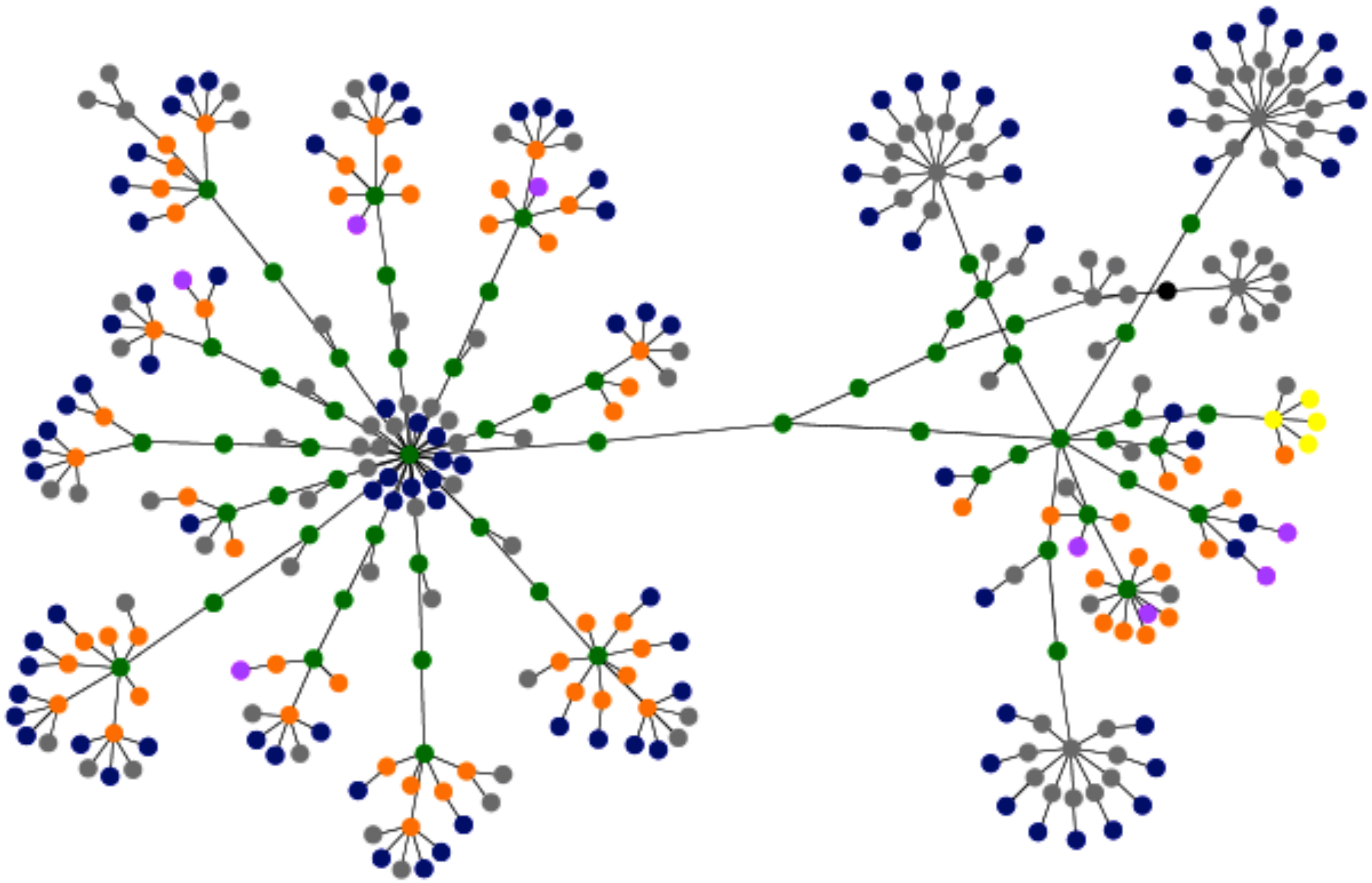
Understand dependencies!!

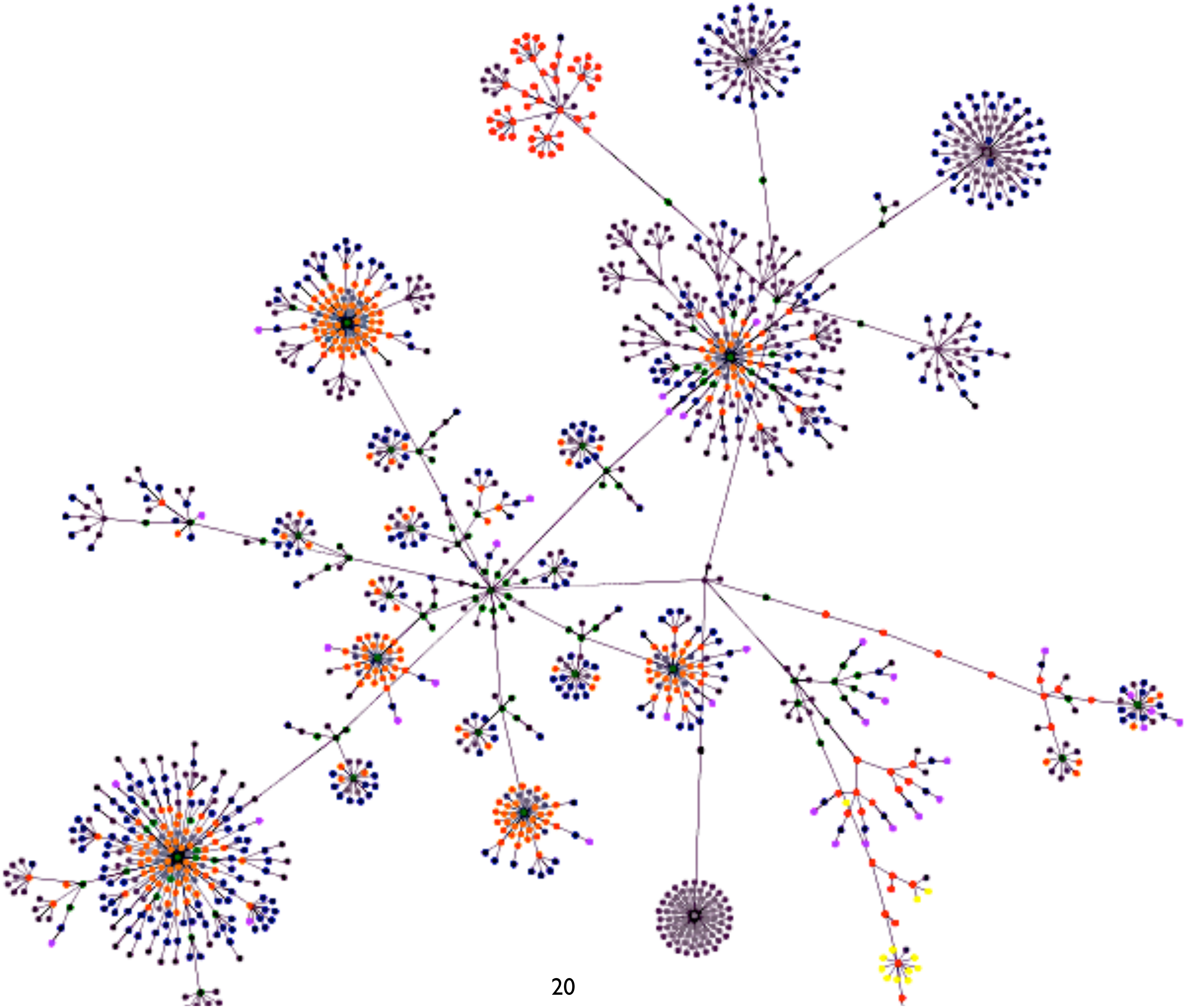
FOCUS

Network science



Which vertex would proof most crucial?





Which vertex would proof most crucial?

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What percentage of vertices need to be removed to affect connectivity?

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statistical properties of graphs

Why study networks?

- Capture relationships / non-spatial interactions
- Understand structure and function
- Metrics exist that capture structure
- Structure or metrics can be related to important properties of the system or its behavior

Network science modeling approach

1. Show how data on interaction in the real world can be translated into graphs

2. Predict the course of the process from its structure

Real networks are not random!

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1. Possible mechanisms guiding network formation

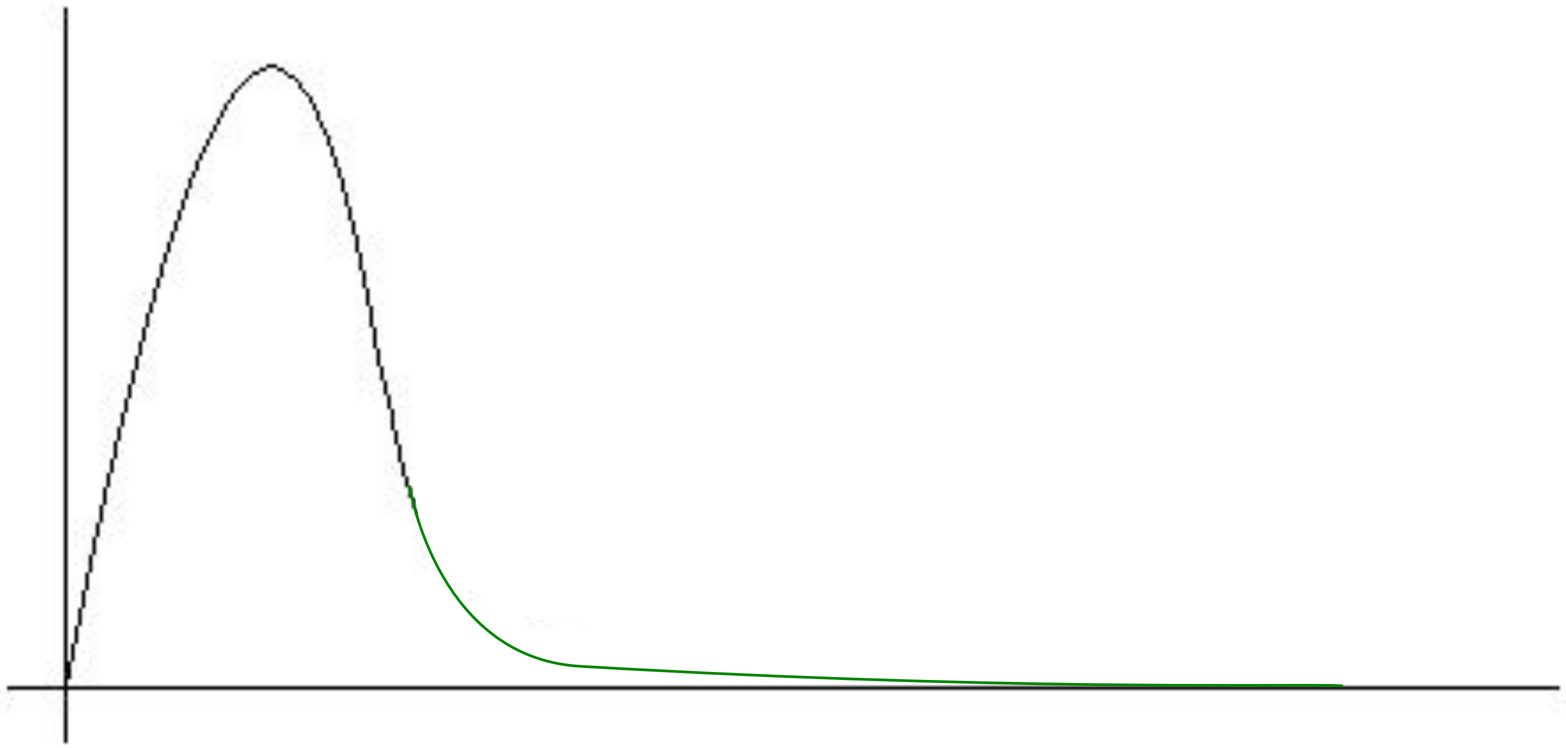
Real networks are not random!

1. Possible mechanisms guiding network formation
2. Ways to exploit structure to achieve certain aims

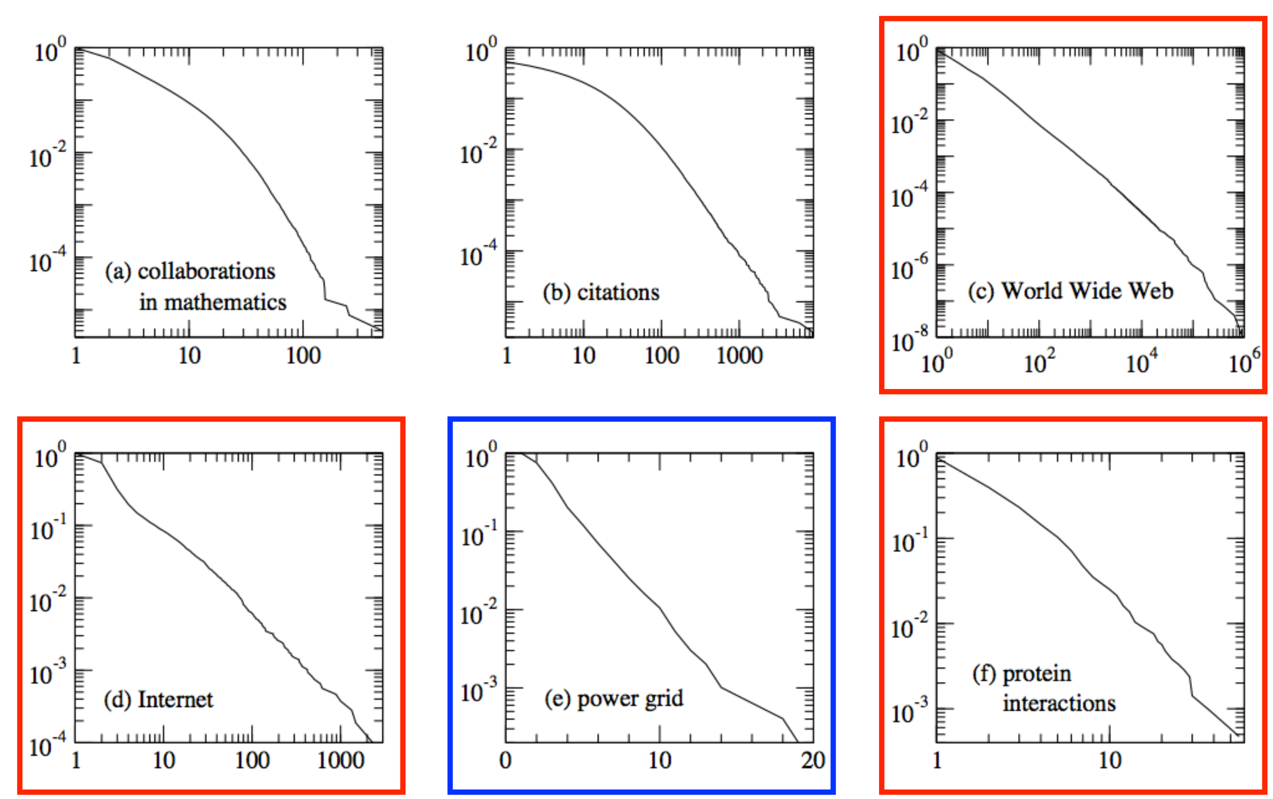
Common network properties

1. degree distribution
2. small-world effect
3. transitivity or clustering
4. network resilience
5. mixing patterns
6. degree correlation
7. community structure

