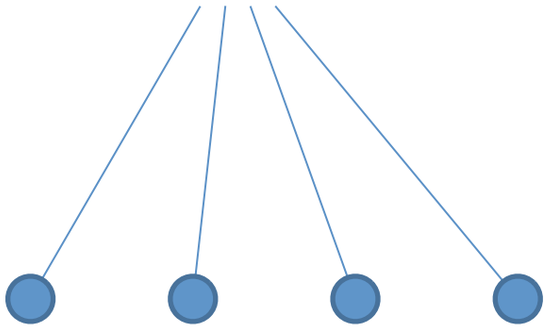
Lecture 10

Review

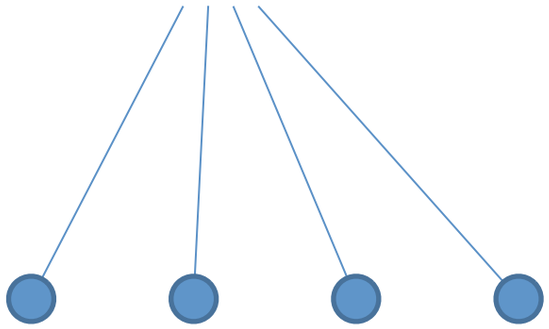
- Mechanisms and model
- Price's model (wealth distribution)