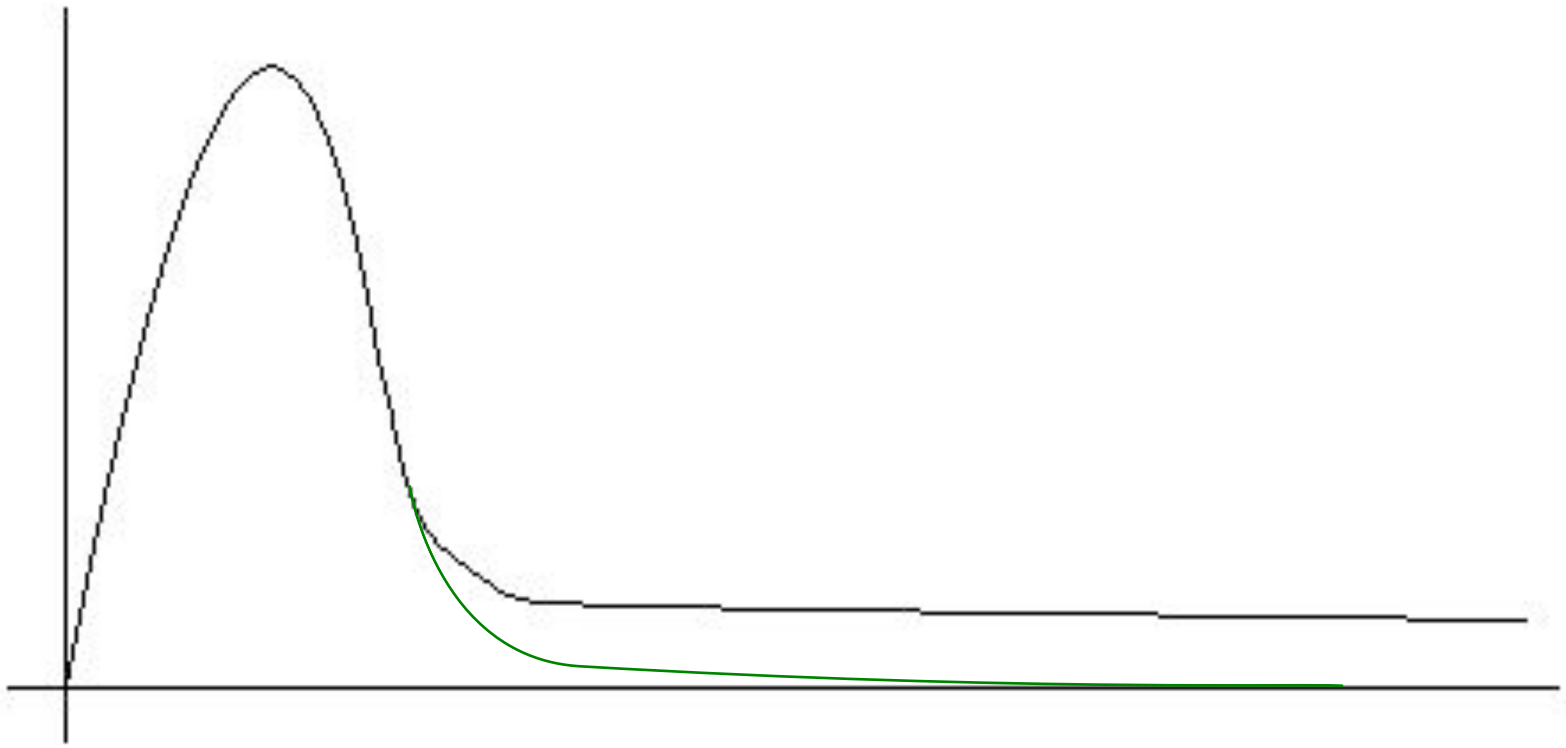
degree distribution



- Poisson
- Heavy-tailed
  - power-law
  - exponential

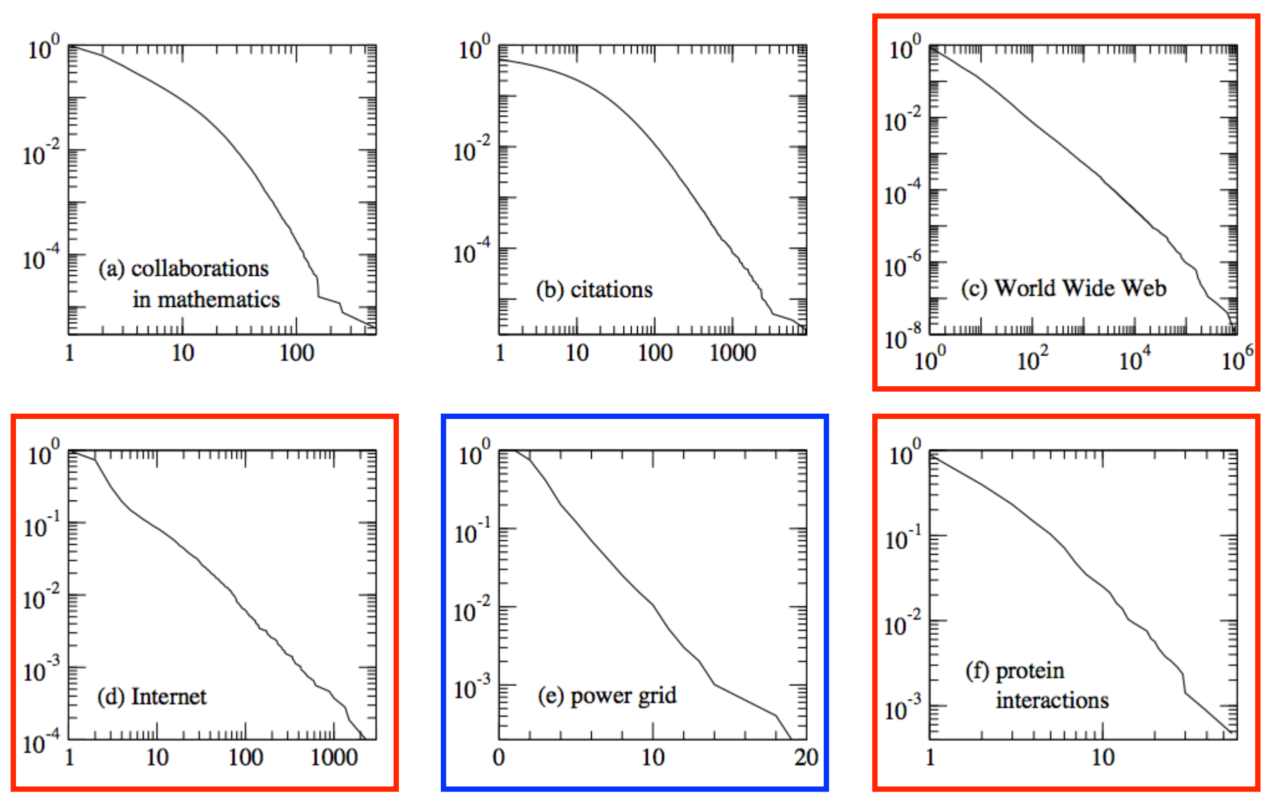


degree distribution

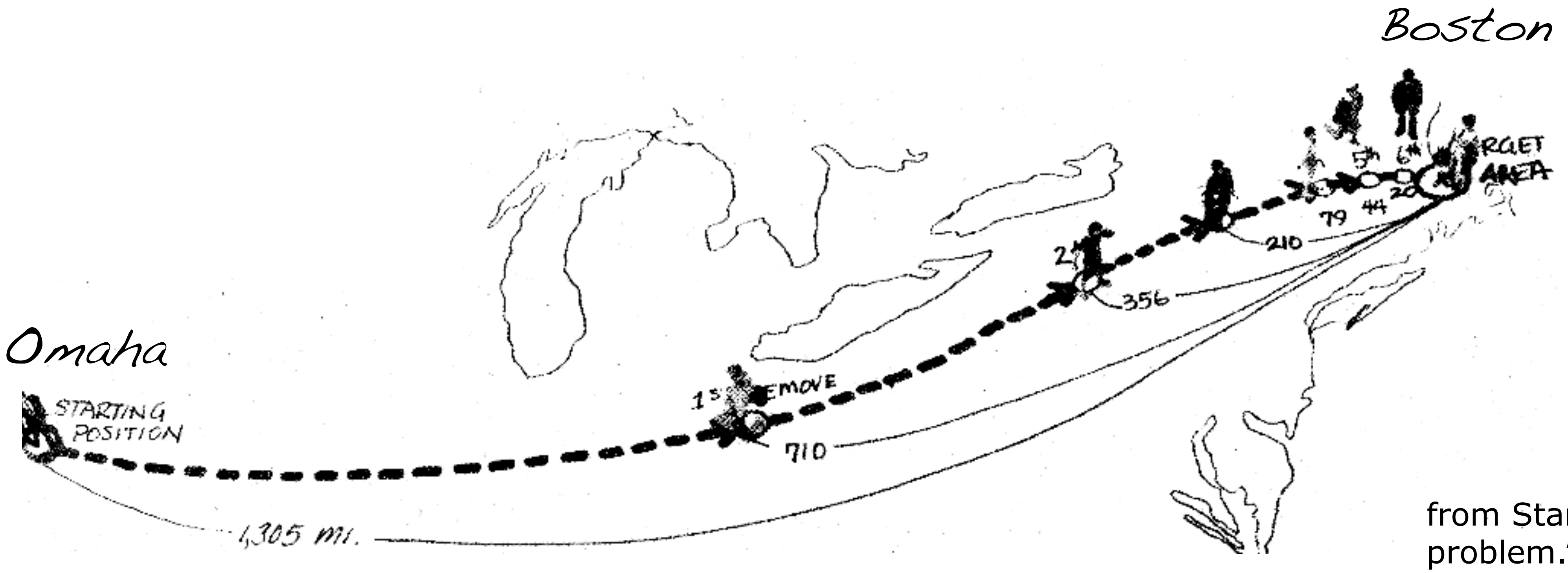


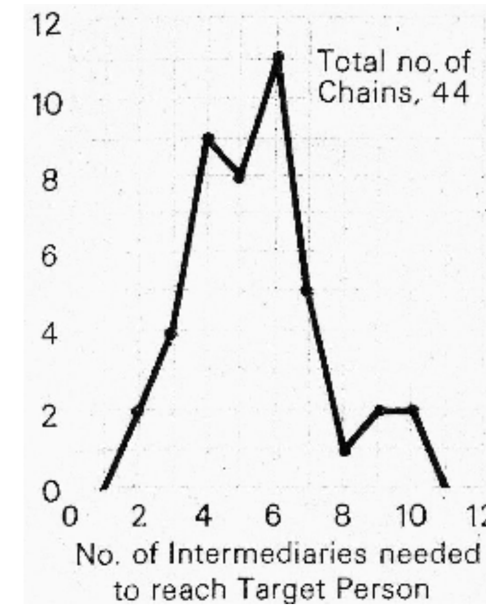
$$\lim_{x\to\infty} e^{\lambda x}\Pr[X > x] = \infty \quad \text{for all } \lambda > 0.$$

- Poisson
- Heavy-tailed
  - power-law
  - exponential



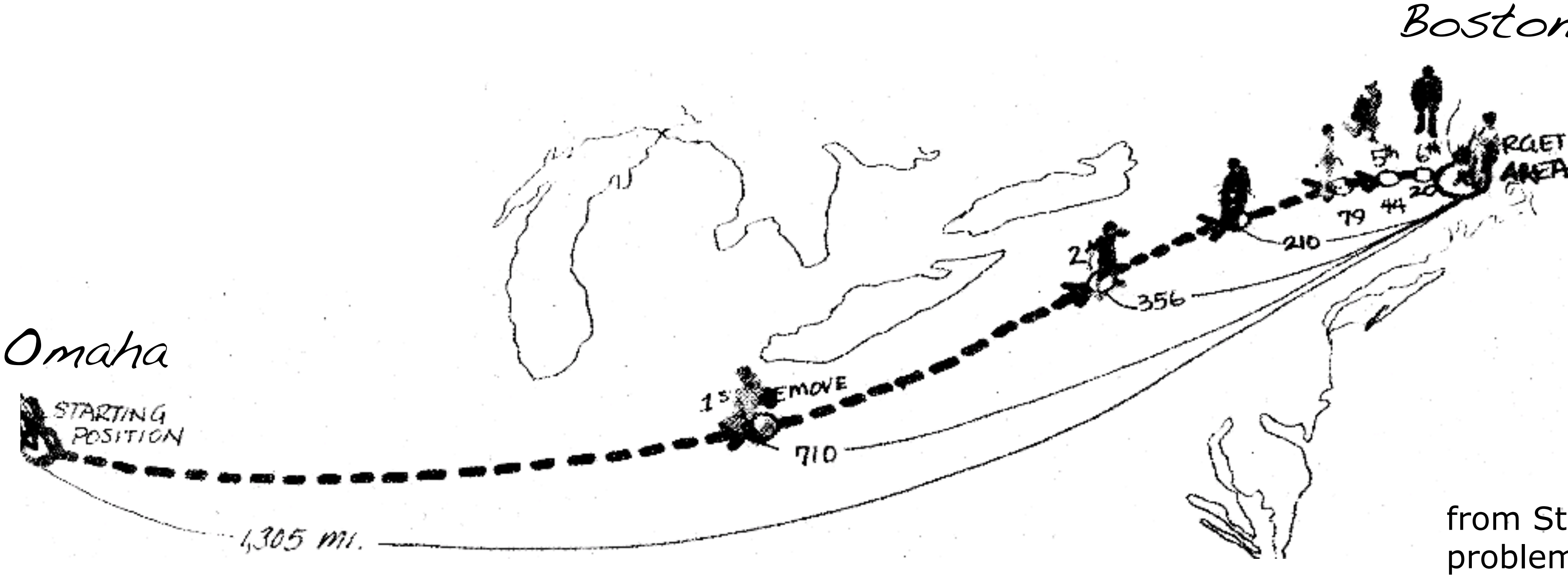
small-world effect

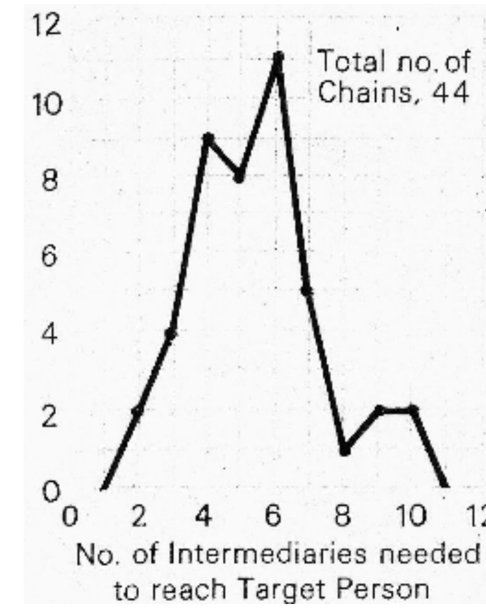




from Stanley Milgram, “The small world problem.” Phycoogy today, 1967

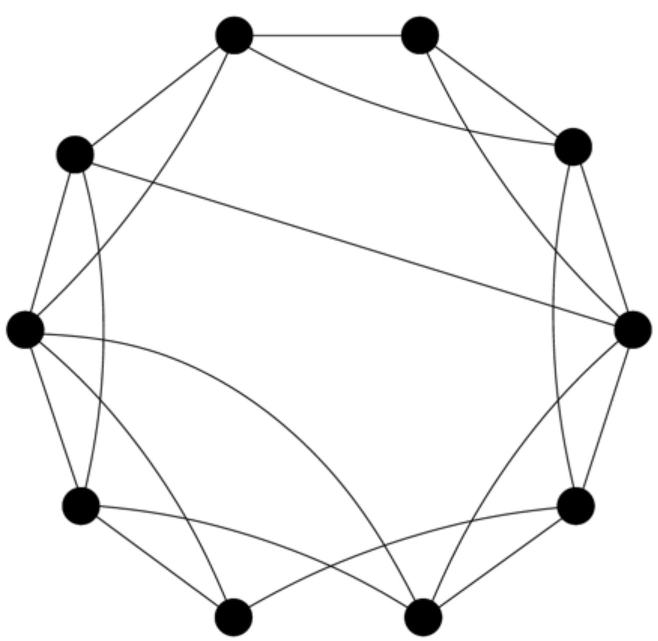
small-world effect





from Stanley Milgram, "The small world problem." Phycoogy today, 1967

Most packages did not arrive = navigation issue ≠ small-world

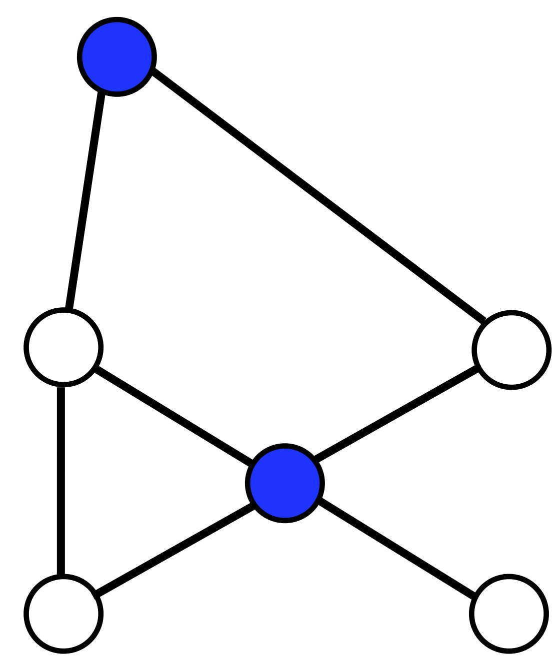


$$\ell = \frac{1}{\frac{1}{2}n(n+1)} \sum_{i \geq j} d_{ij}$$

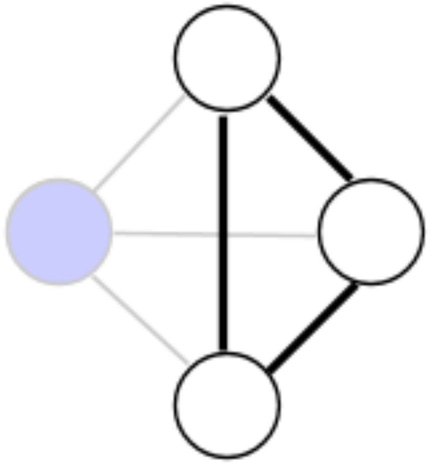
geodesic distance

- The vertex set (of an undirected graph) and the distance function form a metric space, if and only if the graph is connected.
- A metric on a vertex set is a function such that any x, y and z in N satisfy

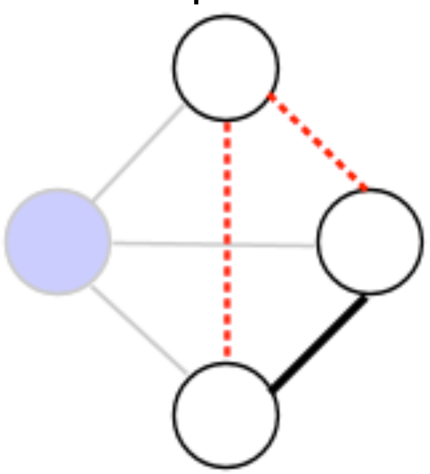
1. non-negativity
   $d(x, y) \geq 0$
2. indiscernible
   $d(x, y) = 0$ if and only if $x{=}y$
3. symmetry
   $d(x, y) = d(y, x)$
4. triangle inequality
   $d(x, z) \leq d(x, y) + d(y, z)$



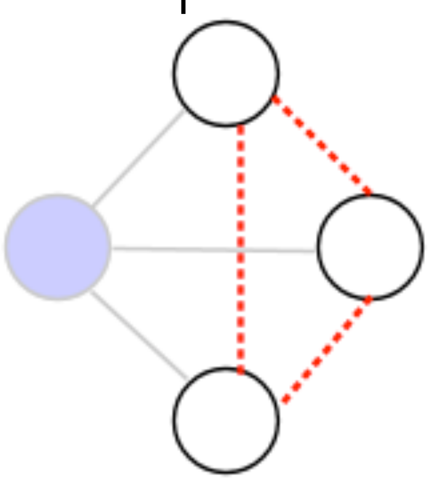
Transitivity



$c_i = 1$

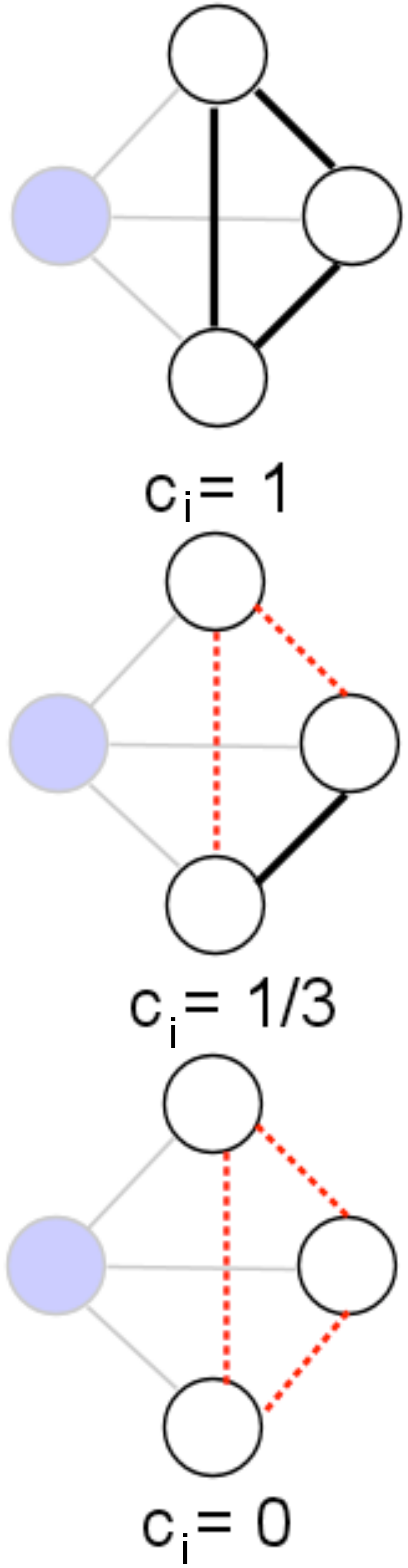


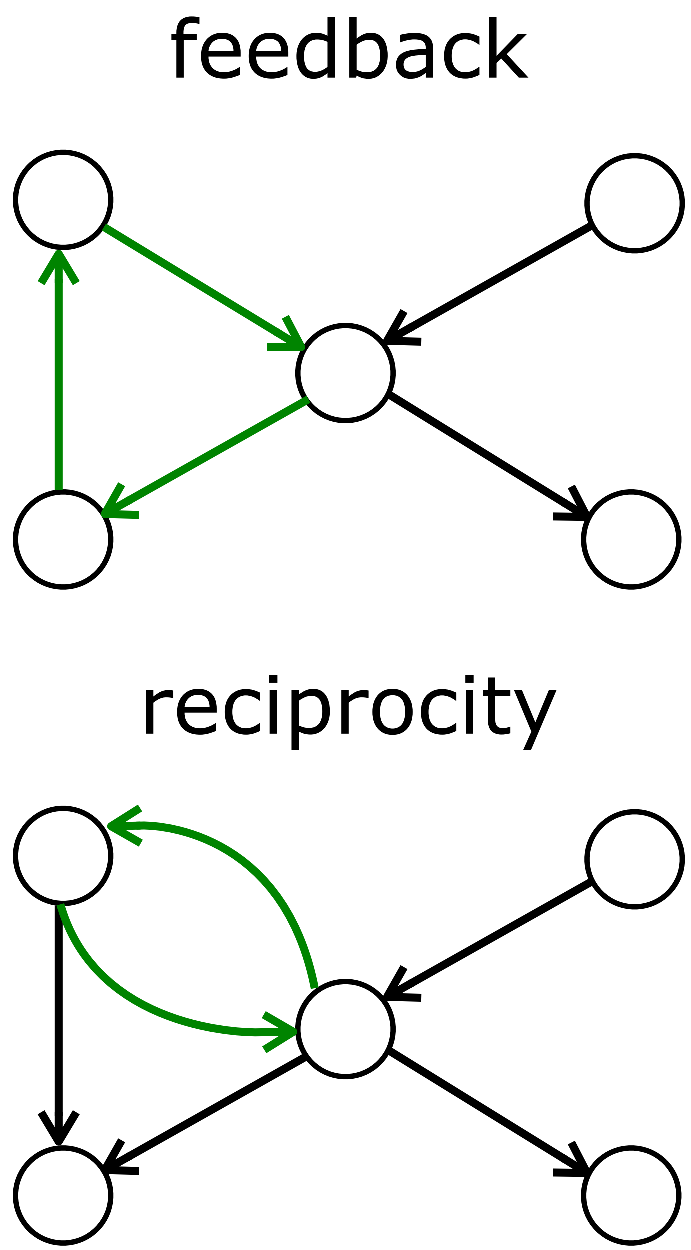
$c_i = 1/3$



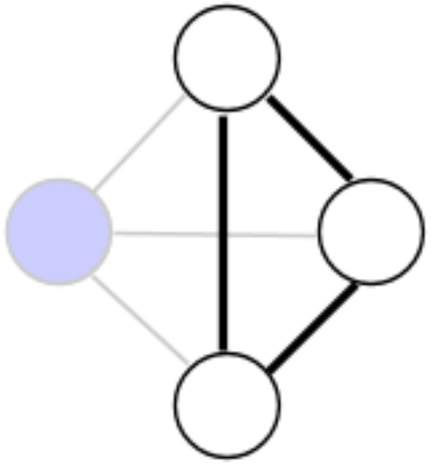
$c_i = 0$

Transitivity

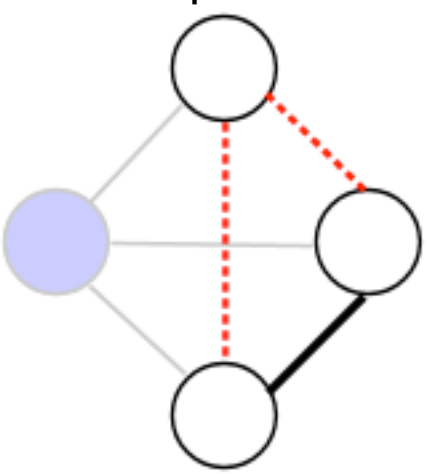




Transitivity



$c_i = 1$



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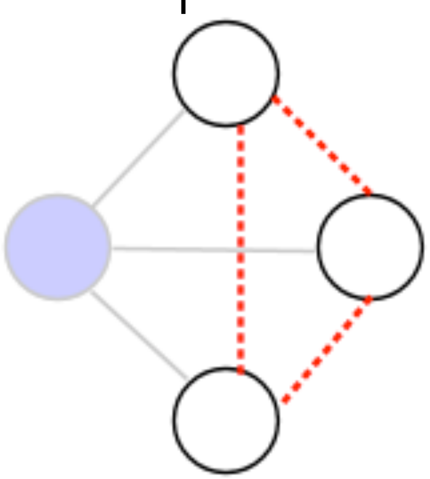
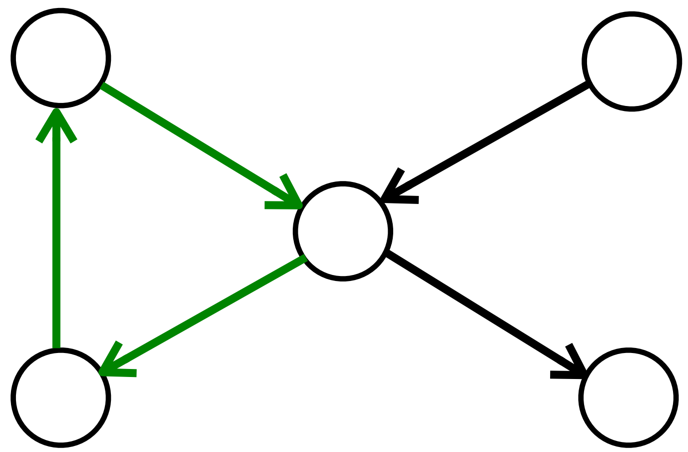
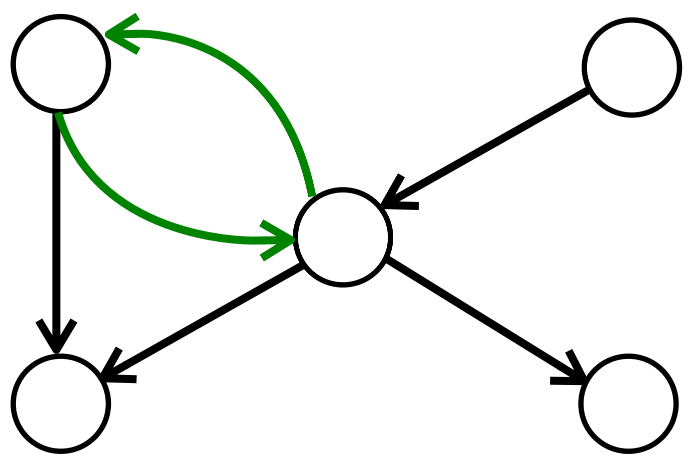


$c_i = 0$

feedback



reciprocity



Ratio of the means

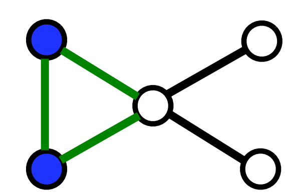
$$C \quad = \quad 3\frac{\text{number of triangles}}{\text{number of triples of vertices}} = \frac{3}{8}$$

vs.

Mean of the ratio

$$C_i = \frac{\text{number of triangles connected to vertex } i}{\text{number of triples centered on vertex } i}$$

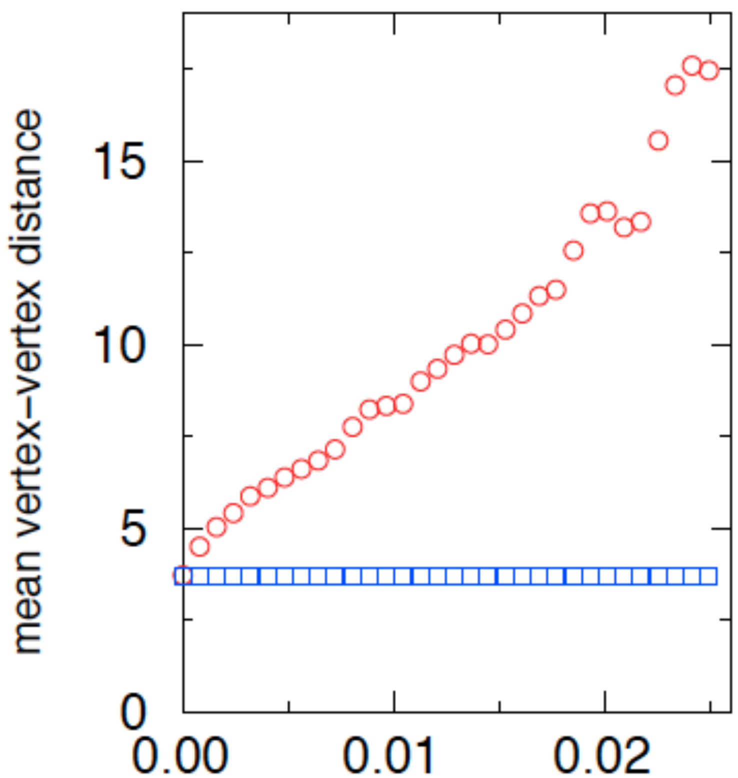
$$C = \frac{1}{n}\sum_i C_i = \frac{13}{30}$$

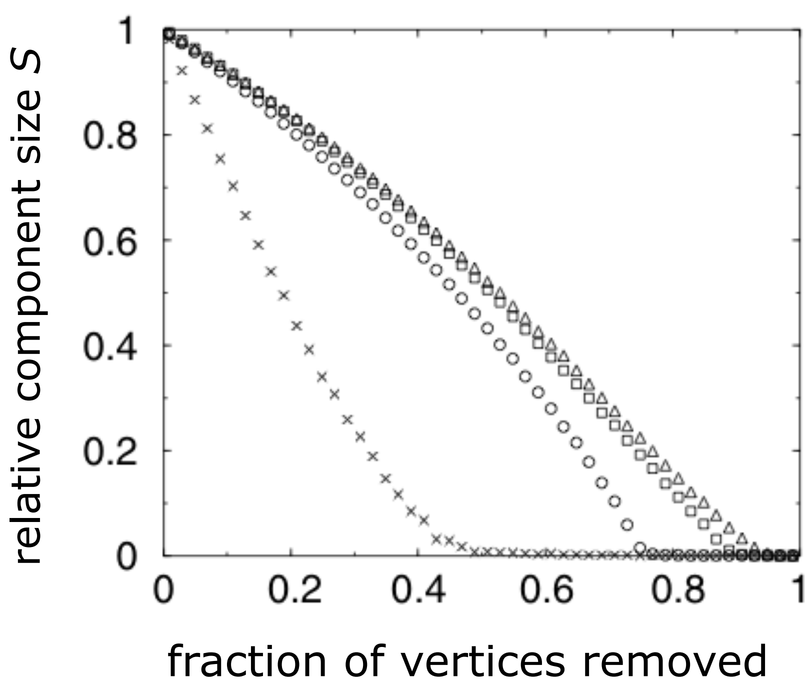


Tends to weight the contribution of low degree vertices more heavily

Network resilience

"Resilience is the ability to provide and maintain an acceptable level of service in the face of faults and challenges to normal operation."

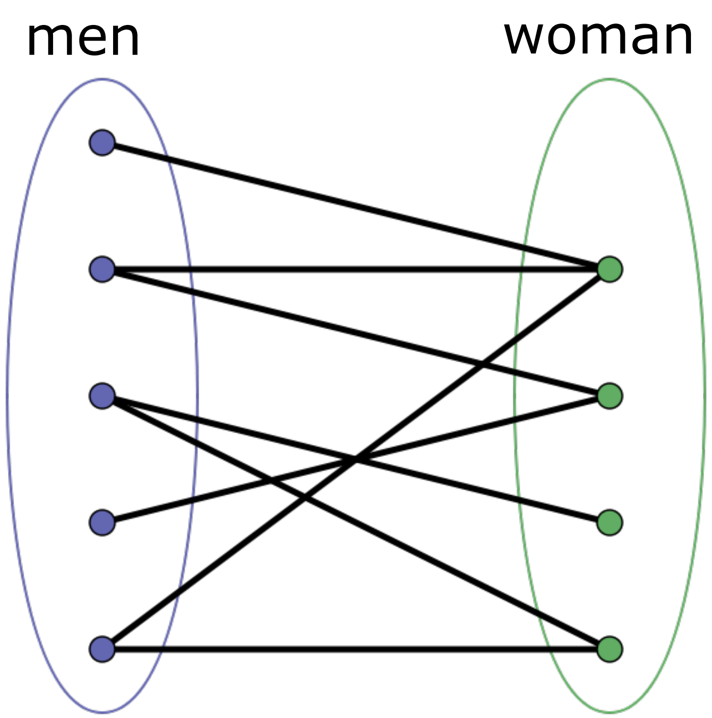




Random and targeted attacks affect other properties:

- small-world
- giant component size

Mixing patterns



E		woman			
		black	hispanic	white	other
men	black	506	32	69	26
	hispanic	23	308	114	38
	white	26	46	599	68
	other	10	14	47	32

e_{ij}		woman			
		black	hispanic	white	other
men	black	0.258	0.016	0.035	0.013
	hispanic	0.012	0.157	0.058	0.019
	white	0.013	0.023	0.306	0.035
	other	0.005	0.007	0.024	0.016

$$Q = \frac{\sum_i P(i|i) - 1}{N-1}$$

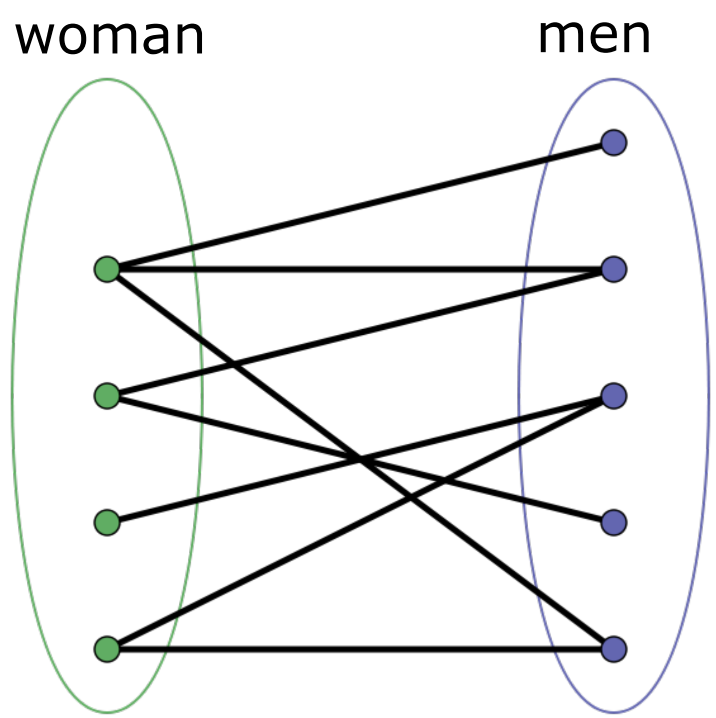
$$\boxed{Q = 0.519}$$

0: randomly mixed network
1: perfectly assorted network

$$P(j|i) = \frac{e_{ij}}{\sum_j e_{ij}}$$

fraction of edges that connect to vertices in group i

Mixing patterns



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		black	hispanic	white	other
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$$\boxed{Q = 0.527}$$

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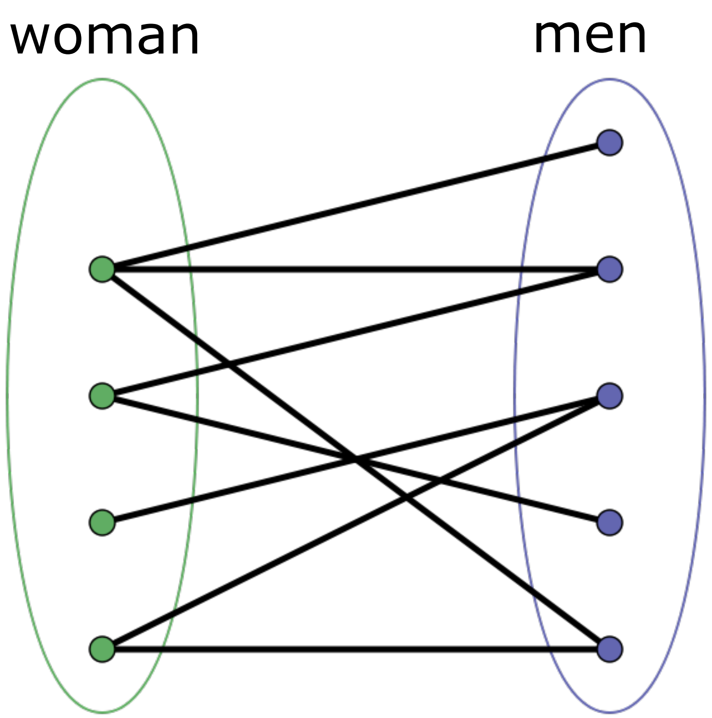
$$P(j|i) = \frac{e_{ij}}{\sum_j e_{ij}}$$

Which one is correct?

Each vertex type weighted equally!