Wealth distribution

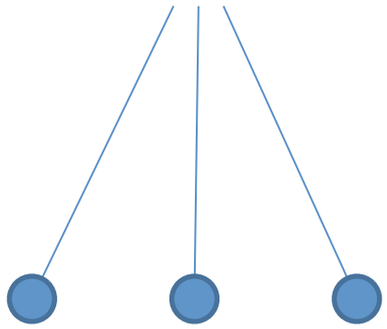
low income



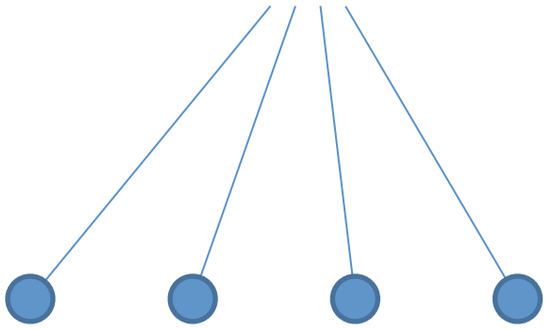
middle class



high income

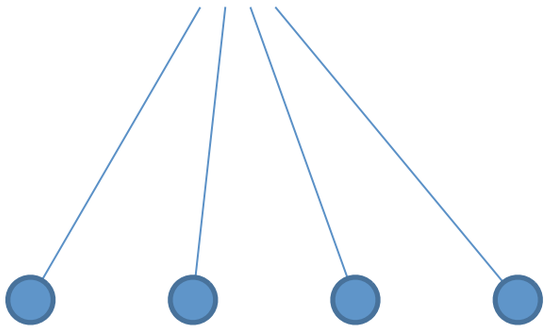


super rich

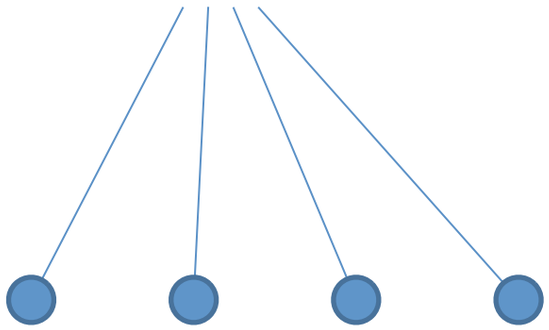


Wealth distribution

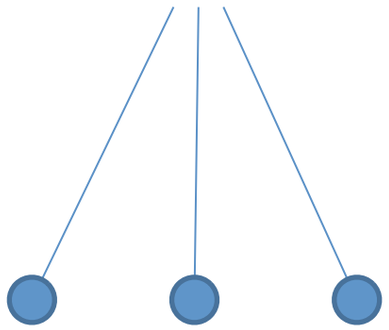
low income



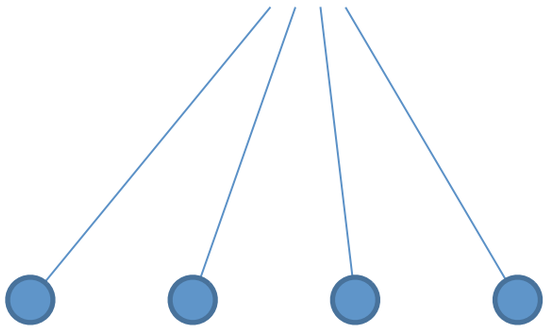
middle class



high income



super rich

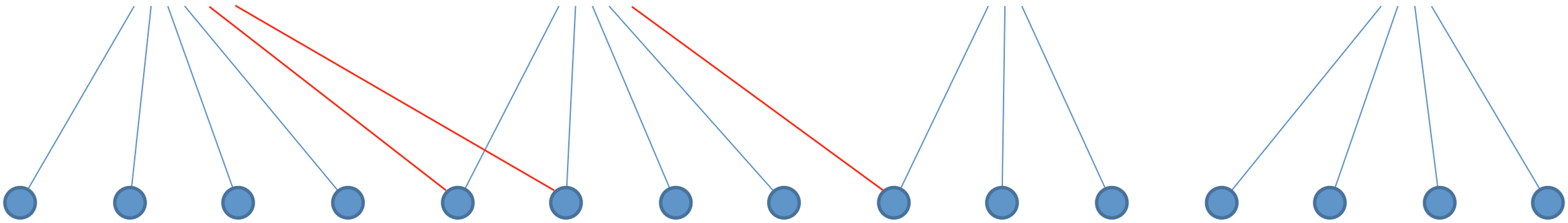


low income

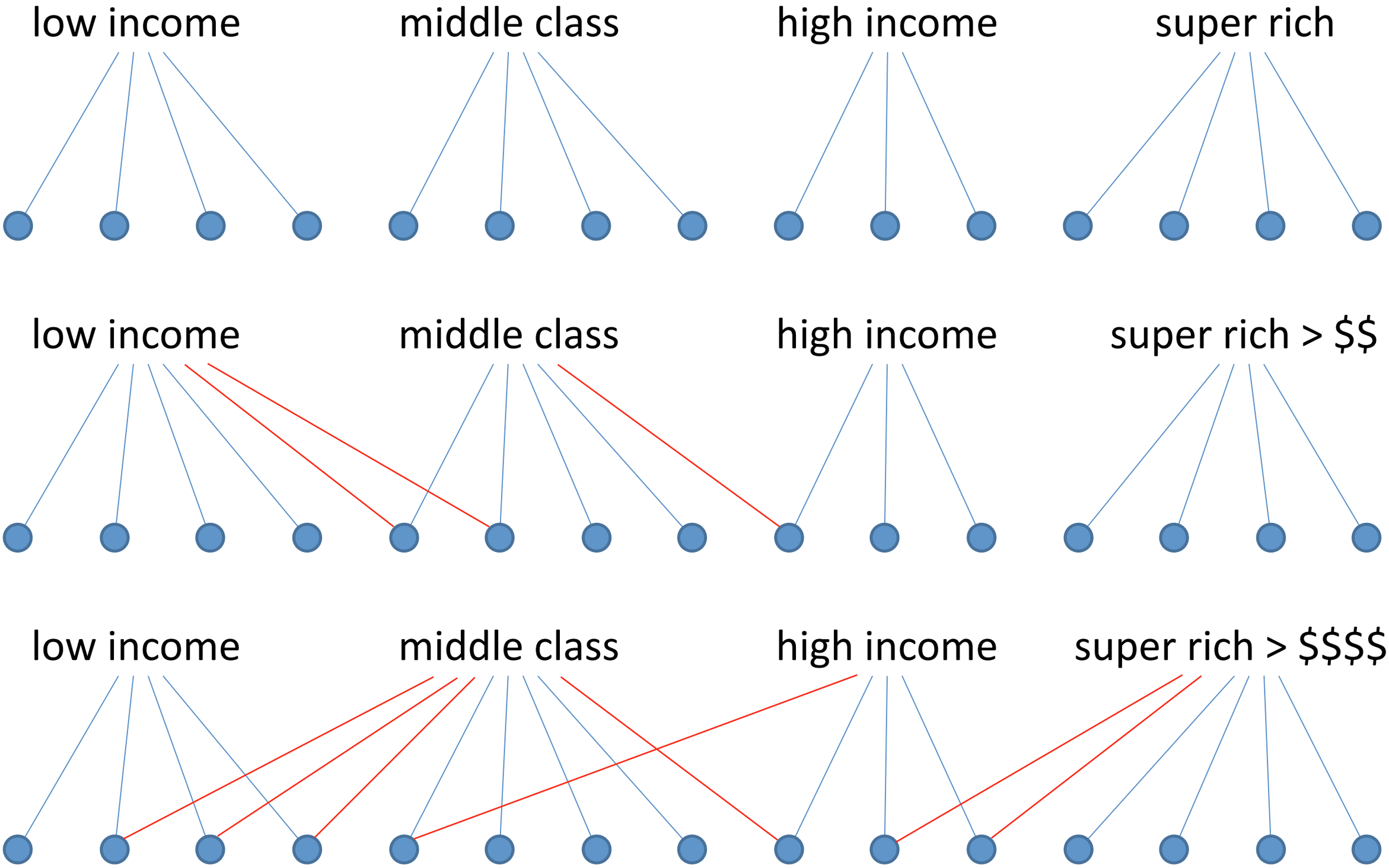
middle class

high income

super rich > $$



Wealth distribution



Today

- Dynamics processes on networks
  - percolation theory and network failure
  - epidemic processes
  - epidemics of corruption
- Focus on economy of influences

Ultimate goal: understand and explain systems build on networks

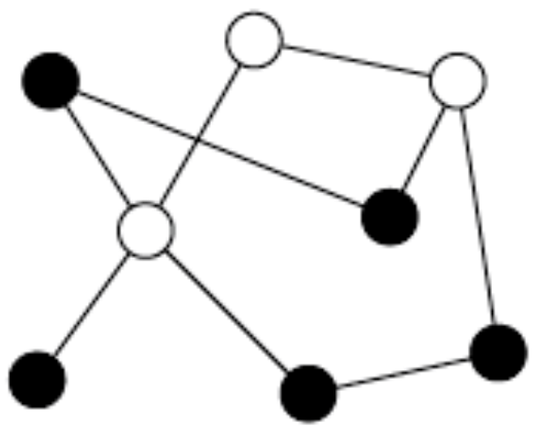
Examples

- How the WWW topology affects web surfing and search engines?
- How the structure of food web affects population dynamics?
- How social structure affects spread of information?
- How social structure affects the dynamics of influences that represent corruption?

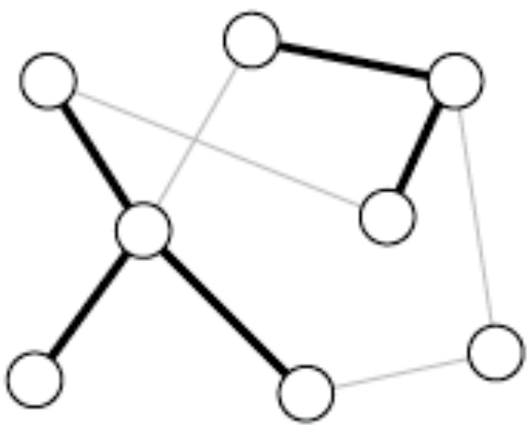
understand the effects of structure on processes

Site and bond percolation

- Site percolation: vertices randomly designated "occupied" or "unoccupied"
- Properties of the patterns of vertices
- Proposed in 1950's to model epidemics
- Site percolation equivalent to network resilience
- Bond percolation