Mixing patterns



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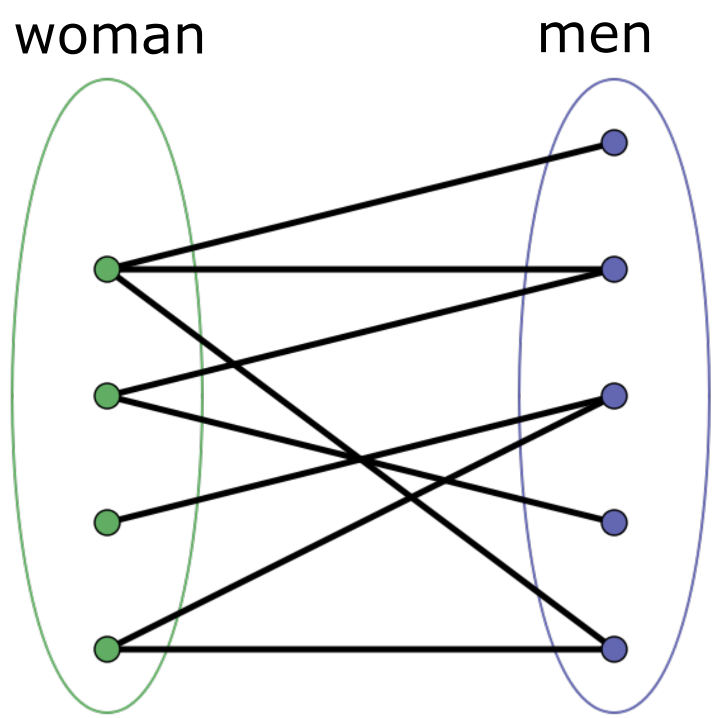
$$Q = \frac{\sum_i P(i|i) - 1}{N - 1}$$

fraction of edges that connect vertices of the same type

$$r = \frac{tr(e) - ||e^2||}{1 - ||e^2||}$$

Mixing patterns



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scalar properties?

$$Q = \frac{\sum_i P(i|i) - 1}{N - 1}$$

fraction of edges that connect
vertices of the same type

$$r = \frac{tr(e) - ||e^2||}{1 - ||e^2||}$$

Degree correlation (undirected)

M. E. J. Newman, Assortative mixing in networks. Phys. Rev. Lett. 89, 208701 (2002)

Pearson correlation:

$$r = \frac{1}{\sigma_q^2} \sum_{jk} jk(e_{jk} - q_j q_k)$$

$$\sigma_q^2 = \sum_k k^2 q_k - \left[\sum_k k q_k\right]^2$$

$$q_k = \frac{(k+1)p_{k+1}}{\sum_j j p_j} = \sum_j e_{jk}$$

	network	n	r
real-world networks	physics coauthorship[a]	52 909	0.363
	biology coauthorship[a]	1 520 251	0.127
	mathematics coauthorship[b]	253 339	0.120
	film actor collaborations[c]	449 913	0.208
	company directors[d]	7 673	0.276
	Internet[e]	10 697	−0.189
	World-Wide Web[f]	269 504	−0.065
	protein interactions[g]	2 115	−0.156
	neural network[h]	307	−0.163
	food web[i]	92	−0.276
models	random graph[u]		0
	Callaway *et al.*[v]		$\delta/(1+2\delta)$
	Barabási and Albert[w]		0

Observable network:

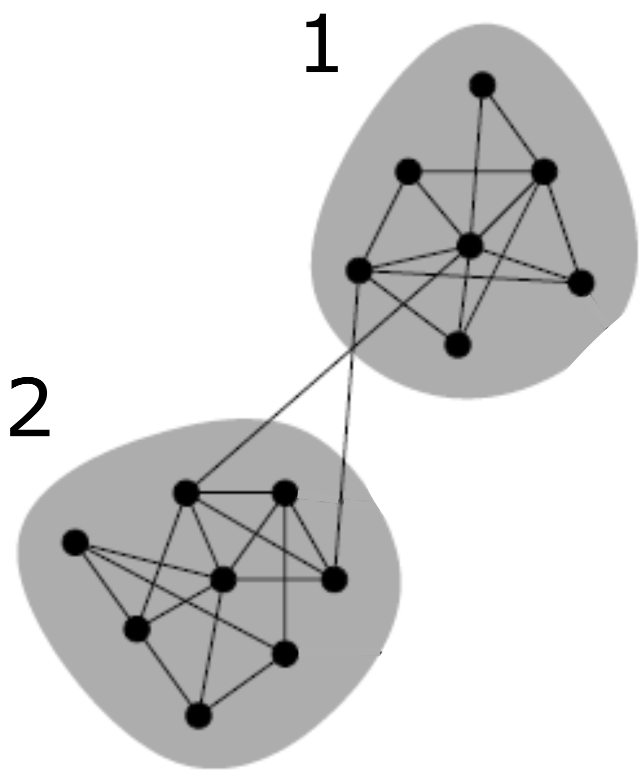
$$r = \frac{\frac{1}{M}\sum_i j_i k_i - \left[\frac{1}{M}\sum_i \frac{1}{2}(j_i + k_i)\right]^2}{\frac{1}{M}\sum_i \frac{1}{2}\left(j_i^2 + k_i^2\right) - \left[\frac{1}{M}\sum_i \frac{1}{2}\left(j_i + k_i\right)\right]^2}$$

$$j_i, k_i \quad : \quad \text{degrees of the vertices at the end of the } i^{th} \text{ edge}$$

$$i = 1, \ldots, M$$

$-1 \leq r \leq 1$, indicates measure of linear dependency

Community structure



Quantified using modularity

s_i=*1* : vertex i belongs to group 1

s_i=-*1* : vertex i belongs to group 2

merely conventional

number of edges between i and j

expected number of edges for random graph

$$Q = \frac{1}{4m}\sum_{ij}\left(A_{ij} - \frac{k_i k_j}{2m}\right) s_i s_j$$

$$m = \frac{1}{2}\sum_i k_i$$

total number of edges

Q>0 indicates positive presence of community structure

network	type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r
film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208
company directors	undirected	7 673	55 392	14.44	4.60	–	0.59	0.88	0.276
math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15	0.34	0.120
physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45	0.56	0.363
biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088	0.60	0.127
telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1			
email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0		0.16	
email address books	directed	16 881	57 029	3.38	5.22	–	0.17	0.13	0.092
student relationships	undirected	573	477	1.66	16.01	–	0.005	0.001	−0.029
sexual contacts	undirected	2 810				3.2			
WWW `nd.edu`	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	−0.067
WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7			
citation network	directed	783 339	6 716 198	8.57		3.0/–			
Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	–	0.13	0.15	0.157
word co-occurrence	undirected	460 902	17 000 000	70.13		2.7		0.44	
Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	−0.189
power grid	undirected	4 941	6 594	2.67	18.99	–	0.10	0.080	−0.003
train routes	undirected	587	19 603	66.79	2.16	–		0.69	−0.033
software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.070	0.082	−0.016
software classes	directed	1 377	2 213	1.61	1.51	–	0.033	0.012	−0.119
electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	−0.154
peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	−0.366
metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	−0.240
protein interactions	undirected	2 115	2 240	2.12	6.80	2.4	0.072	0.071	−0.156
marine food web	directed	135	598	4.43	2.05	–	0.16	0.23	−0.263
freshwater food web	directed	92	997	10.84	1.90	–	0.20	0.087	−0.326
neural network	directed	307	2 359	7.68	3.97	–	0.18	0.28	−0.226

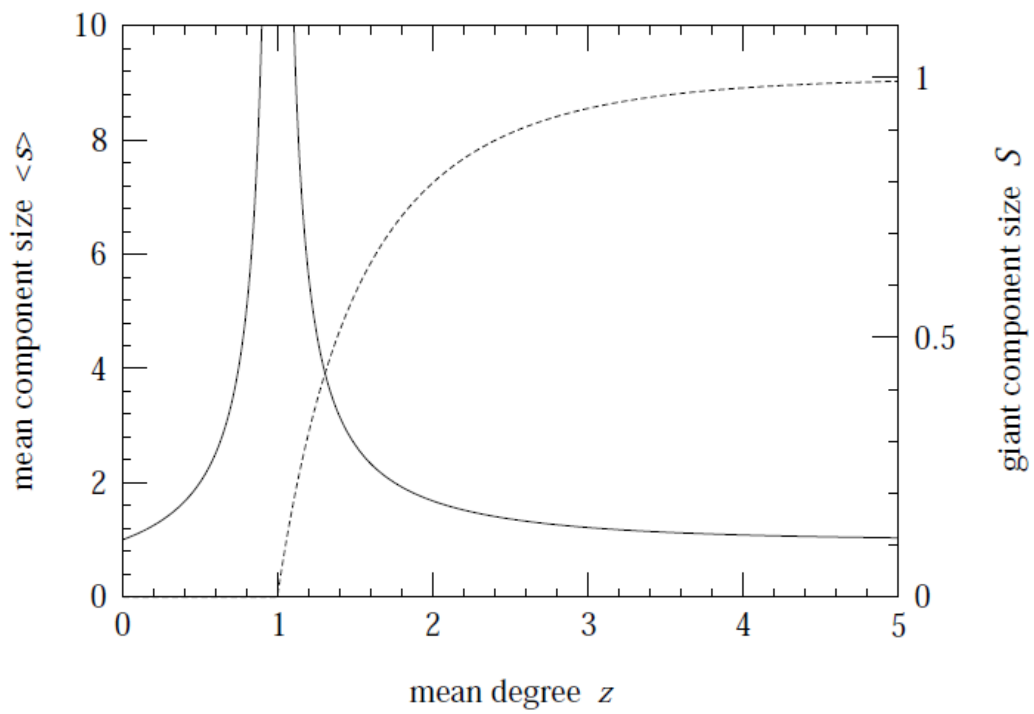
Network models

1. Poisson random graph
2. Configuration model
3. Bipartite graphs
4. Strogatz-Watts (small-world)
5. Albert-Barabasi (scale-free)*
6. Price model*

Poisson random graph

Poisson distribution (unlikely)

$$
\begin{aligned}
p_k &= \binom{n}{k} p^k (1-p)^{n-k} \\
&\approx \frac{z^k e^{-z}}{k!}, \quad z = p(n-1)
\end{aligned}
$$



Mean size + giant component

$$S = 1 - e^{-zS}$$

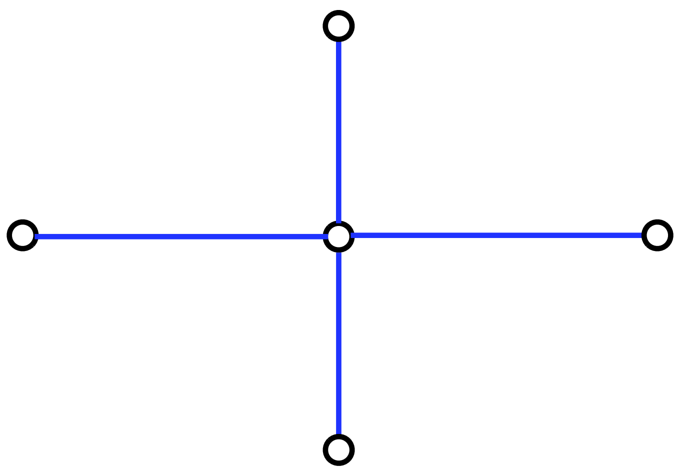
$S=0$ is the only solution for $z<1$

$$< s > = \frac{1}{1 - z + zS}$$

phase transition at $z=1$

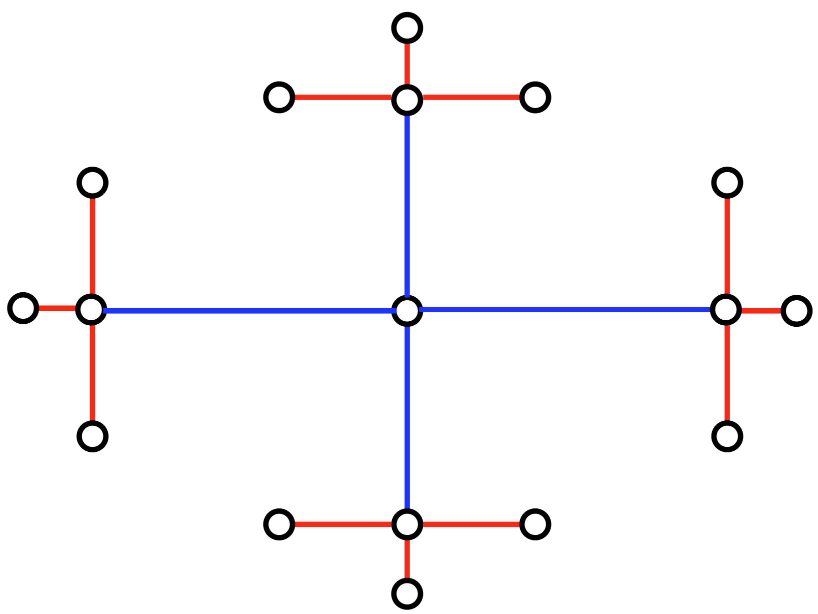
Expected structure varies with the value of p

mean number of neighbors within distance d:



$d=1,\ l^1=z^1=4$

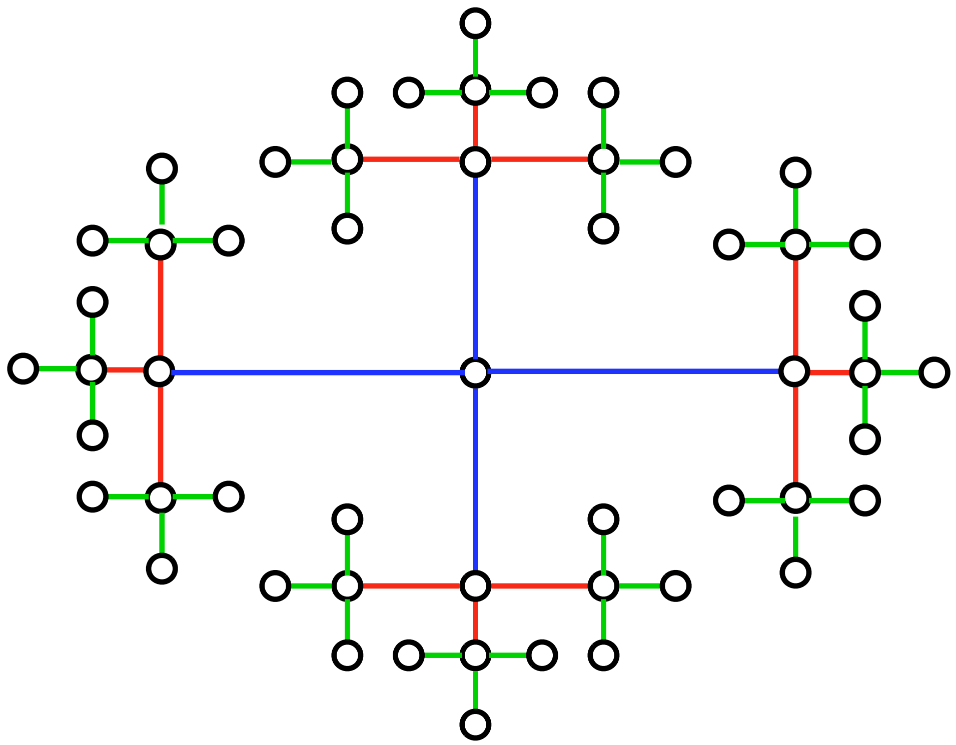
mean number of neighbors within distance d:



$d=1,\ l^1=z^1=4$

$d=2,\ l^2=z^2=16$

mean number of neighbors within distance d:

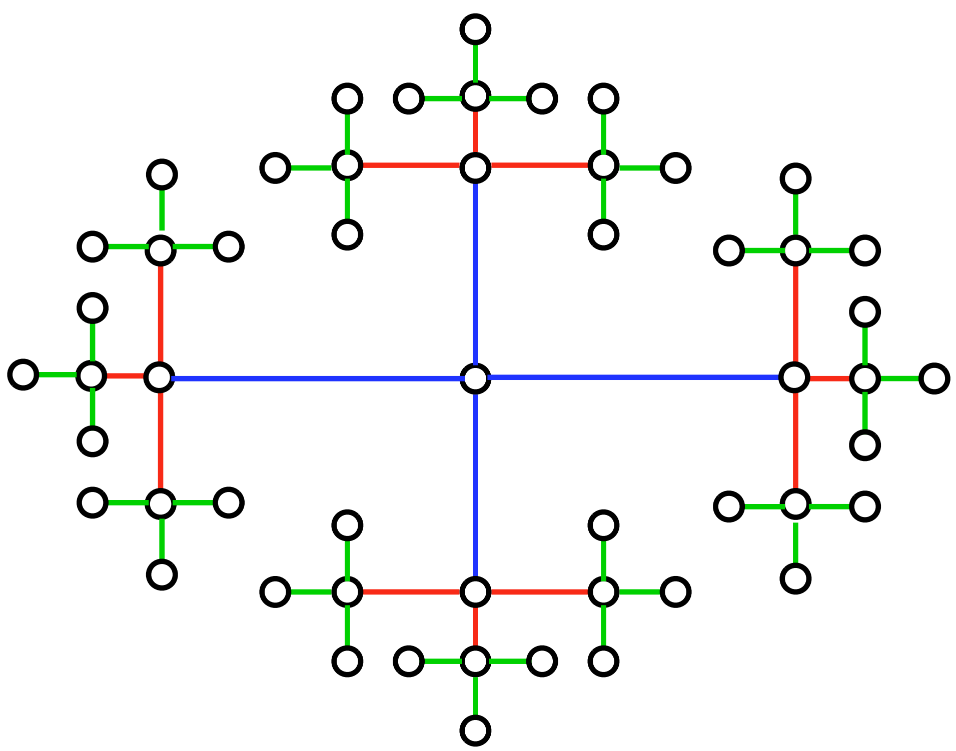


$d=1,\ l^1=z^1=4$

$d=2,\ l^2=z^2=16$

$d=3,\ l^3=z^3=64$

mean number of neighbors within distance d:



$d=1,\ l^1=z^1=4$

$d=2,\ l^2=z^2=16$

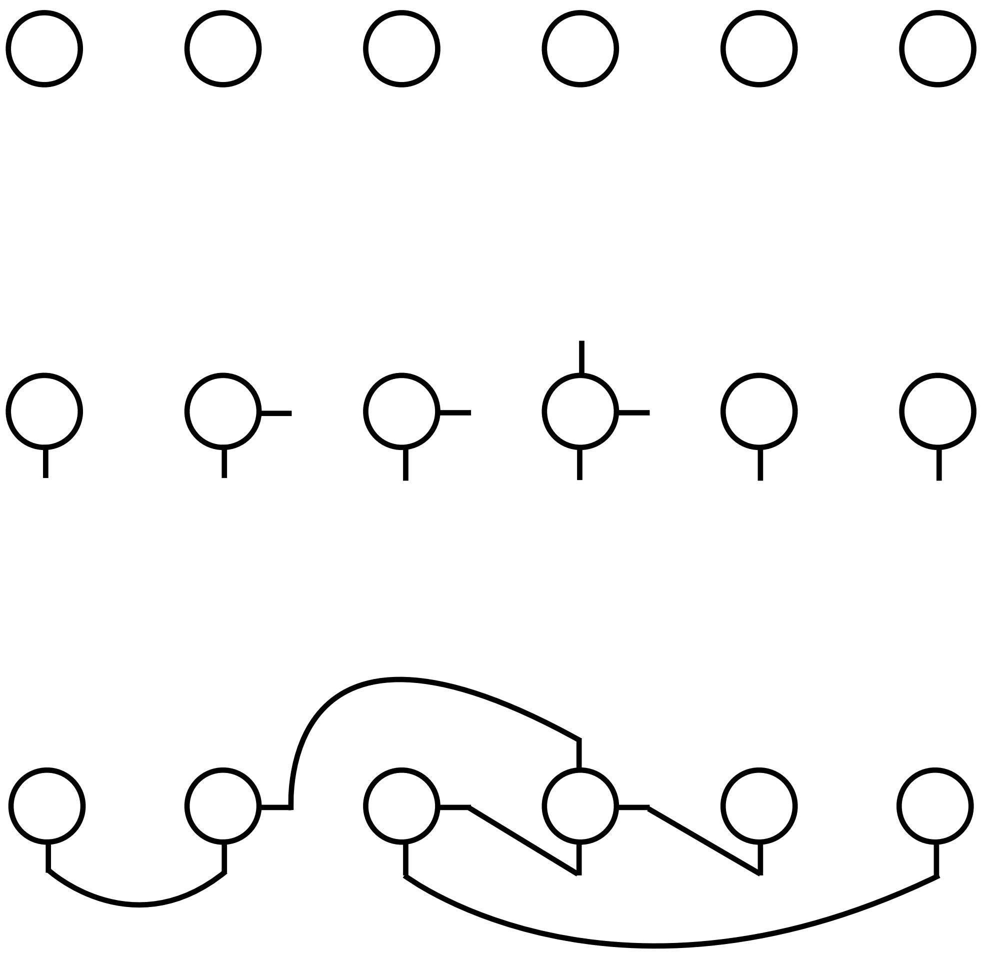
$d=3,\ l^3=z^3=64$

$z^l=n?\ l=logn/logz \longrightarrow$ small world!