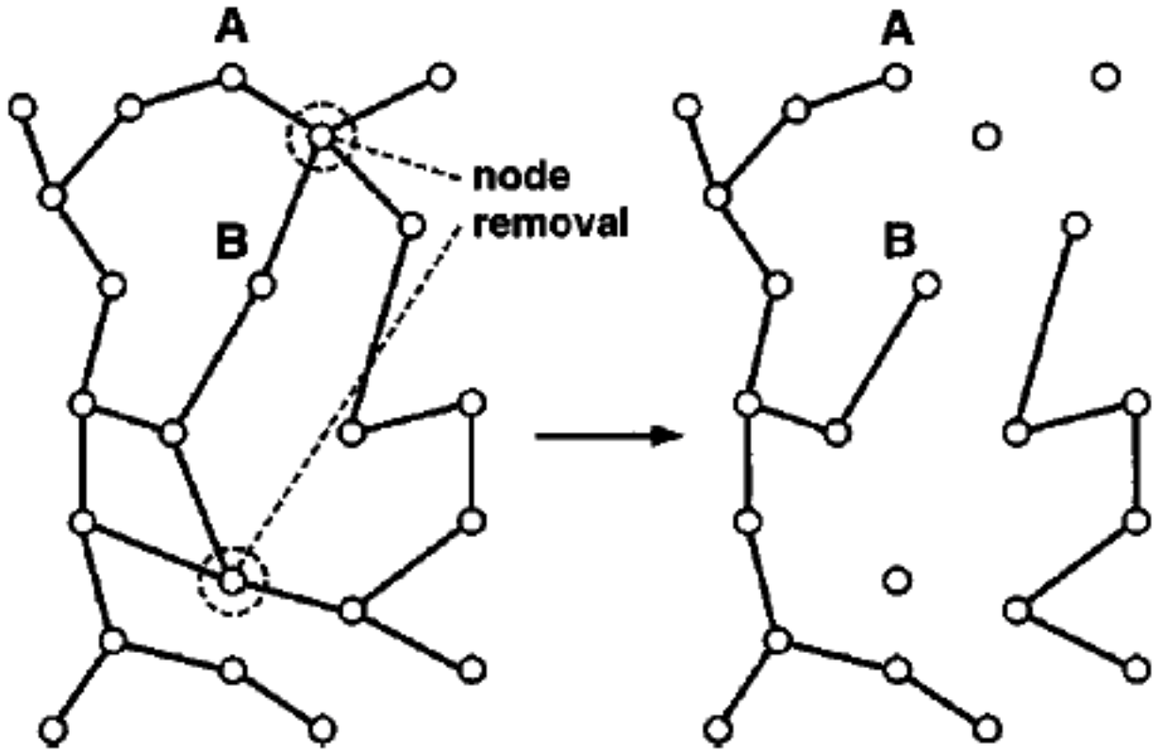


site percolation



bond percolation

Resilience



1. removal of a node implies the removal of all connected edges

2. removal disrupts some of the paths between remaining nodes

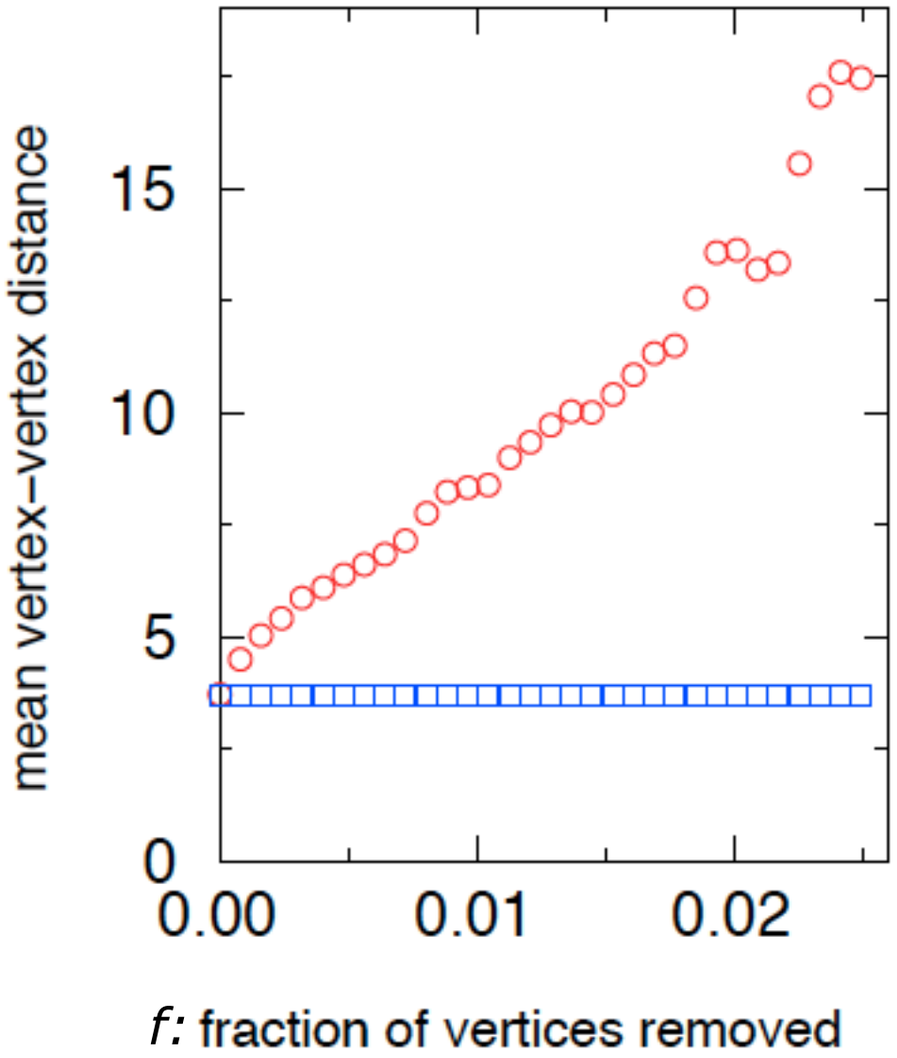
$l=2$

1 giant component

$l=6$

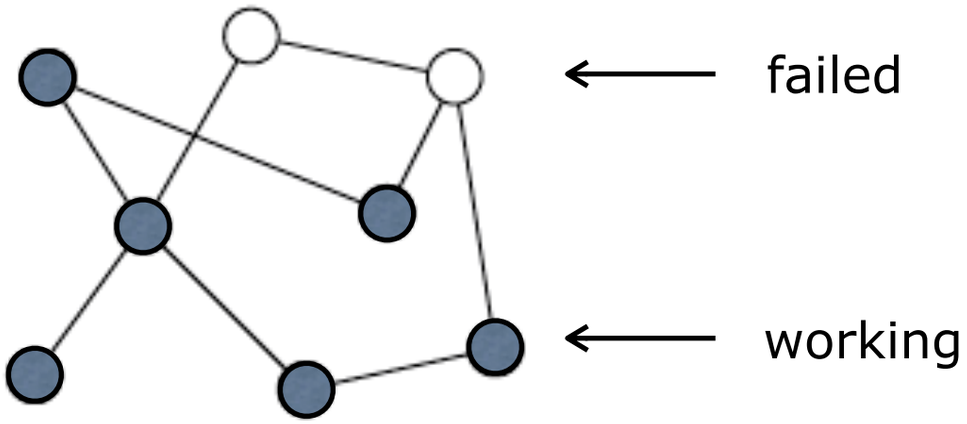
6 components

mean vertex-vertex distance



relative size of the largest cluster

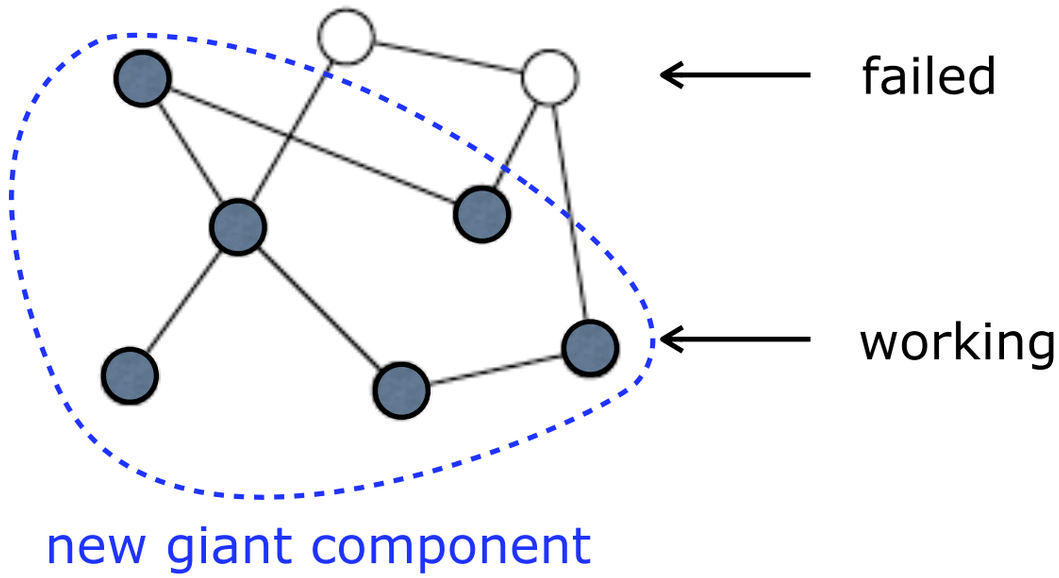
Alternative indicator: lack of variation in the fraction of vertices in the largest component of the network