- caveat -
In almost all other respects, the properties of random networks do not match those of real world networks

baseline
phase transition
giant component

Configuration model



"degree sequence"

choose pairs of stubs at random

connect them

Random in all respects other than
their generalized degree distribution

Probability generating functions:

$$G_0(x) = \sum_{k=0}^{\infty} p_k x^k$$

generates probability distribution of the degree of a node

$$G_1(x) = \frac{G_0'(x)}{G_0'(1)} = \frac{G_0'(x)}{z}$$

generates probability distribution of outgoing edges attached to a vertex

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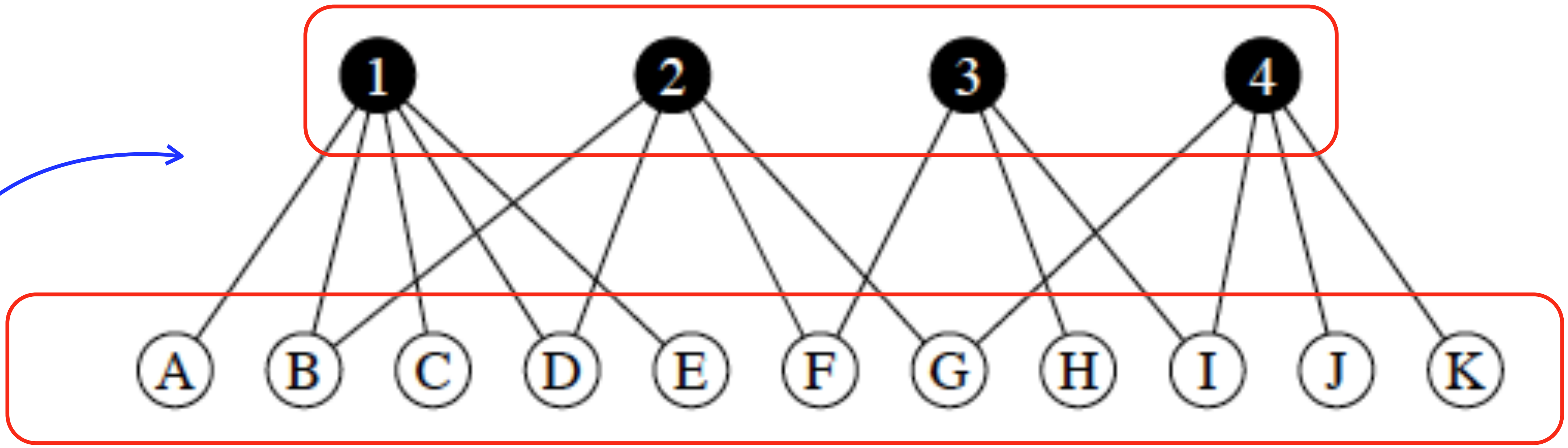
$$\begin{aligned} S &= 1 - e^{-zS} \\ < s > &= \frac{1}{1 - z + zS} \end{aligned}$$

$\xrightarrow{\text{generalize}}$

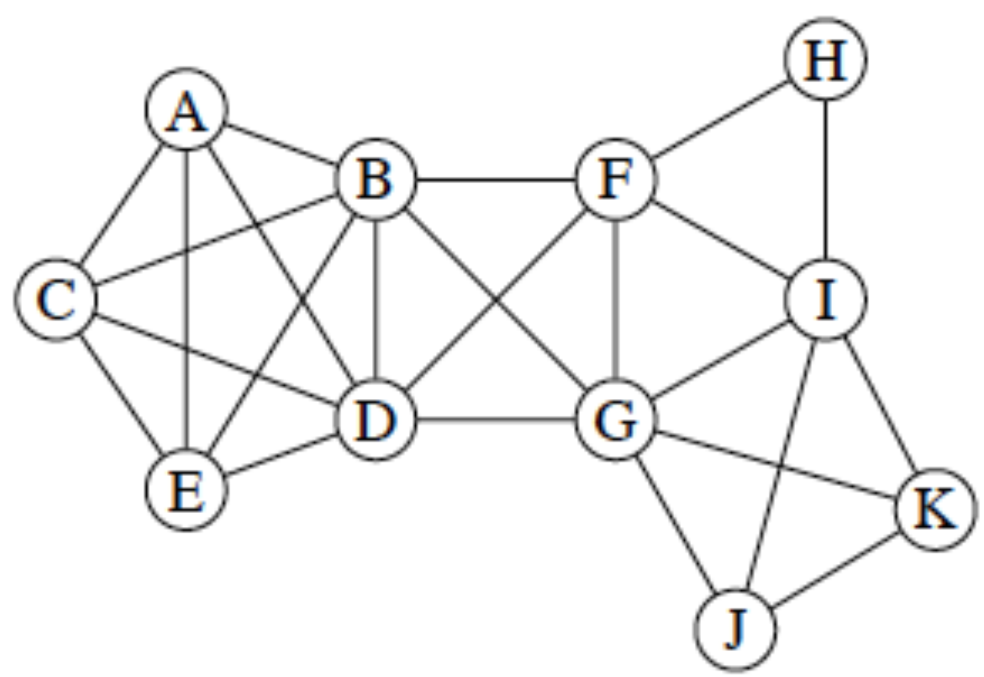
$$\begin{aligned} S &= 1 - G_0(u), \quad u = G_1(u) \\ < s > &= 1 + \frac{G_0'(1)}{1 - G_1(1)} \end{aligned}$$

Bipartite graphs



links running only between vertices of unlike kind

discarding some information

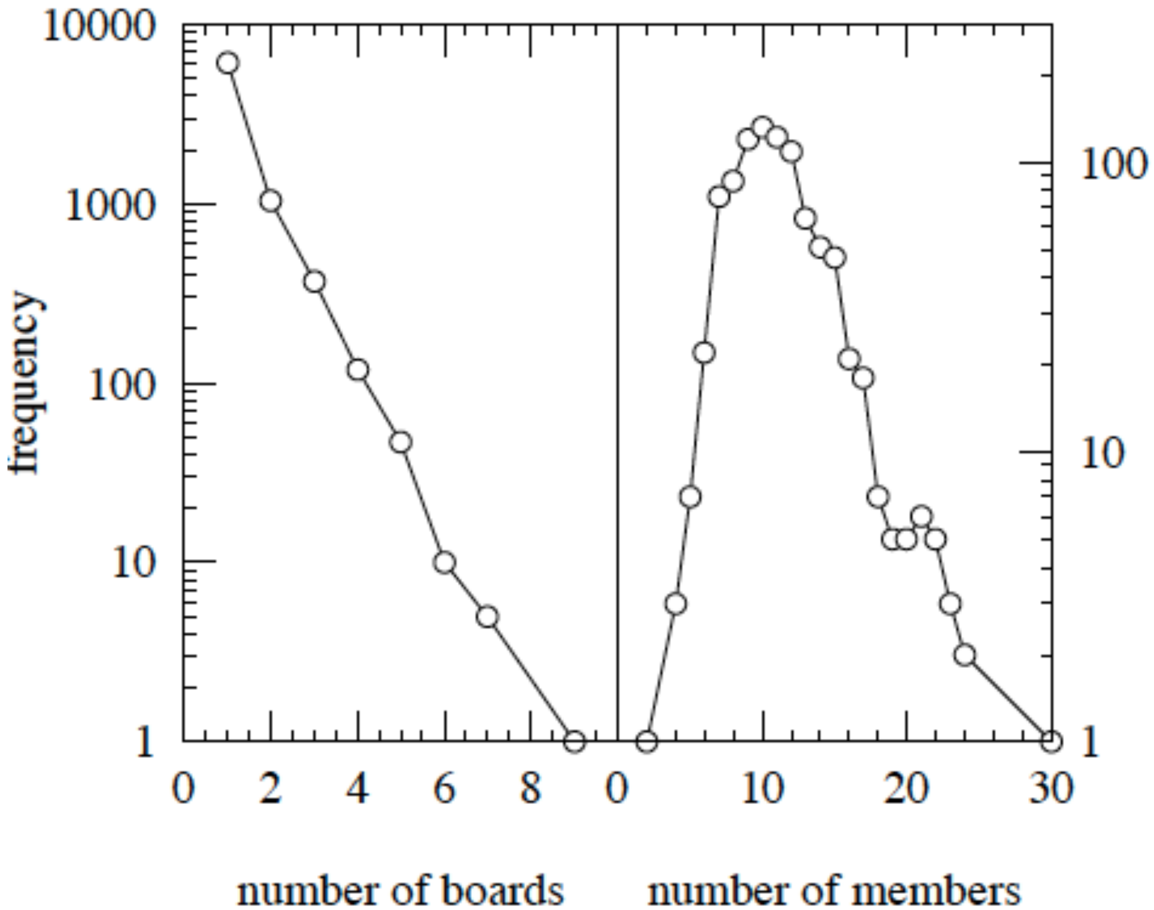


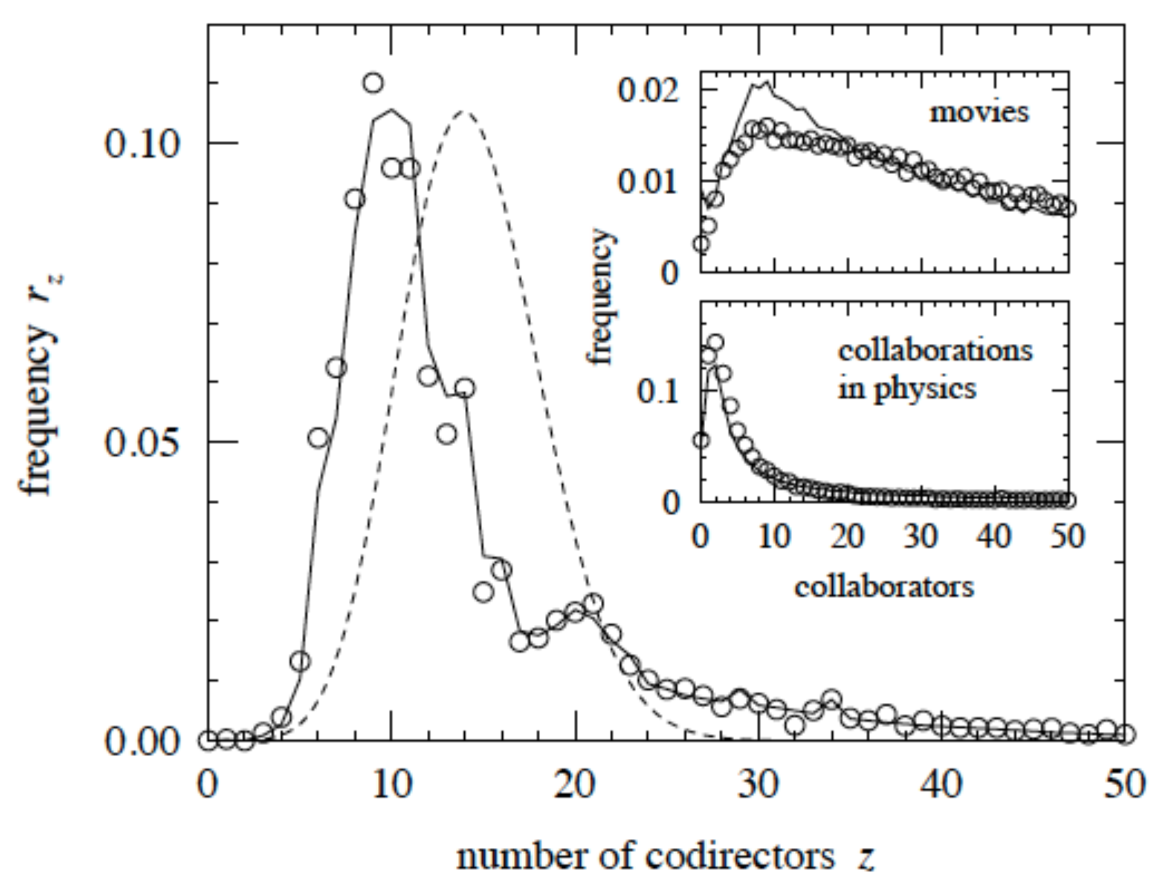
from Random graphs with arbitrary degree distributions and their applications by Newman, Strogatz and Watts

Review

- Generalized random graphs
- Bipartite graphs

generating function formalism



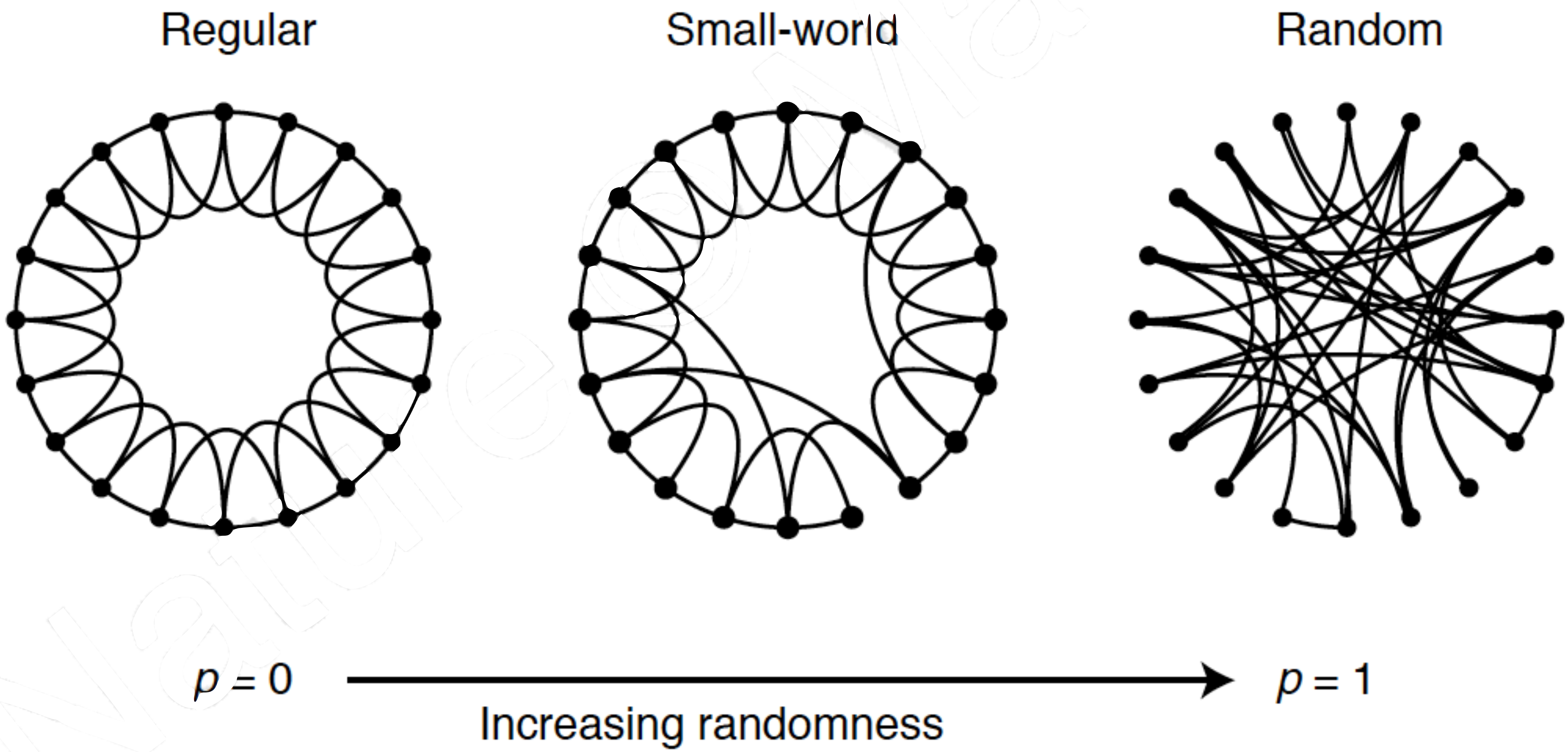


address unrealistic degree distributions

In <u>some</u> cases random graphs with appropriate distributions of vertex degree predict with surprising accuracy the behavior of the real worlds

Strogatz-Watts (small world)