S: relative size of the largest cluster that remains connected

relative size of the largest cluster

Alternative indicator: lack of variation in the fraction of vertices in the largest component of the network



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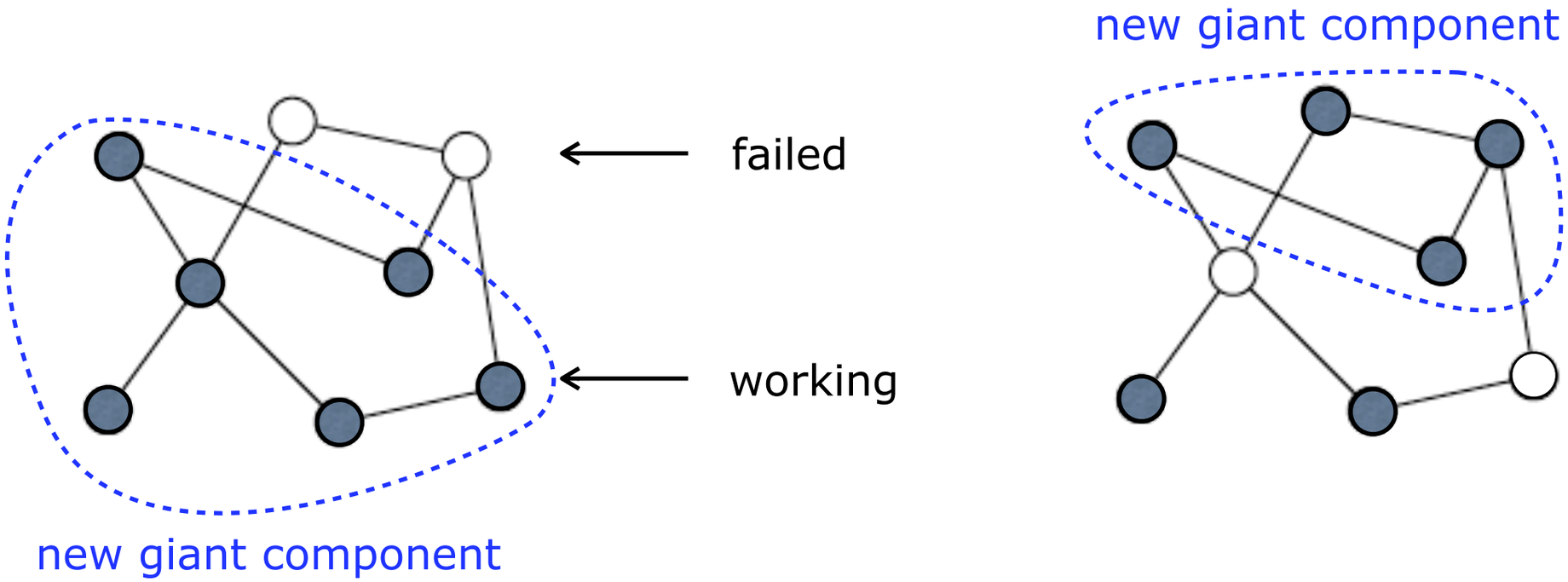

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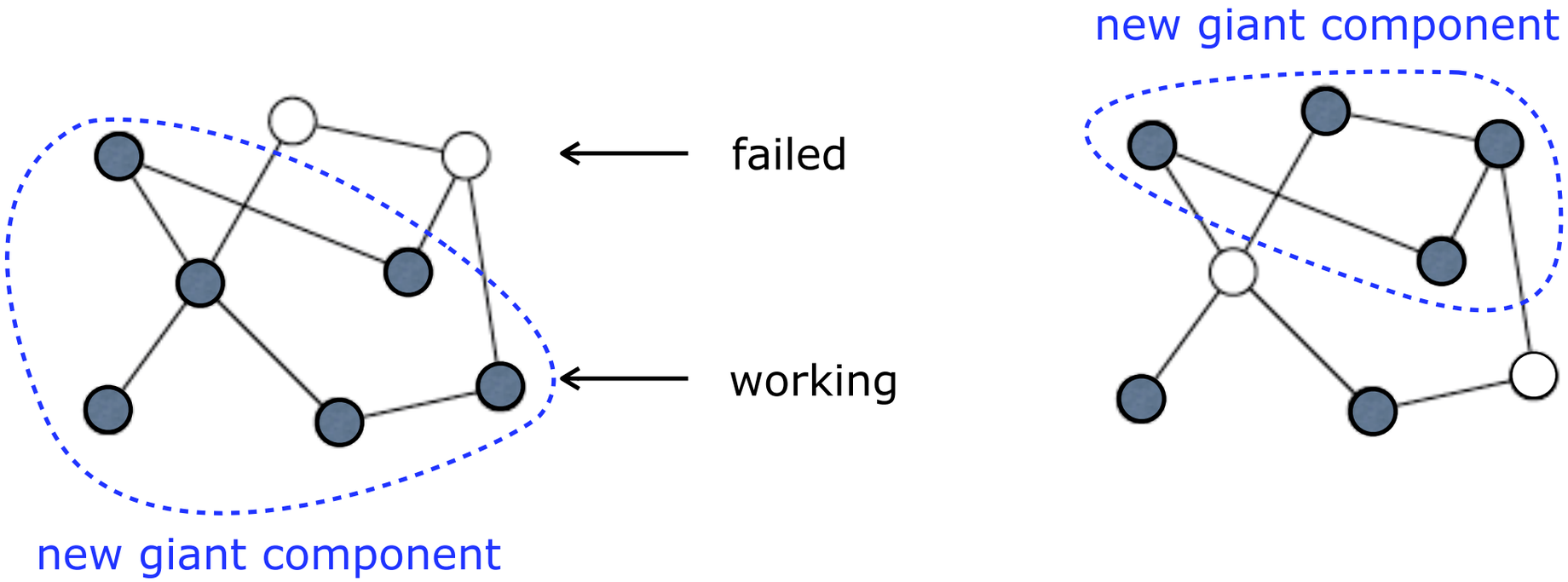