Rewire with probability *p (n=20, z=4)*



z»ln(n) guarantees that the graph remains connected

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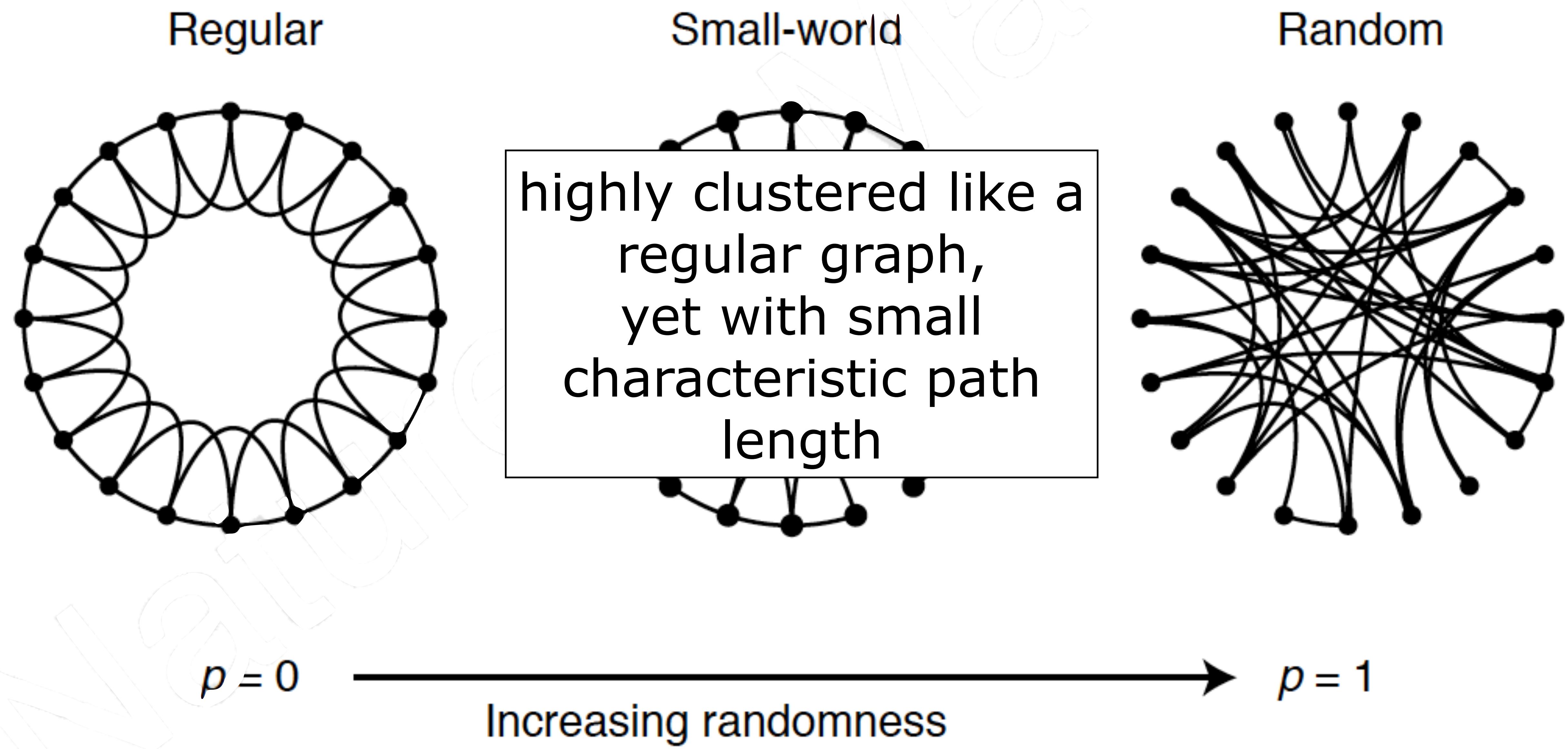
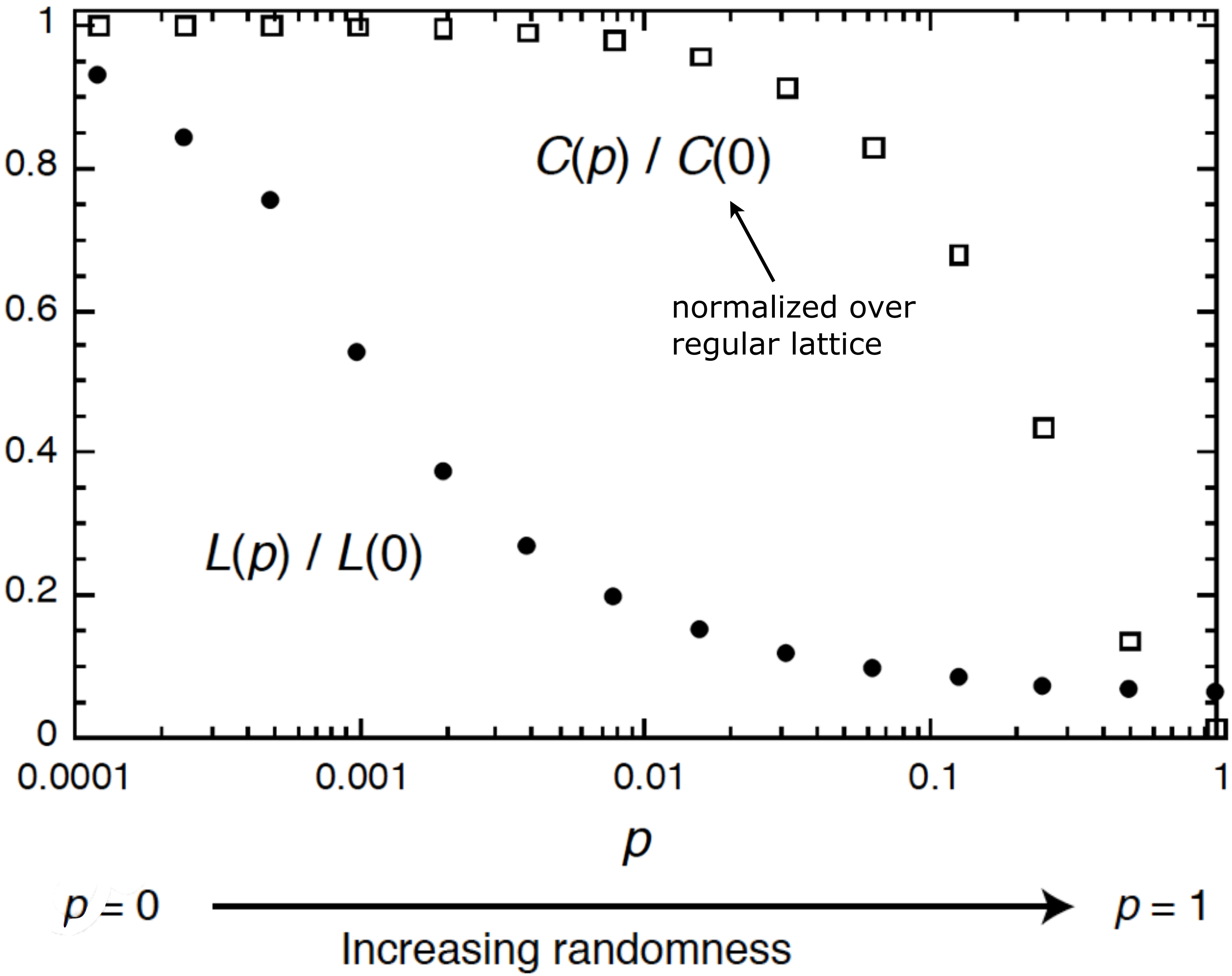
Rewire with probability *p (n=20, z=4)*



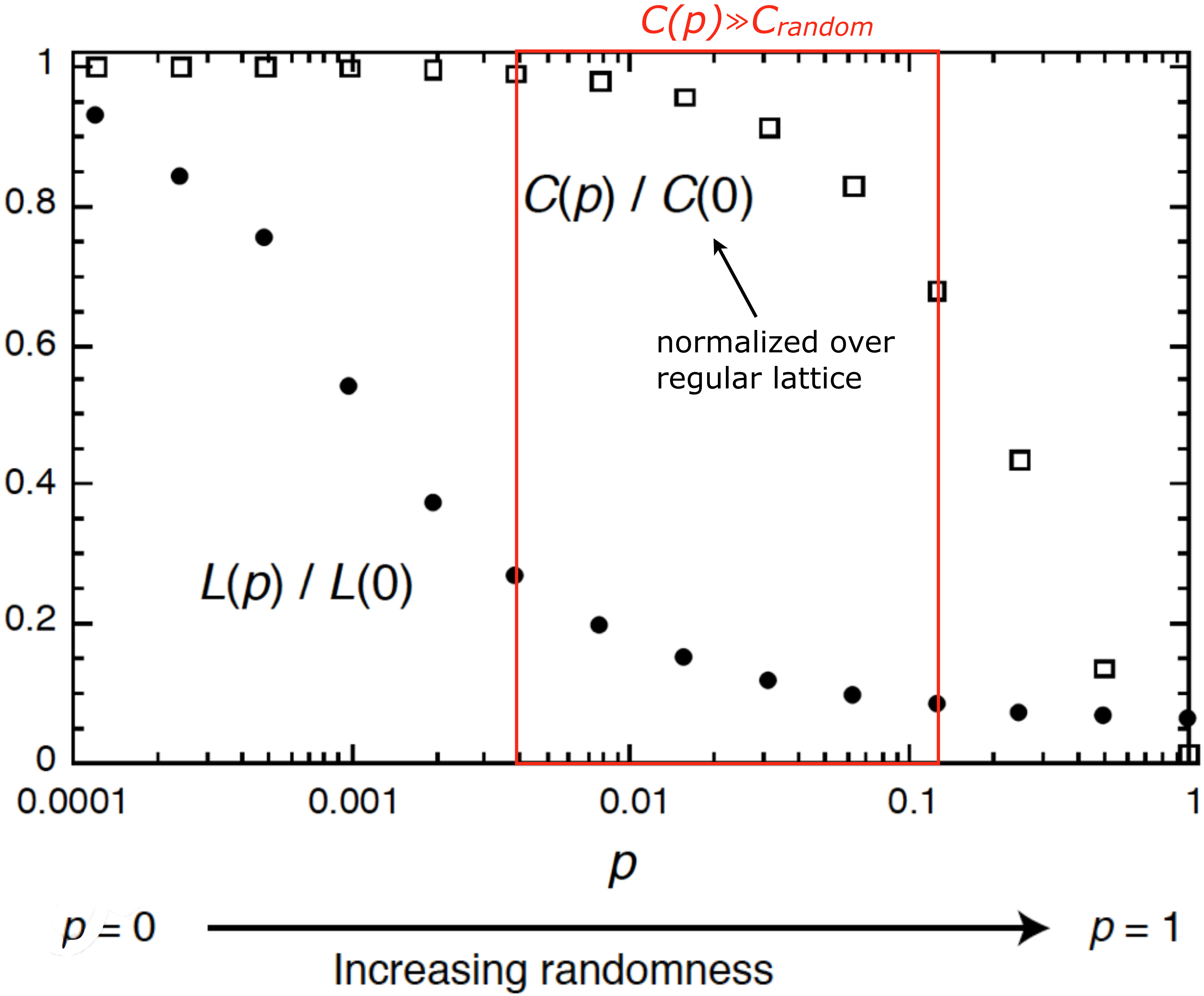
highly clustered like a regular graph, yet with small characteristic path length

z»ln(n) guarantees that the graph remains connected



n=1000, z=10, 20 random realizations

Collective dynamics of ‘small-world’ networks
by Ducan J. Watts and Steven H Strogatz



n=1000, z=10, 20 random realizations

Collective dynamics of ‘small-world’ networks
by Ducan J. Watts and Steven H Strogatz

Remarks

- For small *p*, each short cut has a highly nonlinear effect → $L(p)$ drops quickly
- An edge rewired from a clustered neighborhood has, at most a liner effect → $C(p)$ drops slowly
- Similar results for different initial regular graphs random rewiring

At local level, transition to a small world is undetectable

Albert-Barabasi

- Continuum Theory
- Simple and elegant

$$\Pi(k_i) = \frac{k_i}{\sum_j k_j}$$

- Capture the process
  → topology as byproduct

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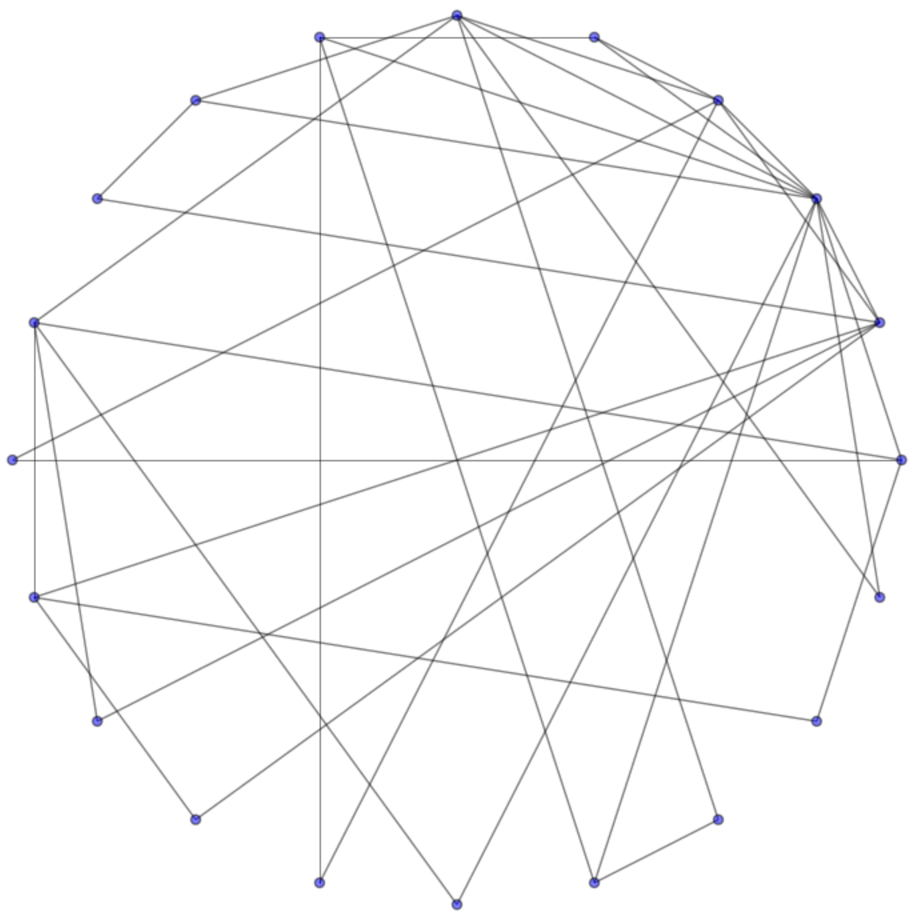

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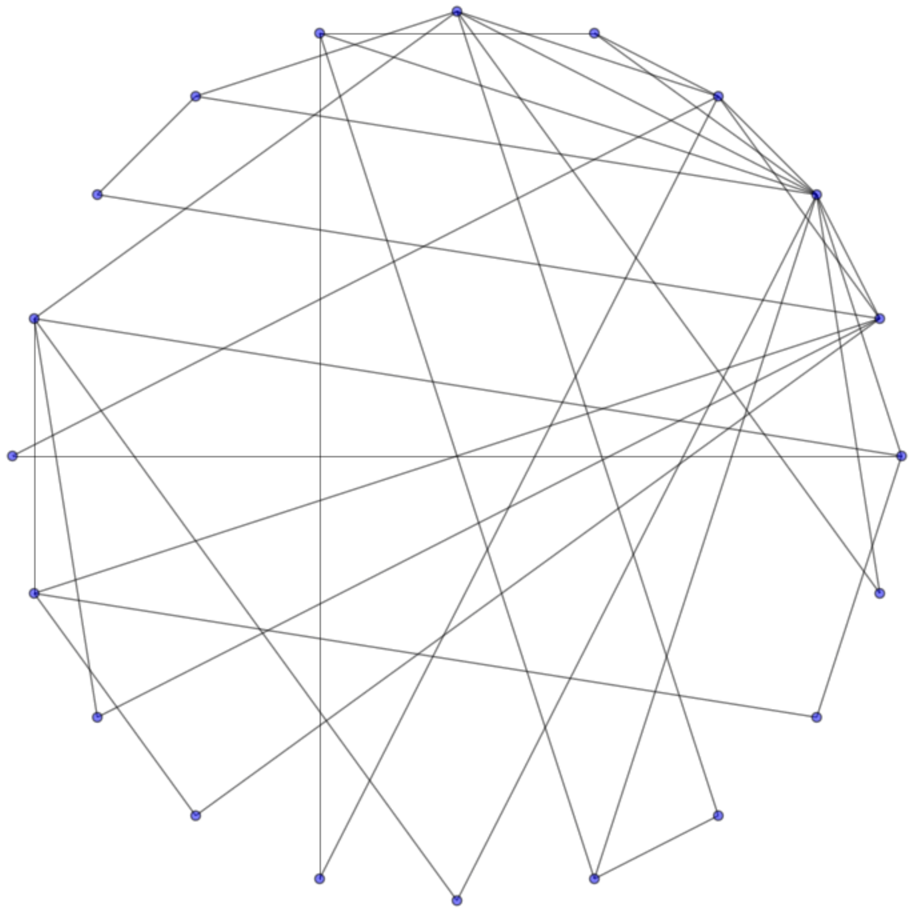

precise mathematical characterization
v. network generating mechanisms (why?)