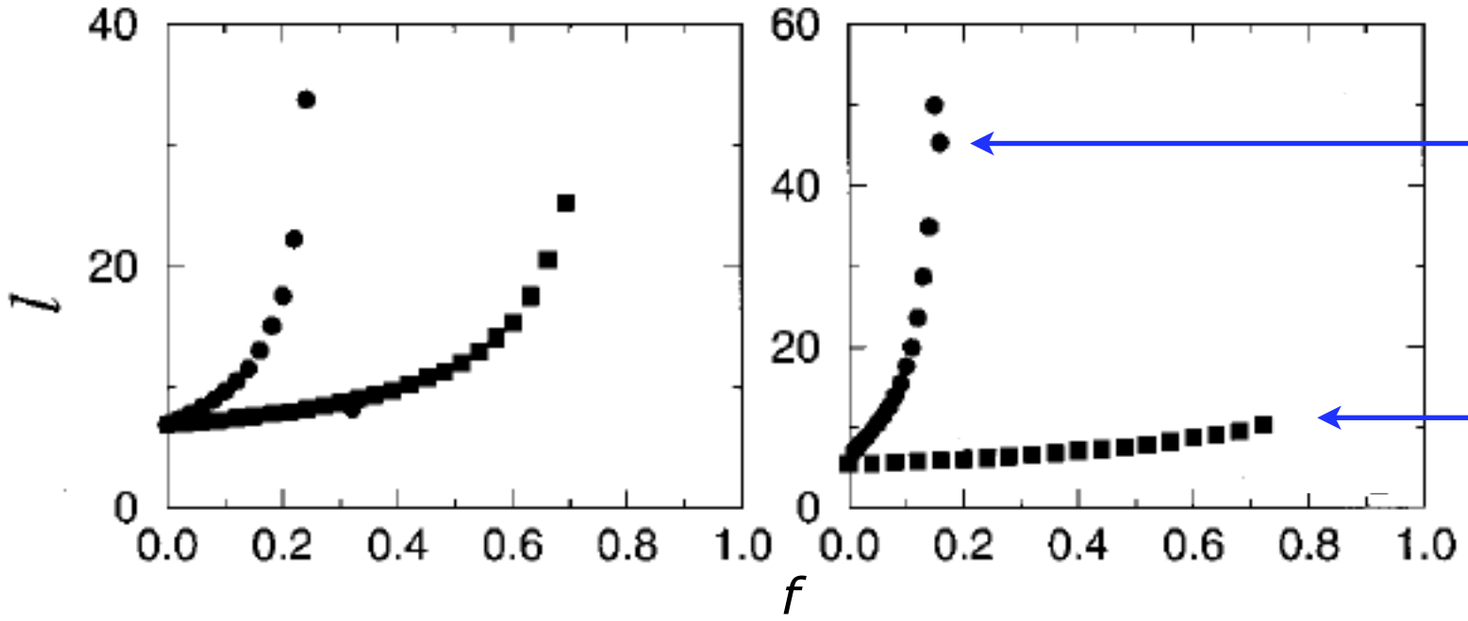
S: relative size of the largest cluster that remains connected

Does the network disintegrate into smaller, disconnected parts when $f > f_c$?

numerical results (N=10000, <k>=4)

random

scale-free



preferential node removal of the most connected nodes

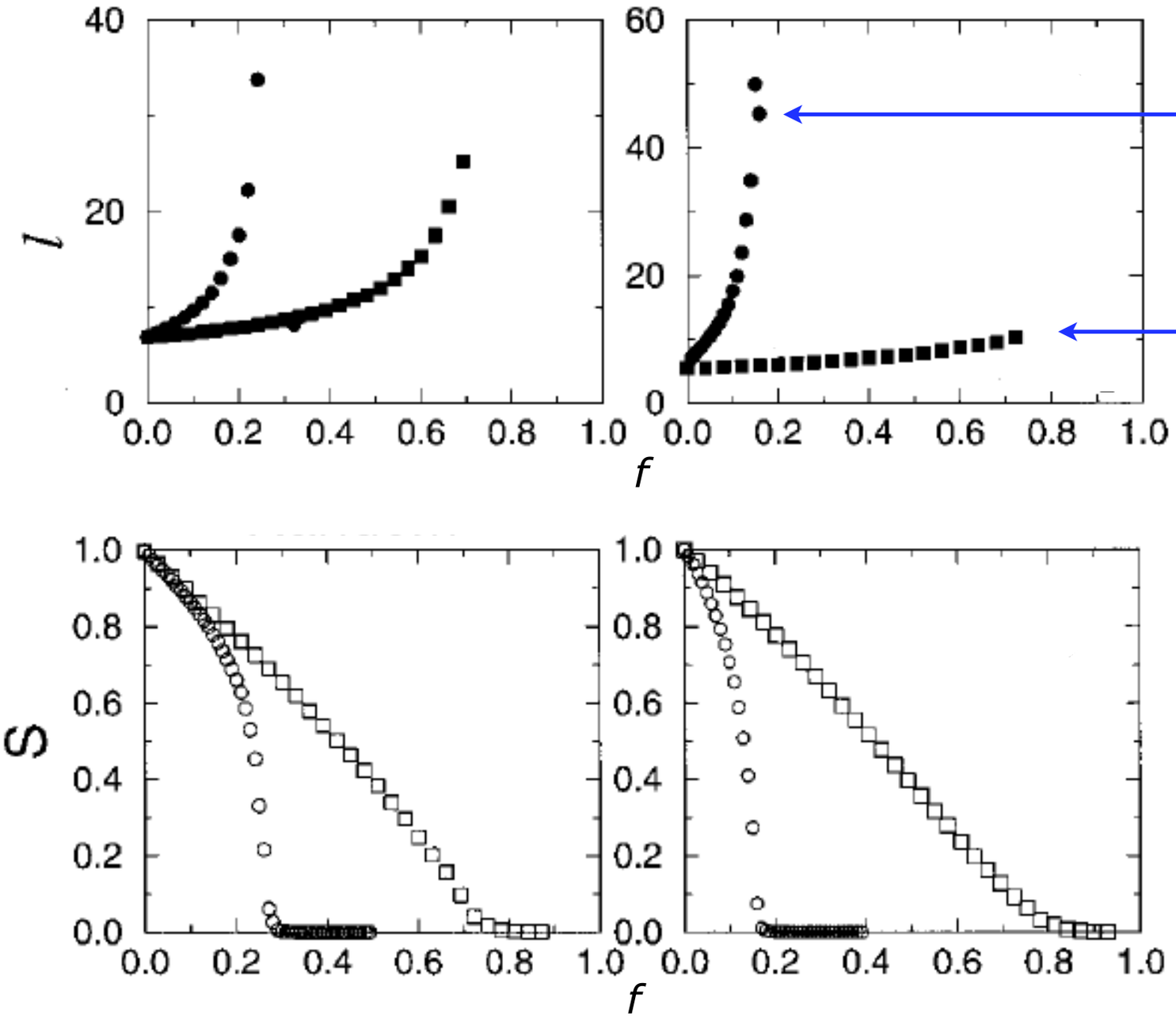
random node removal

Albert, Jeong, Barabasi,
Error and attack tolerance of complex networks.
Nature 406, 378, 2000

numerical results (N=10000, <k>=4)