Price model

- directed graph
- cumulative advantage
- Master-equation or rate-equation method
- Other growth models
  - growth constraints
  - gradual aging
  - fitness

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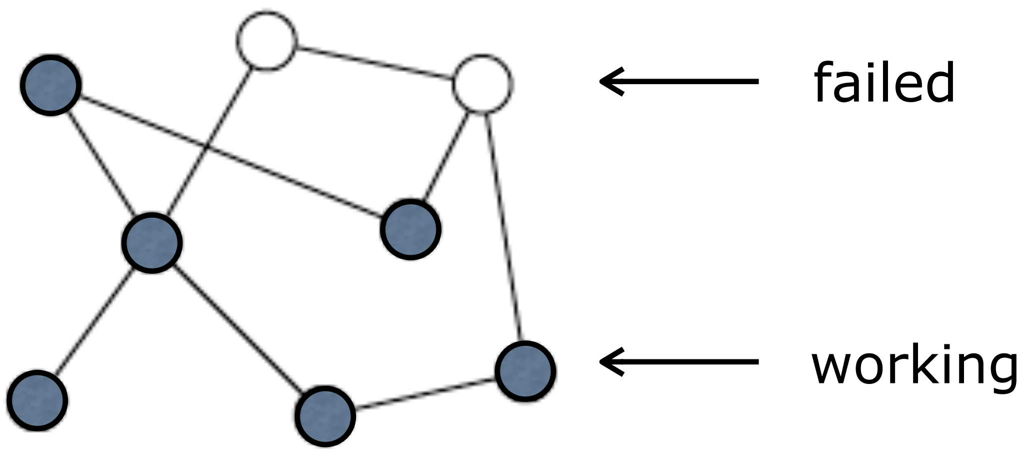
models inspired by real-world network properties

Processes on networks

1. Resilience
2. Epidemic processes
3. Epidemics of corruption

Resilience

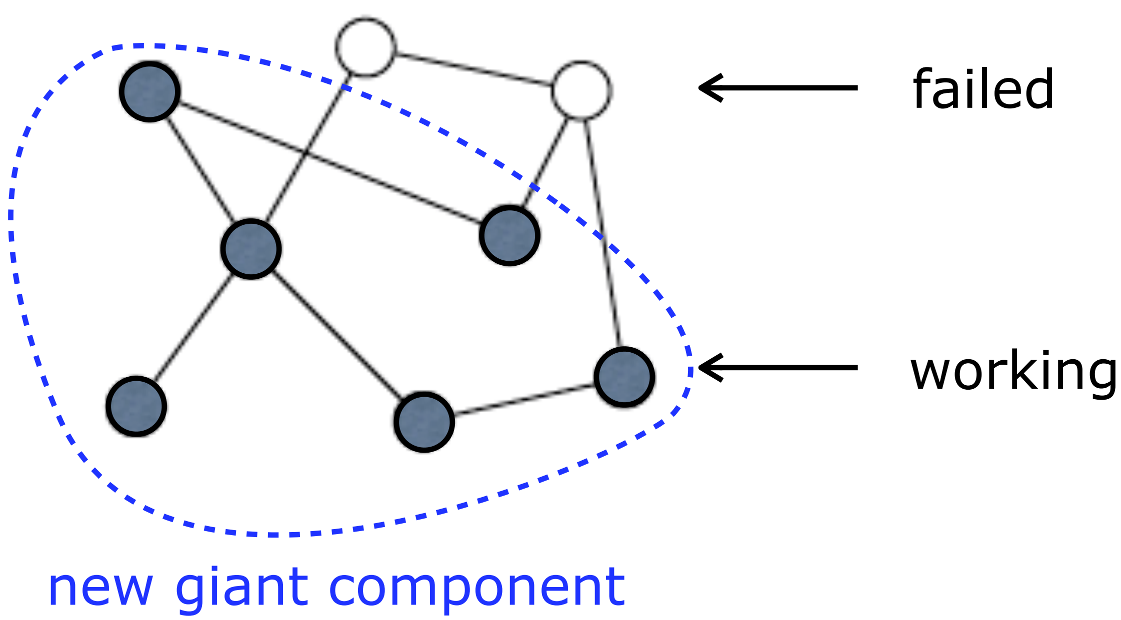
Alternative indicator: lack of variation in the fraction of vertices in the largest component of the network



S: relative size of the largest cluster that remains connected

Resilience

Alternative indicator: lack of variation in the fraction of vertices in the largest component of the network



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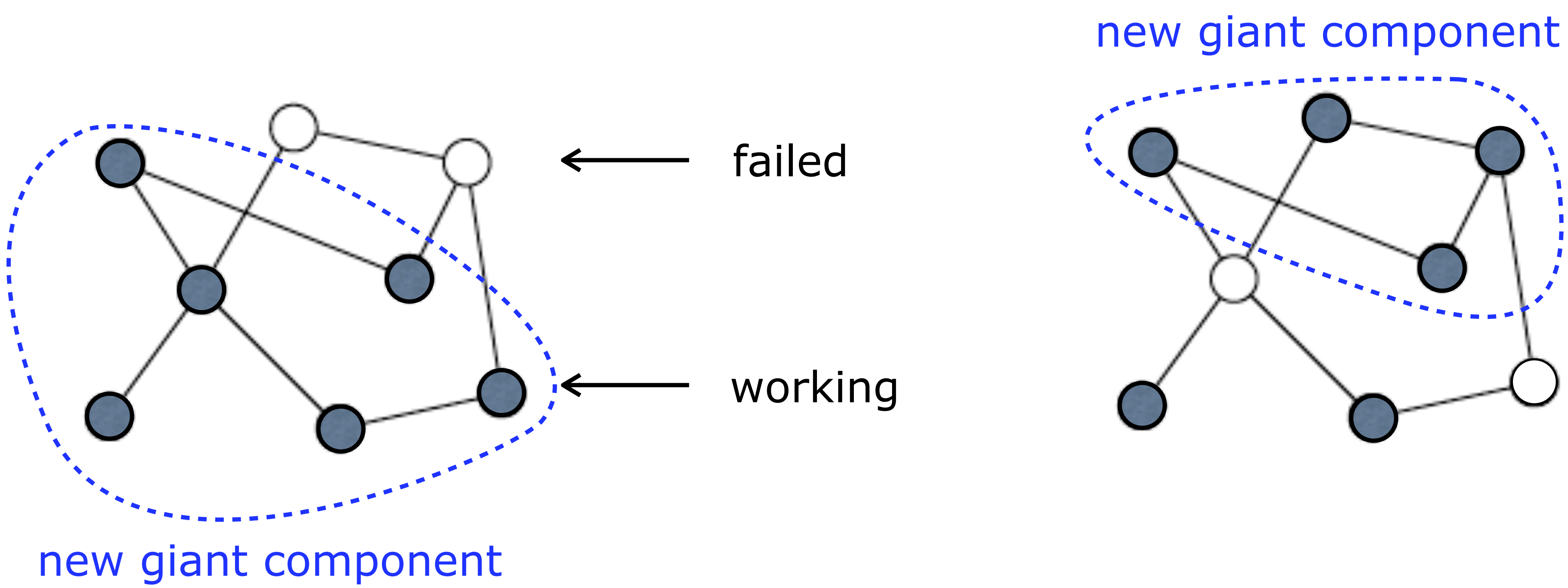

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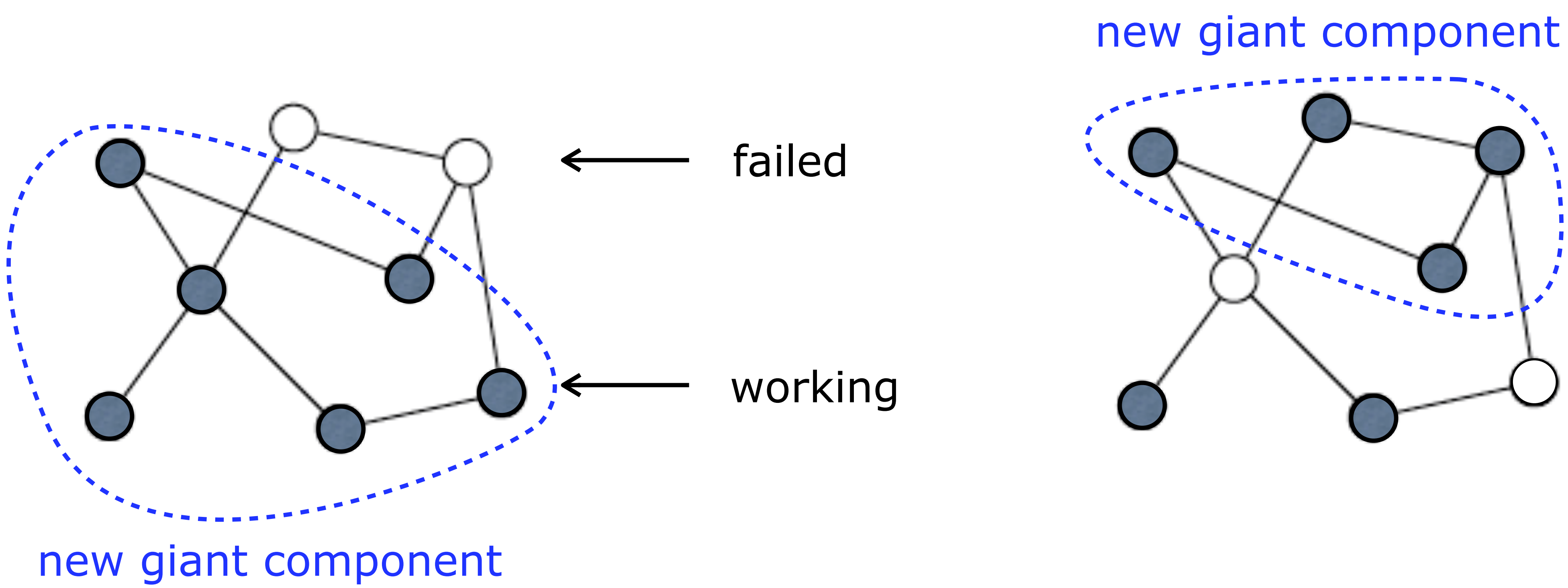
Resilience

Alternative indicator: lack of variation in the fraction of vertices in the largest component of the network



S: relative size of the largest cluster that remains connected

Does the network disintegrate into smaller, disconnected parts when $f > f_c$?

Criterion

"A giant cluster, with a size proportional to he size of the original network, exists if an arbitrary node i, connected to a node j in the giant cluster, is also connecter to at least one other node."

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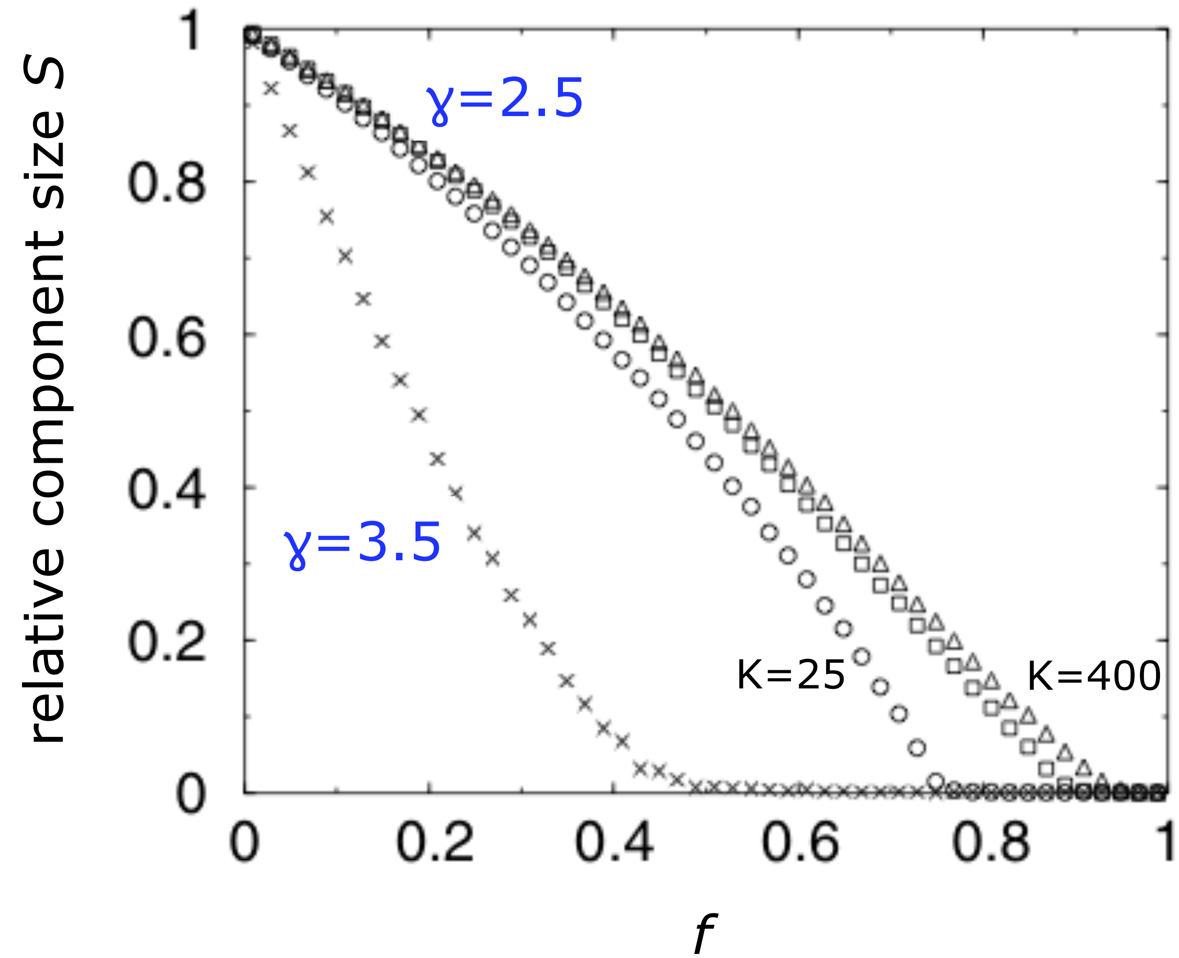
in other words...

$$\frac{< k^2 >}{< k >} = 2$$



Analytical results:
when γ≤3 the critical fraction for disintegration is 1

-power-law distributions-
a connected cluster of nodes survives even for arbitrary large fractions of random crashed sites

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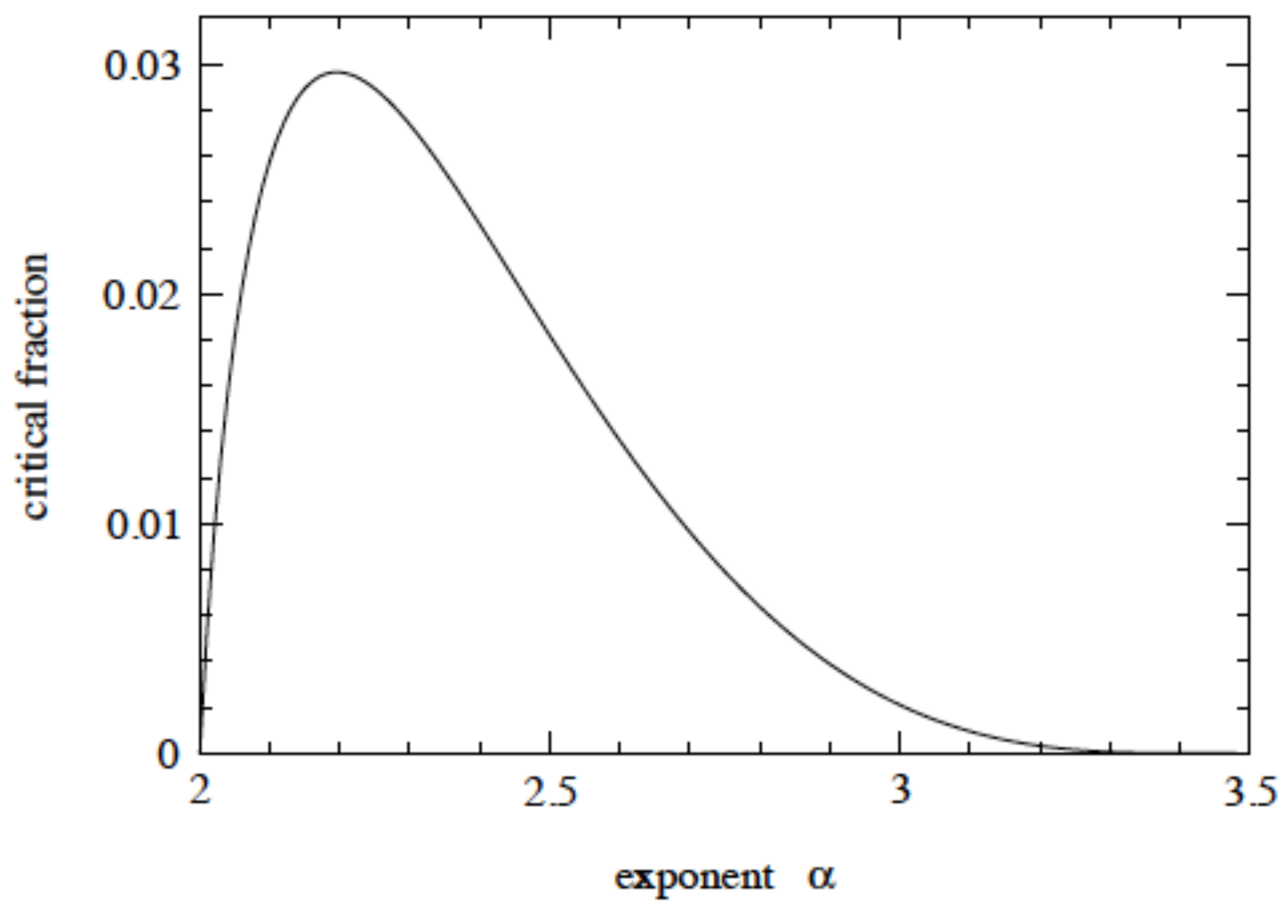


Dorogovtsev, S. N. and Mendes, J. F. F., Comment on “Breakdown of the Internet under intentional attack,” Phys. Rev. Lett., 87:219801 (2001)

-power-law distributions-
a connected cluster of nodes is highly susceptible to targeted attacks

	network	type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r
social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208
	company directors	undirected	7 673	55 392	14.44	4.60	–	0.59	0.88	0.276
	math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15	0.34	0.120
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45	0.56	0.363
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088	0.60	0.127
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1			
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0		0.16	
	email address books	directed	16 881	57 029	3.38	5.22	–	0.17	0.13	0.092
	student relationships	undirected	573	477	1.66	16.01	–	0.005	0.001	−0.029
	sexual contacts	undirected	2 810				3.2			
information	WWW `nd.edu`	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	−0.067
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7			
	citation network	directed	783 339	6 716 198	8.57		3.0/–			
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	–	0.13	0.15	0.157
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7		0.44	
technological	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	−0.189
	power grid	undirected	4 941	6 594	2.67	18.99	–	0.10	0.080	−0.003
	train routes	undirected	587	19 603	66.79	2.16	–		0.69	−0.033
	software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.070	0.082	−0.016
	software classes	directed	1 377	2 213	1.61	1.51	–	0.033	0.012	−0.119
	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	−0.154
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	−0.366

Epidemic processes

- Susceptible: don't have disease but can catch it
- Infective: have the disease and can pass it on
- Recovered: have permanent immunity (removed, refractory)
- s, i, r - fractions of individuals in each state