random

scale-free



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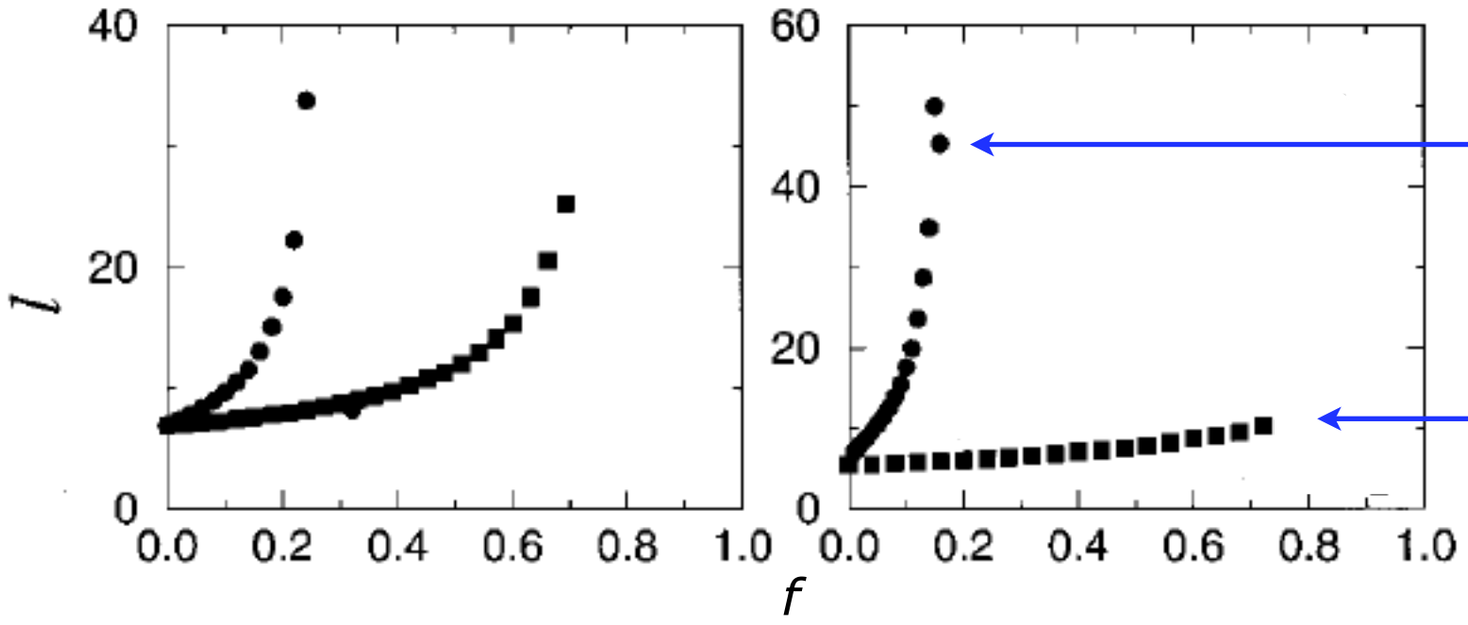

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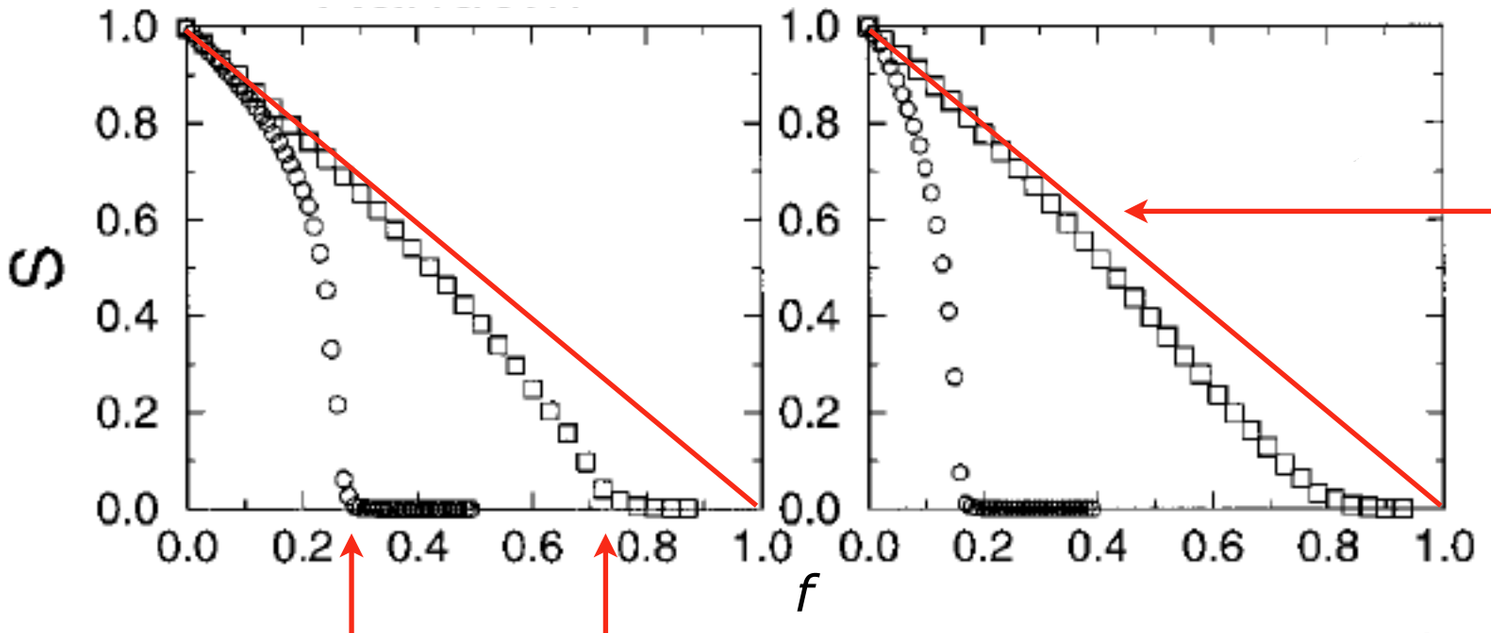

if the removed nodes were missing from the largest cluster

network breaks into tiny isolated clusters

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numerical results (N=10000, <k>=4)

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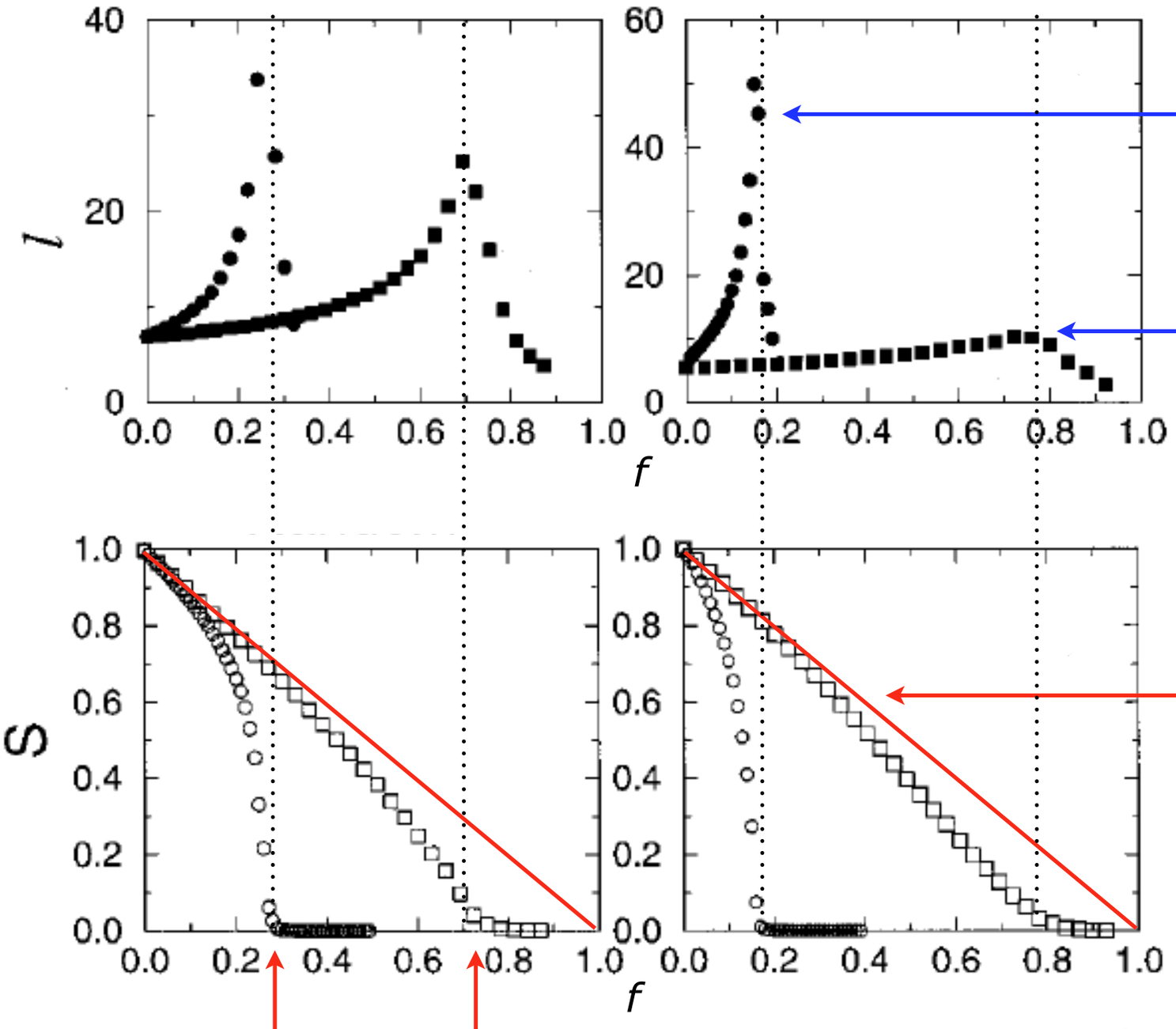