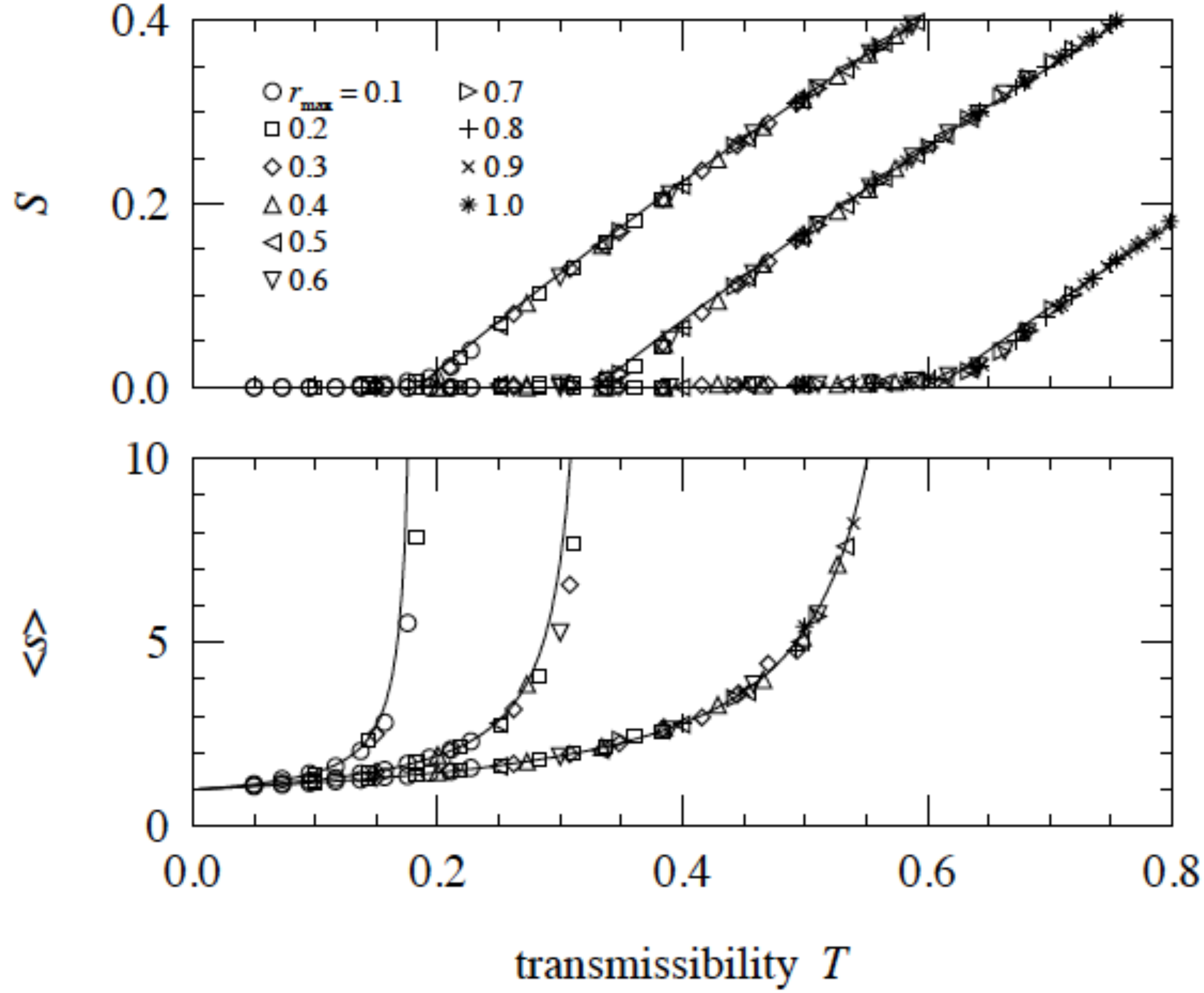
$$\frac{ds}{dt} = -\beta is$$

uniform probability per unit time of catching the disease

$$\frac{di}{dt} = \beta is - \gamma i$$

$$\frac{dr}{dt} = \gamma i$$

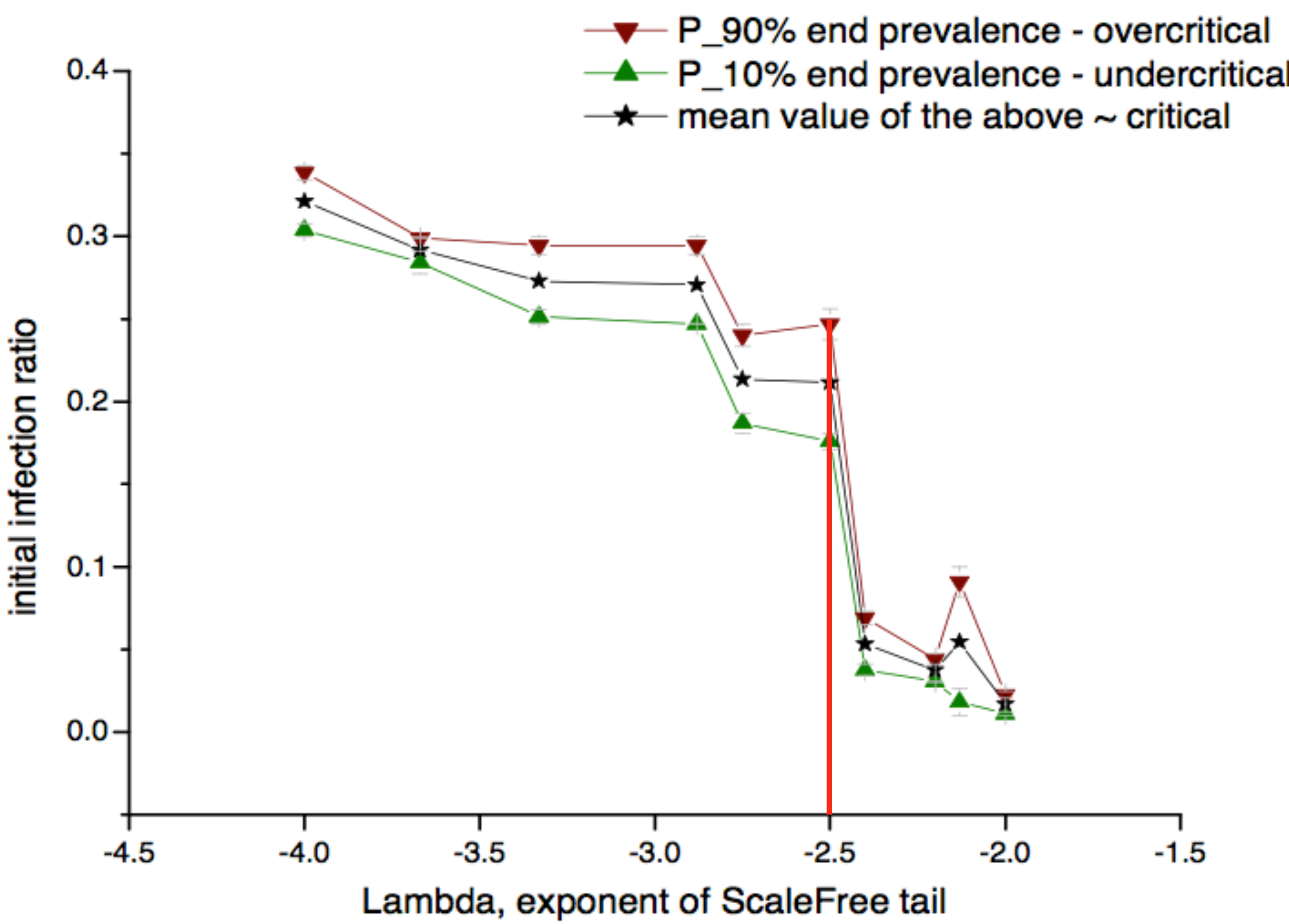
constant rate at which any infective one becomes immune



M. E. J. Newman. The spread of epidemic disease on networks.

Epidemics of corruption

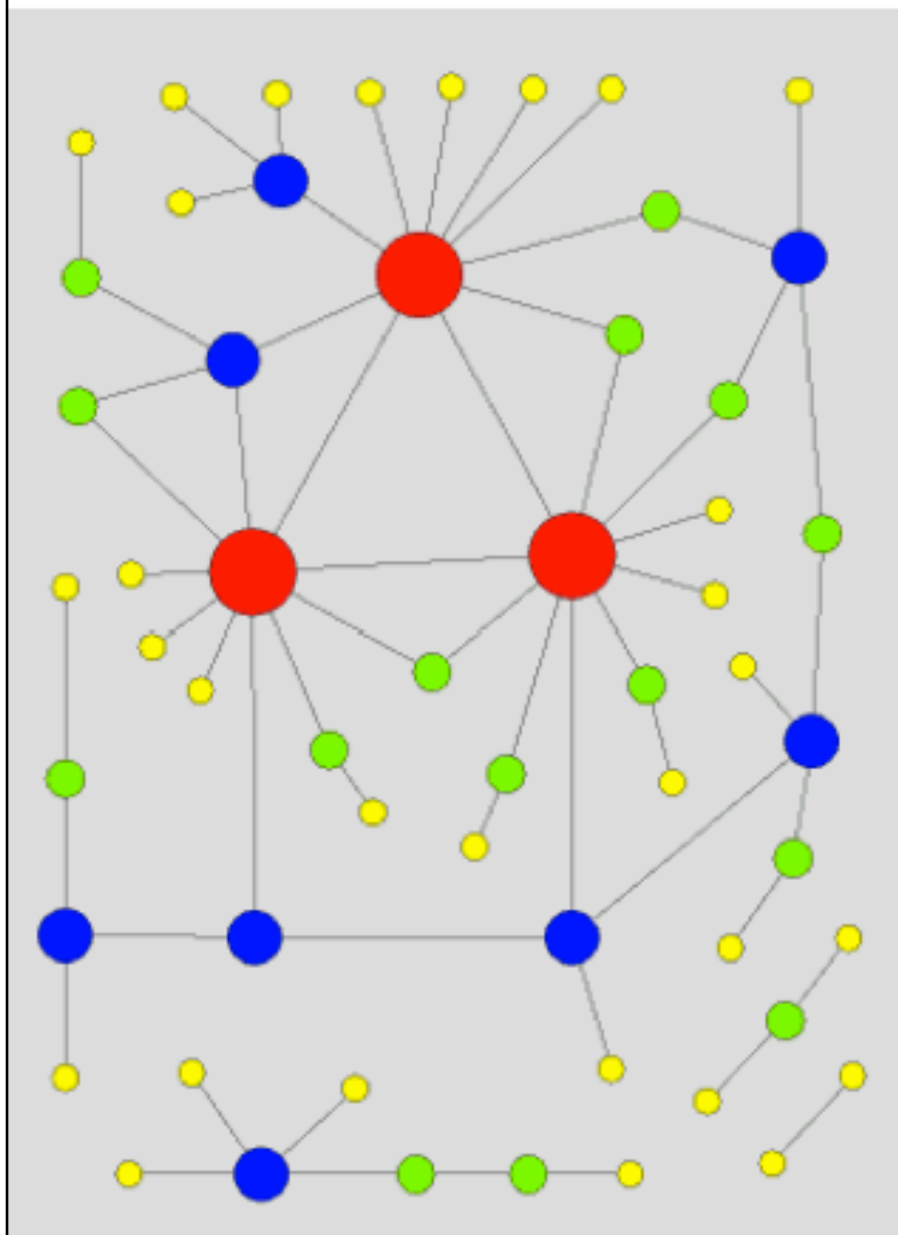
process name	characteristic	typical value
α - process	the local transmission process for # of corrupt neighbors $\geq \Delta$	$\alpha >> \varepsilon, \beta, \gamma$
β - process	the mean field transmission process due to the total prevalence or perception of corruption	$\varepsilon<\beta<\gamma$
γ - process	the corruption recover/elimination process due to the fight of the society against corruption	$\beta\leq\gamma<\alpha$
ε - process	the classical local epidemic process for # of corrupt neighbors $< \Delta$	$\varepsilon << \alpha, \beta, \gamma$



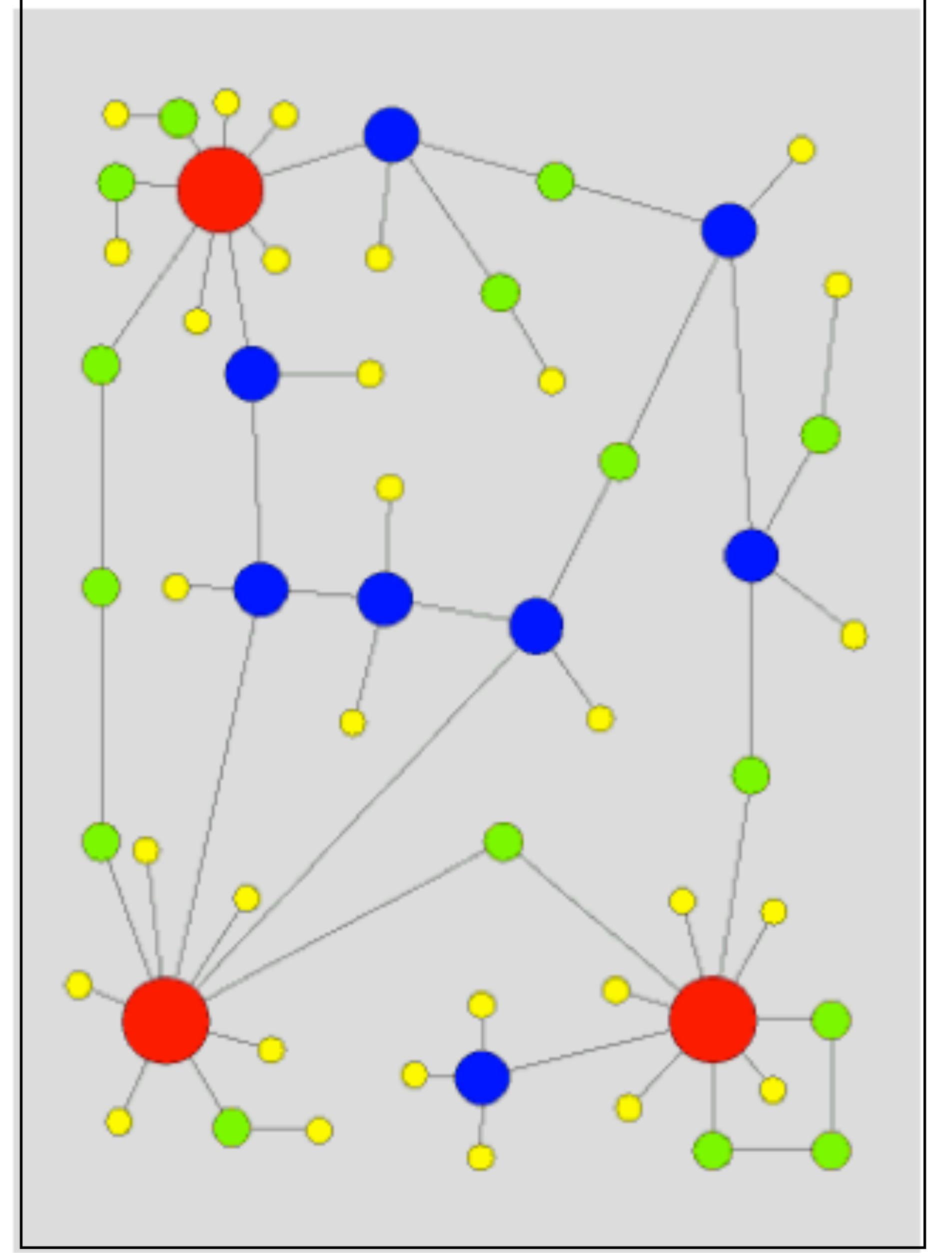
Δ=5; ε=0; α=0.35; β=0.08; γ=0.04; N=20000; M=50000

Additive vs. Multiplicative Degree Correlation

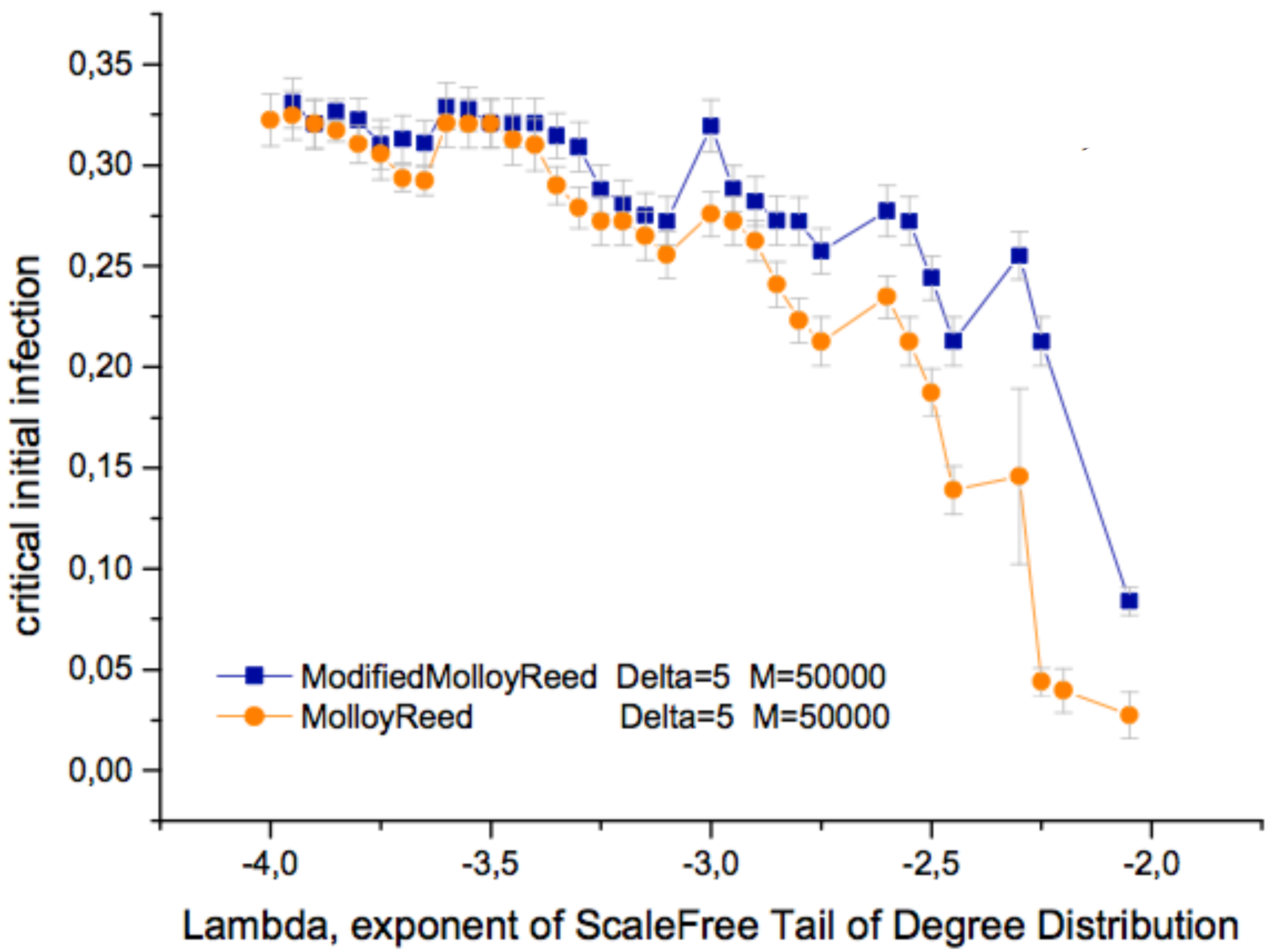
$$P[y \in B(x) | d(x) = k, d(y) = k']$$
$$\sim k \cdot k'$$



$$P[y \in B(x) | d(x) = k, d(y) = k']$$
$$\sim k + k'$$



N.B.: identical degree *distribution:*
3 reds (degree 10); 8 blues (degree 4), 15 greens (degree 2), …



$$\Delta=5;\ \varepsilon=0;\ \alpha=0.35;\ \beta=0.08;\ \gamma=0.04;\ N=20000;\ M=50000$$

Remarks

Similar type of modeling applied to other processes

- opinion formation
- doping usage
- social disorders
- innovation dynamics

Take-home

tech•nol•o•gy |tekˈnäləjē|

noun (pl. **-gies**)

the application of scientific knowledge for practical purposes, esp. in industry

- machinery and equipment developed from such scientific knowledge.
- the branch of knowledge dealing with engineering or applied sciences.

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1. Halfway...
2. Potential of data + models with emphasis on networks