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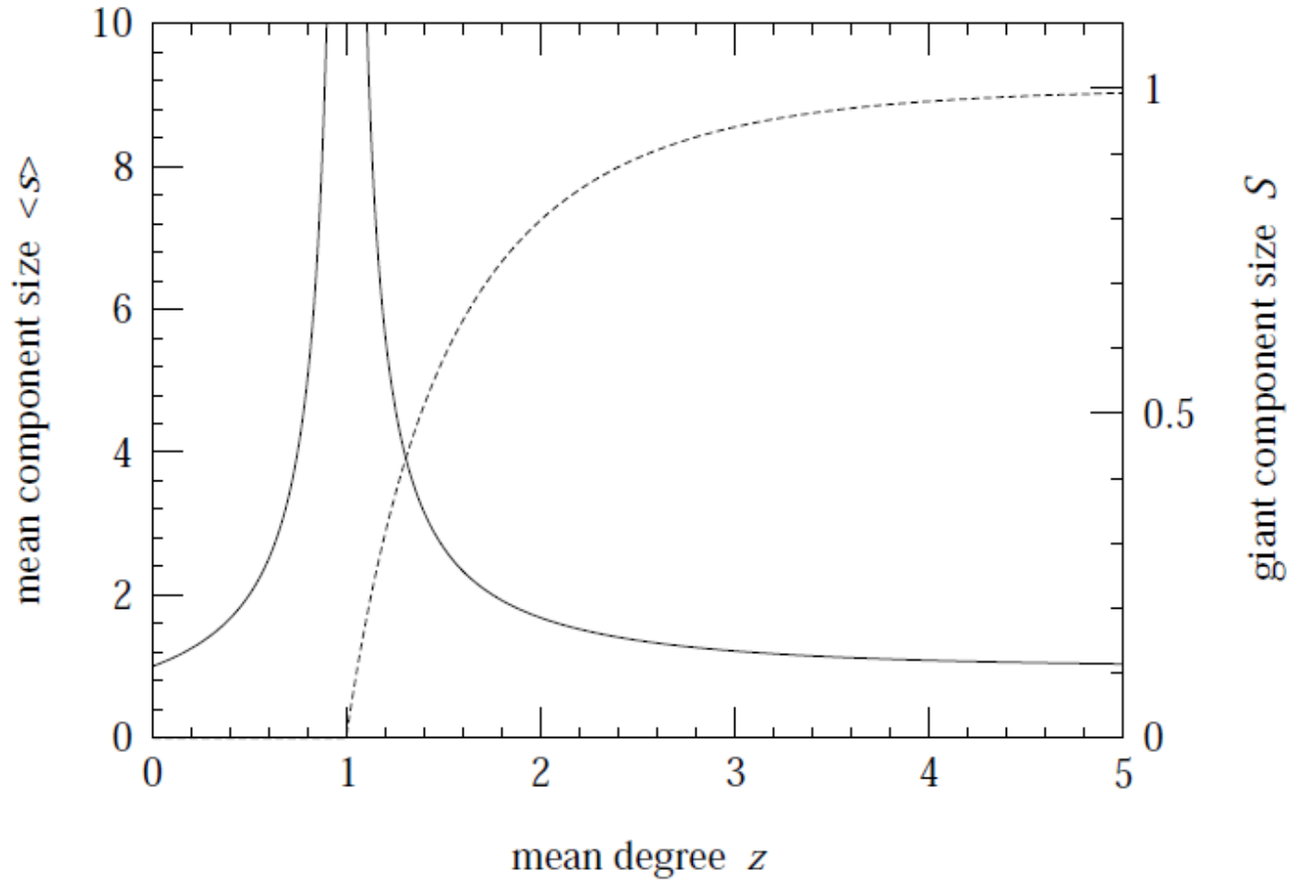
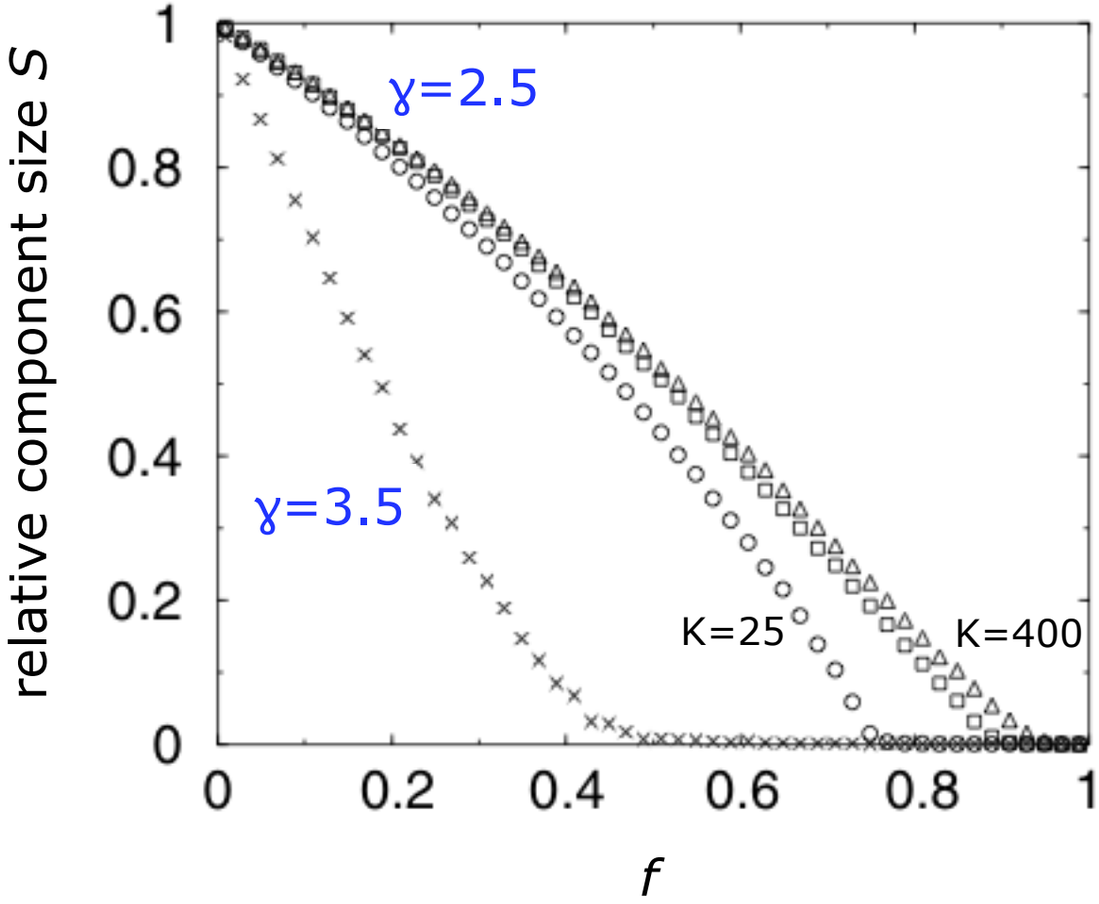
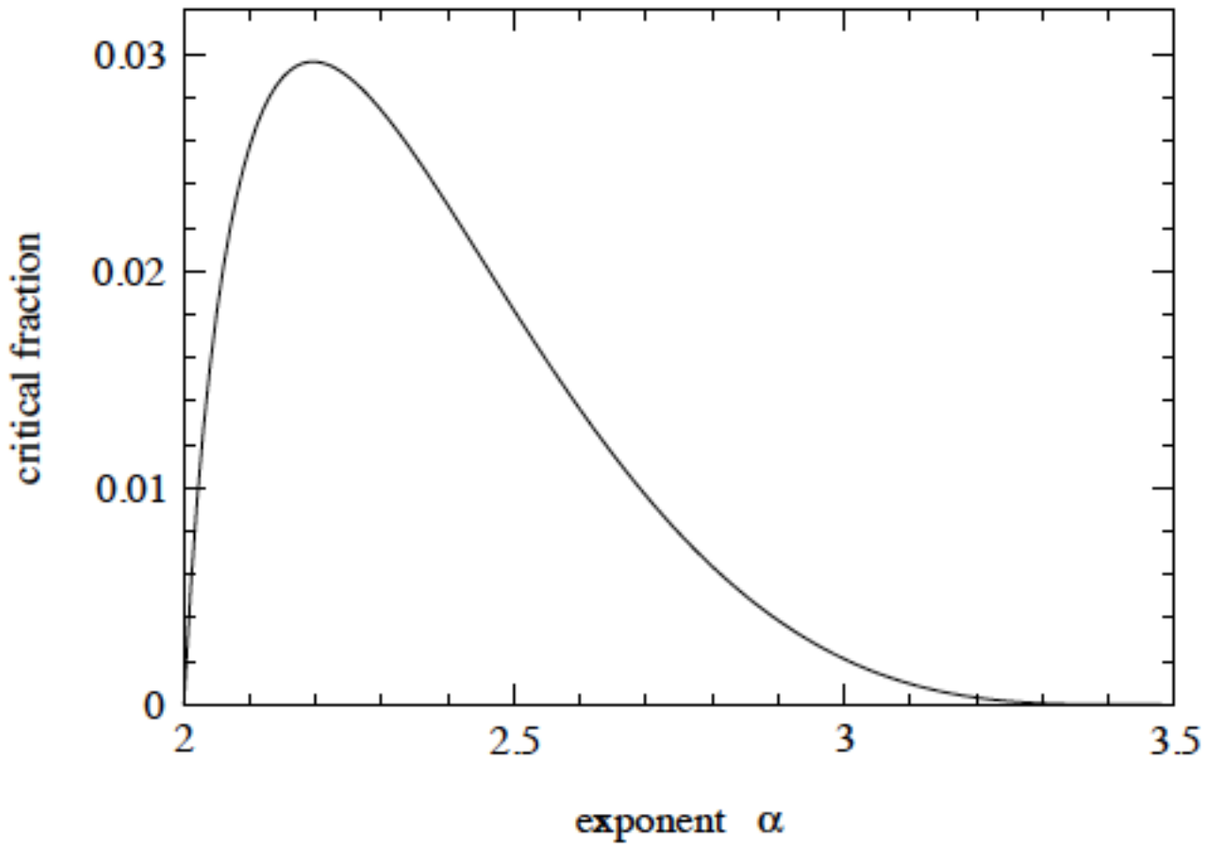
Analytical Results


10
8
6
4
2
0
0
1
2
3
4
5
1
0.5
0
mean component size <s>
giant component size S
mean degree z



when γ≤3 the critical value for giant component formation is zero

-power-law distributions-
a connected cluster of nodes survives even for arbitrary large fractions of random crashed sites



Dorogovtsev, S. N. and Mendes, J. F. F., Comment on "Breakdown of the Internet under intentional attack," Phys. Rev. Lett., 87:219801 (2001)

-power-law distributions-
a connected cluster of nodes is highly susceptible to targeted attacks

	network	type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r
social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208
	company directors	undirected	7 673	55 392	14.44	4.60	–	0.59	0.88	0.276
	math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15	0.34	0.120
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45	0.56	0.363
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088	0.60	0.127
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1			
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0		0.16	
	email address books	directed	16 881	57 029	3.38	5.22	–	0.17	0.13	0.092
	student relationships	undirected	573	477	1.66	16.01	–	0.005	0.001	−0.029
	sexual contacts	undirected	2 810				3.2			
information	WWW nd.edu	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	−0.067
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7			
	citation network	directed	783 339	6 716 198	8.57		3.0/–			
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	–	0.13	0.15	0.157
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7		0.44	
technological	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	−0.189
	power grid	undirected	4 941	6 594	2.67	18.99	–	0.10	0.080	−0.003
	train routes	undirected	587	19 603	66.79	2.16	–		0.69	−0.033
	software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.070	0.082	−0.016
	software classes	directed	1 377	2 213	1.61	1.51	–	0.033	0.012	−0.119
	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	−0.154
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	−0.366

Remarks

- Real-world implication?
- Modeling dynamics on a fixed topology when the time scales describing network topology and the dynamic process differ widely

Next class

- Epidemic processes
  - the SIR model
  - the SIS model